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選擇權之評價：Ornstein-Uhlenbeck 股價變動過程下

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摘要

Black-Scholes(1973)選擇權評價模型假設在一個完美市場下，包括沒有稅賦、沒有交易成本、沒有賣空的限制等。B-S 另假設標的資產服從幾何布朗寧運動(Geometric Brownian Motion)，其特性為股價報酬是連續的，且服從對數常態分配。但事實上，現實環境中，金融市場並非完美市場，包括交易成本、賣空限制、漲跌幅限制等，而股價並非完全遵循幾何布朗寧運動的特性。Goldenberg(1986)探討標的資產服從 Ornstein-Uhlenbeck 過程，而非幾何布朗寧運動的情況。本研究推導出標的資產服從 Ornstein-Uhlenbeck 過程下選擇權的評價公式。經蒙地卡羅模擬發現，日報酬的相關係數越大時，則此時選擇權的價值低於 B-S 越多。本研究也討論標的資產有漲跌幅的限制時，本文所推導出選擇權的評價公式之適用。

關鍵字: 幾何布朗寧運動，漲跌幅限制，蒙地卡羅模擬

Abstract

In this paper we extend Goldenberg (1986) to use this process to describe the stock prices in imperfect markets, and derive the closed form of European call value under risk neutral probability measure. The option pricing model under the assumption of Ornstein-Uhlenbeck position process could express the correlation of underlying asset, and is more consistent with the behavior of observed stock prices. . Monte Carlo simulation is used to verify the formula. We find that high correlation will reduce the value of the call option. We also compare this model to the Black-Scholes model, and find when the time to maturity is longer or the correlation is less, the difference between both models will be less. Besides, the larger moneyness, interest rate, and the less variance would cause the difference of both models be less. We also discuss the option pricing formula when the stock prices are subject to price limit.   The stock prices generated by Ornstein-Uhlenbeck position process would be consistent with these characteristics in markets with price limits. When the range of price limits becomes narrower, the European call value will be less

Keywords: Geometric Brownian Motion, Ornstein-Uhlenbeck process, Price Limits, Monte Carlo Simulation

1. Introduction

Black-Scholes (1973) constructed an option pricing model in the perfect market, where is no tax, transaction cost, limits on borrowing or lending, constraints on short selling, and price limits. Besides, the risk-free rate and variance of the return on the stock are assumed to be constant. They use Geometric Brownian Motion to model the stock price. The returns of stock prices in Brownian Motion are independent and vary with an infinite velocity as the time interval is infinitesimal, because of the not differentiable property of Brownian Motion. While in the real world, the stock prices are not so like that in Geometric Brownian Motion. The observed stock prices are not always vary sharply in a very short time interval, and the returns of stock prices aren't independent. It could be caused by the impacts of frictional elements, such as the existence of tax, transaction cost, and other constraints. Hence, Geometric Brownian Motion couldn't resolve the problem of modeling observed market prices in real markets well. In this paper, we follows Goldenberg (1986) to use an alternative stochastic process, called Ornstein-Uhlenbeck position process, for modeling the stock prices, which could better describe the behavior of stock prices in imperfect markets than Geometric Brownian Motion, and derive the closed from of European call value. After deriving the option formula under Ornstein-Uhlenbeck process, Monte Carlo simulation is used to verify the accuracy of the formula. Furthermore, we also discuss the application of this process on considering price limits. The stock prices generated by Ornstein-Uhlenbeck position process have some properties consistent with the characteristics of the prices with the imposition of price limits.

2. Sample path properties of stock price follows Ornstein-Uhlenbeck Position Process

As we know, the return of the stock price of the Geometric Brownian Motion under the perfect market assumption is independent. Besides, because Brownian Motion is not differentiable, the return under the assumption varies with an infinite velocity as the time interval is infinitesimal. However, in the real world the market is not so prefect, there are some frictional elements, such as tax, transaction cost, and price limits. Empirical literatures showed that stock returns are not independent, and the changes of stock prices are not infinite as a very short term, especially under the constraints of price limits. These facts violate the properties of Geometric Brownian Motion. Goldenberg (1986) stated that there is an alternative better stochastic process, called Ornstein-Uhlenbeck position process, for the sample path of the future prices in the imperfect markets than the conventional assumption of Geometric Brownian Motion. Ornstein-Uhlenbeck position process is the integral of Ornstein-Uhlenbeck process, which is derived in 1930 by Ornstein and Uhlenbeck, and later by S. Bernstein (1934), and Krutkow(1934).

Doob(1942) applied the methods and results of modern probability theory to the analysis of Ornstein-Uhlenbeck position process distribution, its properties, and its derivation. The process was defined by Doob(1942) as the following properties:

Definition: Let $u(t)$ $(-\infty < t < +\infty)$ *be a one-parameter family of chance variables, determining a stochastic process with the following properties.*

1. *The process is temporally homogenous*
2. *The process is a Markov process*
3. *If s, t are arbitrary distinct numbers, u(s), u(t) have a (non-singular) bivariate Gaussian distribution.*

Define m and σ_0^2 *by*

$$m = E\left[u(t)\right] \qquad \sigma^2 = E\left\{\left[u(t)-m\right]^2\right\} \qquad (1)$$

Then the given process is one of the following two types.

(A) If $t_1 < t_2 < t_3 < ...... < t_n$*, the random variables* $v_{t1}, v_{t2}, ...v_{tn}$ *are mutually independent Gaussian variables.*

(B) (O. U. process) There exist a friction coefficient β*>0. If* $t_1 < t_2 < t_3 < ...... < t_n$*, the random variables* $v_{t1}, v_{t2}, ...v_{tn}$ *have an n-variate Gaussian distribution with common mean m , variance* σ_0^2 *and covariance as:*

$$E[(u_t - m)(u_s - m)] = \sigma_0^2 e^{-\beta|t-s|} \qquad (2)$$

We could find that Ornstein-Uhlenbeck position process could express the correlation between stock prices, which is investigated in observed stock prices. The correlation of two log stock price increments decreases as the distance of these two increments increases. When the correlation is zero, the returns of stock prices are independent, and would approach that generated by Brownian motion. Besides, the volatility of the log stock price inherits the property of the Ornstein-Uhlenbeck position process. In the long term, the volatility of the log stock price generated by Ornstein-Uhlenbeck position process is like that generated by Brownian motion, while in short term, the volatility of the stock return generated by Ornstein-Uhlenbeck position process is less than that generated by Brownian motion.

3. Option pricing when underlying asset follows Ornstein-Uhlenbeck position process

Goldenberg(1986) stated that we could apply Ornstein-Uhlenbeck position process to describe the return of the future price considering the existence of frictional elements. He also said we could consider using this stochastic process on option pricing model. However, he didn't go further to provide the closed form of the option value. Here, we derive the closed form of European call option value under the risk neutral probability measure when the underlying asset follows Ornstein-Uhlenbeck position process instead of geometric Brownian motion. As the definition, the final payoff of an European call option is :

$$C_T = Max(0, S_T - K) \tag{3}$$

so the value of the call at initial time is:

$$C = e^{-rt} E^Q[(S_T - K) I_{\{S_T > K\}}] \tag{4}$$

where: $I_A = \begin{cases} 0 & \text{if } \omega \in \text{A} \\ 1 & \text{if } \omega \notin \text{A} \end{cases}$

Hence, we can get a closed form of European call option value as following:

$$C = S_0 N\left(\frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) - Ke^{-rT} N\left(\frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2 v^2}{2T})T}{\sigma v}\right) \tag{5}$$

where:

S_0 : stock price at the initial time
K : exercise price
r : interest rate
T: time to maturity

$$v^2 = \frac{2}{\beta^2}(e^{-\beta T} - 1 + \beta T)$$

β is correlation parameter
σ^2 is the parameter

We verify the closed form by simulate the value of European call option with Monte Carlo simulation. In table (1) , we let K=50, r=2%, T=10/250, correlation of daily returns=20%, variance=50%, and compare both values of closed form and simulation. We find that both values in different ways are almost the same. It shows that our resolution is correct. Besides, the standard deviation of numerical solution is very small as we simulate 10000 times.

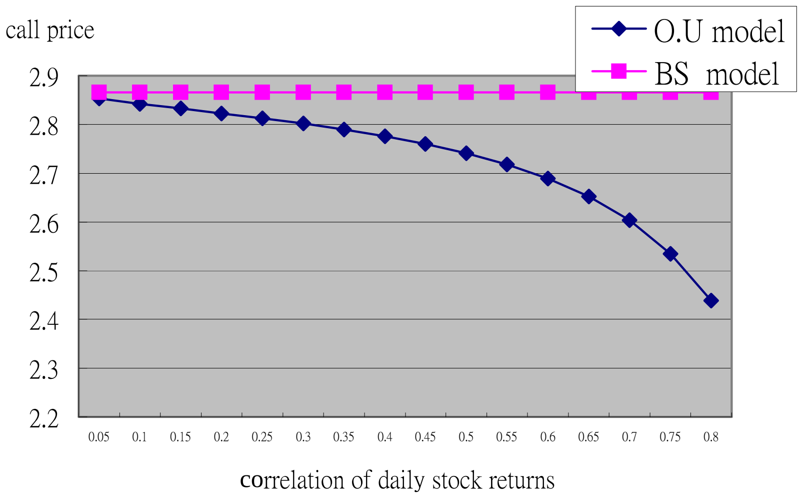
Table 1   Numerical solution and closed form solution of European call value

S	40	45	50	55	60	65	70
closed form	0.1455	0.8635	2.7943	6.0988	10.3755	15.1299	20.0609
Monte Carlo	0.1523	0.8768	2.8396	6.1329	10.4270	15.1532	20.0630
STD	0.0100	0.0281	0.0372	0.0907	0.0718	0.0876	0.0753

As we mentioned in previous section, OU position process will approximate GBM process when the correlation is zero or the time to maturity is long enough, so we first investigate how the difference between both call values changes with different levels of correlation and time to maturity. We assume the initial stock price and exercise price are both 50, the annual interest rate is 5%, and the annual variance for comparing with GBM is 50%.

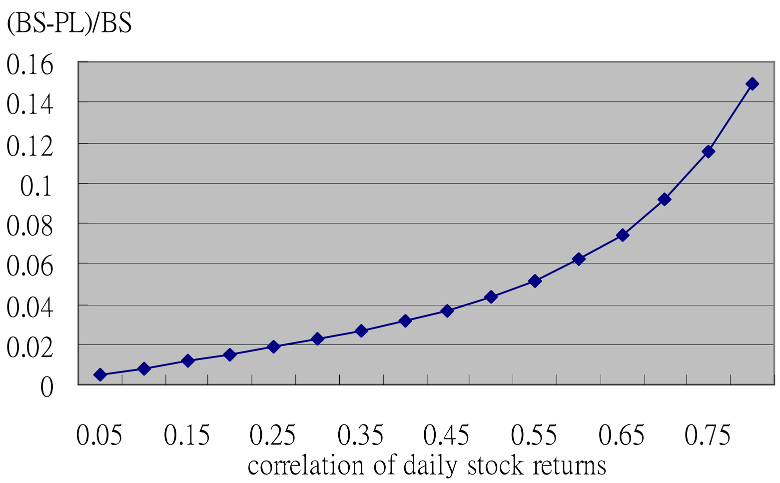
In Figure1 and Figure 2, we let T=10/250, and the correlation is the relation among daily stock returns, and from 0.05 to 0.8. In table 4.1, we investigate the relation between the daily correlation and call value. Under the condition of invariant variance 50%, the larger correlation will cause the less value of the call option. This is because in the given short period (T=10/250), the higher correlation will reduce the volatility of the stock, and cause the less call value.

Fig.1 The call prices of BS and OU position process model with different levels of correlation



In Fig 2, we show the relation between daily correlation and the difference ratio, which is divide the difference between BS call value and OU position process call value by the BS call value in order to see more clearly the changing of the difference between two model in percentage. We could find that the difference ratio of the call option values between two models will be larger as the correlation increasing.

Fig 2 The difference ration between BS model and OU position process model with different levels of correlation



4. The application to option pricing considering price limits

The behavior of stock price under price limits is not like that in the prefect market. The stock price hits the bounds when there is imbalanced demand or supply of the stocks. As soon as the price hits the bounds, the higher or lower price trades will be suspended to the next day. The imposition of price limits will stop the stock price form unbounded variation each day. Here we conclude three important and interesting characteristics of the stock price under price limits.

Characteristic 1:

Under the regulation of price limits, there is the constraint of stock prices such that the stock prices would unlikely vary with the infinite velocity as the time interval is infinitesimal, while which is the property of the stock prices of the assumption of Geometric Brownian Motion.

Characteristic 2:

The regulation of price limits has the impact of "delay price discovery hypothesis", which induces the positive correlation among the returns of stock prices.

Characteristic 3:

In the short term, the volatility of stock returns with the constraints of price limits would be less than that without price limits. While in the long term, both the volatilities with and without price limits would be consistent

We investigate the volatilities of three stocks listed in Taiwan Stock Exchange Corporation and their ADRs (American Depositary Receipt). The stocks in Taiwan Stock Exchange Corporation are constrained by price limits, while the ADRs representing the same share of the companies are traded without limits. We define the difference ratio as dividing the difference between volatilities of stock and ADR by the volatility of ADR. Ignoring the little effect of exchange rates, we can find that there is a trend that the difference ratio is declining as the time interval increases. This results support our inference. However, the difference between both volatilities of the two markets is still significant even if the time interval is large enough. It could be caused by the different properties of both markets. Even though, the trend of declining difference between stock prices with and without limits is still investigated.

Table 2 The volatilities of the return of the stock price in

Taiwan Security Market and Nasdaq in different measure period.

	Volatility	Daily	Weekly	Monthly
TSM	stock in Taiwan	0.12%	0.57%	2.78%
	ADR	0.21%	0.84%	3.91%
	difference ratio	43.10%	31.54%	28.99%
	Volatility	Daily	Weekly	Monthly
UMC	stock in Taiwan	0.12%	0.62%	2.53%
	ADR	0.25%	0.93%	4.04%
	difference ratio	53.08%	33.26%	37.31%
	Volatility	Daily	Weekly	Monthly
ASX	stock in Taiwan	0.14%	0.79%	2.92%
	ADR	0.23%	0.99%	2.76%
	difference ratio	38.69%	20.60%	-5.77%

From the above discussion, the Geometric Brownian Motion seems not a proper process for the stock price under price limits. The Geometric Brownian Motion could vary with an infinite velocity in an infinitesimal time interval, and is independent between each particle. These two properties are obviously different with the behavior of stock price we investigated under the institution of price limits. In addition, in the short term, the volatility of the returns of the stock prices with price limits is less than that without limits. If we assume the stock price without price limits follows

In the stock market with price limits, there is the “delay price discovery effect”, which means the stock prices of each day could be correlative, and as the time distance is longer, the delaying effect is lower. The stock price with Ornstein-Uhlenbeck position process can express the correlation. If the stock price frequently hits the limits, the delaying price discovery effect would be larger, so that the correlation coefficient will also be higher. On the other hand, if the stock price rarely hits the limits, it means the stock price could always reflect all the information immediately, and the correlation coefficient will be very small. When the correlation coefficient approaches to zero, the stock price generated by Ornstein-Uhlenbeck position process could be the same as the stock price generated by Brownian motion process.

In the modified price-limits option pricing model, there are six parameters to be estimated. Except the parameter $\overline{V}$, the other five parameters could be found directly form observed data. The parameter $\overline{V}$ stands for the instantaneous variance of the return of the stock price without price limits, while it is difficult to be estimated from the data under price limits. Before implementing the model, we need to define how to estimate the value of this parameter. Although we know that $\overline{V}T$ is also the variance of the return of the stock price under price limits in the long-term, adopting long-term data to find the parameter would not reflect the recent situation of the stock. As alternative way, we could use the technique of modeling the stock price under price limits in the empirical literatures to find the intrinsic instantaneous variance of the return of the stock without price limits through the observed stock prices under price limits. Ghiang & Wei (1995) suggested the method GMM to estimate the parameter $\overline{V}$.

5. Conclusion

In this paper we extend Goldenberg (1986) to use this process to describe the stock prices in imperfect markets, and derive the closed form of European call value under risk neutral probability measure. The option pricing model under the assumption of Ornstein-Uhlenbeck position process could express the correlation of underlying asset, and is more consistent with the behavior of observed stock prices. We find when other things being equal, the higher correlation will reduce the value of the call. We also compare this model to the Black Scholes model, and find when the time to maturity is longer or the correlation is less, the difference between both models will be less. Besides, the larger moneyness, interest rate, and the less variance would cause the difference of both models be less.

We apply this option pricing model in markets with price limits. First we investigate the behavior of stock prices under price limits. Under this imposition, stock prices couldn’t vary with an infinite velocity as an infinitesimal time interval. They would comply with the constraints derived by Goldenberg(1986). From many empirical literatures, the regulation of price limits has the effect of "delay price discovery", which means there exists the correlation among stock returns. Besides, in the short term, the volatility of stock return with price limits would be less than that without price limits. While in the long term, for the sake of having enough time to reflect relative information, the volatility with price limits would be like that without limits. The stock prices generated by Ornstein-Uhlenbeck position process would be consistent with these characteristics in markets with price limits, whereas those generated by Geometric Brownian Motion are not. We use Ornstein-Uhlenbeck position process into the option pricing model in markets with limits, and find when the range of price limits become narrower, the European call value will be less.

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