2. Literature Review

This section reviews related literatures of this paper. First of all, we introduce the concept of Fund of Funds (FoF). Then we review the literatures related to the Markowitz mean-variance model. Furthermore, we review the literatures related to the Black Litterman Model.

2.1 Introduction of Fund of Funds

Now Fund of Funds (FoF) sector is one of the largest institutional categories in the private equity industry, with 120 firms managing funds of around $170 billion. It becomes a driving force in the globalization of the asset class. The number of FoF managers has grown more than ten-fold over the past two decades. The pace has now become unsustainable, particularly in the context of fund-raising dramatic downturn in the private equity industry as a whole.

2.1.1 Definition

A fund of funds (FoF) is an investment fund that uses an investment strategy of holding a portfolio of other investment funds rather than investing directly in shares, bonds or other securities. This type of investing is often referred to as multi-manager investment.

There are different types of fund of funds, each investing in a different type of collective investment scheme (typically one type per FoF), for example, mutual fund FoF, hedge fund FoF, private equity FoF or investment trust FoF.

2.1.2 The advantages of Fund of Funds

Investing in a collective investment scheme will increase diversity compared to a small investor directly holding a range of securities. Investing in a fund of funds arrangement will achieve even greater diversification.

An investment manager may actively manage your investment with a view to selecting the best securities. A FoF manager will try to select the best performing funds to invest in based upon the managers past performance and other factors. If the FoF manager is skillful, this additional level of selection can provide greater stability and take on some of the risk relating to the decisions of a single manager. As in all other areas of investment, there are no guarantees. Some advantages of fund of funds are as following:

1. Diversification

Going by the conception of "do not keep all eggs in one basket", a FoF manager would invest in various mutual fund sectors which give investors benefits of diversification. Experience of the past few years shows that the top performers are usually different from year to year. However, there are certain funds, which have been consistent performers. Therefore, a FoF that invests in 5 of the top ten funds today, is expected to yield better returns than investing in the top performing fund of the day. Secondly, investors get a chance to diversify across various fund managers and investing styles.

2. Convenience

As a prudent investor, one would like to diversify his investment across both equity and debt funds. By choosing a suitable FoF, investors get a chance to invest across different classes of funds within just one investment. Thus, it becomes very convenient for investing and monitoring.

3. Rebalancing

Suppose some investor has to invest and his debt-equity allocation is 30:70. After one year the debt value of 30% in debt has grown to 32.40 (8% of return rate), and the equity value of 70% has grown to 94.50% (35% of return rate). The debt-equity allocation has now become 25.5:74.5. Thus the portfolio has become riskier than the original profile of 30:70. Therefore, he needs to sell position of 5.5% in the equity and put in debt to bring back the debt-equity ratio to 30:70.

This rebalancing will involve tax, when the investor rebalances its portfolio of stock. For instance, it costs investors tax of 0.3% per sell transaction in Taiwan. When a FoF manager rebalances its portfolio, there is no tax, since there is no tax on transaction of mutual fund in many countries such as US and Taiwan. This is a big benefit.

2.1.3 The disadvantages of Fund of Funds

Management fees for funds of funds are typically higher than those on traditional investment funds because they include part of the management fees charged by the underlying funds.

Since a fund of funds buys many different funds which themselves invest in many different securities, it is possible for the fund of funds to own the same stock through several different funds and it can be difficult to keep track of the overall holdings.

Funds of funds are often used when investing in hedge funds, as they typically have a high minimum investment level compared to traditional investment funds which precludes many from investing directly. In addition, hedge fund investing is more complicated and higher risk than traditional collective investments; this lack of accessibility favors a FoF with a professional manager and built in spread of risk.

2.2 Markowitz Mean-Variance Portfolio Selection Model

2.2.1 The Model

An investor aims to make a positive return on his investments. Therefore, the expected return of the portfolio of assets would seem to be to a logical objective function. To maximize the expected return of the portfolio, one has to simply invest in the single asset with the highest expected return. Addition of other assets with a lower expected return to the portfolio would lower the expected return of the portfolio. However, investing in a single asset goes against the notion of diversification and the result will be that the performance of the portfolio is based on the performance of the one asset. If the asset performs well, so does the investment. It also means the converse, if the asset performs badly, so does the portfolio. The performance of the investments solely depends on the one asset. This makes the return very erratic and therefore very risky. It would be better if the return could be more steady.

Therefore, it is also important to investigate the risk involved in the portfolio. One should not only maximize the expected return but also take into account the risk of the portfolio; one should balance the risk and the expected return of the portfolio. Markowitz (1952) proposed the mean-variance methodology for the portfolio problem which served as a basis for the development of modern financial theory during the past decades. The core of the Markowitz mean-variance model is to take the expected return of a portfolio as the investment return and the variance of the expected returns of a portfolio as the investment risk. According to the Markowitz mean-variance model, a given specific rate of return can be used to derive the minimum investment risk by minimizing the variance of a portfolio; and, a given risk level which the investor can tolerate can be used to derive the maximum returns by maximizing the expected returns of a portfolio. The assumptions underlying the Markowitz mean-variance Model are as follows:

1. Perfect and competitive markets: no tax, no transaction cost and the securities and assets are perfectly divisible.
2. All investors are risk averse; all investors have the same beliefs
3. Security returns are jointly normal distribution
4. Dominant principle: an investor would prefer more return to less and would prefer less risk to more

The main input data of the Markowitz mean-variance model are expected returns and variance-covariance matrix of expected returns of these assets. Simplifying the number and types of the input data has been one of the main research topics in this field for the last four decades. To derive the set of attainable portfolios (derived from the expected return and the covariance matrix estimated by the investor) that an investor can reach, we need to solve the following problem:

$$\begin{aligned} &\min_{w} w^T \sum w \\ &w^T \bar{r} = \bar{r}_p \end{aligned} \tag{2.1}$$

or

$$\begin{aligned} &\max_{w} w^T \bar{r} \\ &w^T \sum w = \sigma_p^2 \end{aligned} \tag{2.2}$$

w : the column vector of portfolio weights

σ_p^2 : the variance of the portfolio

$\bar{r}_p$: the expected return of the portfolio

$\bar{r}$:the column vector of expected returns

$\sum$:the covariance matrix.

The solution can be found via the method of Lagrange. Often the following equation is solved instead of the above ones:

$$\max_{w} L(w,\delta) = w^T u - \frac{\delta}{2} w^T \sum w \qquad (2.3)$$

$L(w,\delta)$: the Lagrangian equation.

w^* : the Markowitz optimal portfolio

u :the column vector of expected (excess) returns

δ :the risk aversion parameter stated by the investors.

The factor $\frac{1}{2}$ is only introduced for convenience, it does not alter the problem. This is actually the same as Eq.(2.1) and Eq.(2.2). The solution w^* has to fulfill simultaneously $\frac{\partial L}{\partial w} = 0$ and $\frac{\partial L}{\partial \delta} = 0$, then the following solution is achieved:

$$w^* = (\delta \sum)^{-1} u \qquad (2.4)$$

2.2.2 Problems in the Use of Markowitz Model

Although the Markowitz mean-variance model might seem attractive and useful, there are a lot of problems when fund managers use the model in practice. Michaud (1989) thoroughly discusses the practical problems of using the model. He claims that the model often leads to irrelevant optimal portfolios and that some studies have shown that even equal weighting can be superior to Markowitz optimal portfolios. The main problems are that the optimization procedure often results in concentrated portfolios, requirement of much input data, lack of robustness, and corner solutions.

1. Concentrated portfolios

Diversification is one of the important points derived from the mean-variance model. However, the model can result in portfolios with large long and short positions in only a few assets, which is the opposite of the diversification itself. Michaud (1989) proposed the concentrated portfolios are very counterintuitive, which is one of the reasons for the lack of popularity of using unconstrained mean-variance optimizers in making investment decisions.

2. The model is not robust

The main deficiency is that the model is not robust. This means that a small change in the values of the input parameters can cause a large change in the composition of the portfolio. The mean-variance model assumes that the input data is correct, without any estimation error. The model does not address this uncertainty and sets out to optimize the parameters as if they were certain. Michaud (1989) describes mean-variance optimizers even as "estimation-error maximizers". Best and Grauer (1991) have analyzed the behavior of the mean-variance optimizer under changes in the asset mean. They show that a small increase in an asset mean can cause a very different portfolio composition. Half of the assets can be eliminated from the original portfolio, by a small increase in the mean of one asset. They state that for a mean-variance problem with a budget constraint, the rate of change for the vector of weights depends on the change in the mean. More specifically, it depends on the risk-aversion parameter, the inverted variance-covariance matrix and how to specify the change in the asset mean.

Michaud (1989) considers the covariance matrix as the main culprit for the non-robust behavior. The covariance matrix is often estimated from data, but this estimation procedure can produce matrices that are nearly singular or singular. This causes problems when the matrix is inverted during the optimization procedure.

3. Corner solution

One of the most striking empirical problems in using the Markowitz model, is that when running the optimizer without constraints, the model almost always recommends portfolios with large negative weights in several assets (Black & Litterman, 1992). Fund or portfolio managers using the model are often not permitted to take short positions. Because of this, a shorting constraint is often added to the optimization process. What happens then is that when optimizing a portfolio with constraints, the model gives a solution with zero weights in many of the assets and therefore takes large positions in only a few of the assets and unreasonable large weights in some assets. Many investors find portfolios of this kind unreasonable and although it seems, as though many investors are appealed to the idea of mean-variance optimization, these problems appear to be among the main reasons for not using it. In a world in which investors are quite sure about the input to an optimization model, the output of the model would not seem so unreasonable. In reality, however, every approximation about future return and risk is quite uncertain and the chance that it is "absolutely correct" is low. Since the estimation of future risk and return is uncertain, it seems reasonable that investors wish to invest in portfolios which are not prospective disasters if the estimations prove incorrect. Michaud (1989) claims that better input estimates could help bridge problems of the unintuitiveness of Markowitz portfolios. Fisher and Statman(2000) maintain that although good estimates are better then bad, better estimates will not bridge the gap between meanvariance optimized portfolios and "intuitive" portfolios, in which investors are willing to invest, since estimation errors can never be eliminated. It is not possible to predict future expected returns, variances and covariances with 100 % confidence.

In a nutshell, the idea of mean-variance optimization has an intuitive appeal and is very useful for educational purposes. When the optimization procedure is used for practical purposes, the resulting portfolios are counterintuitive and the optimization procedure should be constrained. Furthermore, the model is non-robust and the assumption of normally distributed expected returns is not always a good assumption. Mean-variance analysis has been a good starting point for the development of portfolio selection, but it could be improved on.

2.3 The Black-Litterman Model

This section provides a quick literature review of the Black-Litterman model. Black and Litterman initiated the first paper about Black-Litterman model in 1992, which provides a good discussion of the model along with the main assumptions. The authors present several results and most of the input data required to generate the results. However, they do not document all of their assumptions about the model. As a result, it is not easy to reproduce their results. They provide some of the key equations required to implement the Black-Litterman model, but they do not provide any equations for the posterior variance.

He and Litterman (1999) provide a clearer illustration of the Black-Litterman model. There are still a few fuzzy details in their paper until Satchell and Scowcroft (2000) use the Bayesian rule to derive the Black-Litterman model. Satchell and Scowcroft have presented several examples of Bayesian asset allocation portfolio construction models and showed how they combine judgmental and quantitative views.

Idzorek(2004) recreates the mechanics of the Black-Litterman model based on the source data of He and Litterman, and their assumptions. Idzorek details the process of developing the inputs for the Black-Litterman model, which enables investors to combine their unique views with the implied equilibrium return vector to form a new combined return vector. The new combined return vector leads to intuitive, well-diversified portfolios. The two parameters of the Black-Litterman model that control the relative importance placed on the equilibrium returns versus the view returns, the scalar (τ, the weight-on-views) and the uncertainty in the views (Ω), are very difficult to specify. The Black-Litterman formula with 100% certainty in the views enables one to determine the implied confidence in a view. Using this implied confidence framework, a new method for controlling the tilts and the final portfolio weights caused by the views is introduced. The method asserts that the magnitude of the tilts should be controlled by the user-specified confidence level based on an intuitive 0% to 100% confidence level. Overall, the Black-Litterman model overcomes the most-often cited weaknesses of mean-variance optimization (unintuitive, highly concentrated portfolios, input-sensitivity, and estimation error-maximization), helping users to realize the benefits of the Markowitz paradigm. Likewise, the proposed new method for incorporating user-specified confidence levels should increase the intuitiveness and the usability of the Black-Litterman model.

Liang (2002) studies the out-of-sample performance of the Black-Litterman model on international asset allocation. As to the portion of the invertors' views, with short-run momentum, this research formulates two kinds of investors' views, which are respectively the cumulative return of the previous periods, "Portfolio I" and the average excess return "Portfolio II". G7 is the investment pool of this research of which the data spans from January in 1991 to December in 2000. According to the previous period as well as the same holding period, the asset allocation in the portfolio comes in four periods of 1 month, 3 months, 6 months and 1 year, respectively. The empirical results obtained are as follows:

1. As a whole, the performance of the model, including the cumulative return and Sharpe ratio, is better than that of G7 index and global minimum variance portfolio.
2. As far as the holding period of the portfolio is concerned, the performance of 3 months through 6 months is the best.
3. With regard to the setup of the level of confidence, Sharpe ratio is better at more conservative level of confidence (10~50%).
4. In terms of the comparison between "Portfolio I" and "Portfolio II", mostly the latter is better than the former.

Mankert(2006) analyzes the Black-Litterman model using both mathematical and behavioral finance approach. Mankert makes use of sampling theoretical approach to generate a new interpretation of the model and gives an interpretable formula for the mystical parameter τ, the weight-on-views. Secondly, she draws implications from research results within behavioral finance. One of the most interesting features of the

Black-Litterman model is that the benchmark portfolio, against which the performance of the portfolio manager is evaluated, functions as the point of reference. According to behavioral finance, the actual utility function of the investor is reference-based and investors estimate losses and gains in relation to this benchmark. Implications drawn from research results within behavioral finance explain why the portfolio output given by the Black-Litterman model appears more intuitive to fund managers than portfolios generated by the Markowitz model. Another feature of the Black-Litterman model she proposes is that the user assigns levels of confidence to each asset view in the form of confidence intervals.