4. Simulation Results

4.1 Data

We use the fund of funds (FoF) as our research subject. Usually, there are only 10~ 20 mutual funds held by a FoF portfolio where the number of securities of mutual fund may be more than 100. We choose 11 Fidelity funds as our holding targets, including America Fund, American Growth Fund, European Fund, European Small Companies Fund, Germany Fund, Japan Fund, Japan Smaller Companies Fund, Asean Fund, Latin America Fund, Australia Fund, and Emerging Markets Fund. Table 4-1 shows the market capitalization, size category and value/growth category, which are classified by Morningstar[1].

Our sample consists of 11 Fidelity funds with the period from January 1 2003 to December 31, 2007. We collect daily net value of all funds from Fidelity's website[2]. We chose these funds with the consideration of region, size, growth and value effect. We try to combine all kinds of types in our portfolio. At the same time, all of our target funds are calculated as US dollars. As a result, we do not have to worry about the foreign exchange problem in our simulation.

Each fund has its own investment objective and strategies for reaching that objective. Each fund's portfolio manager invests each fund's asset in a way which the manager believes that it will help the mutual fund achieve its objective. Table 4-2 shows our data and its statistical summary.

[1] Please refer to Morningstar website:http://www.hk.morningstar.com/

[2] Please refer to Fidelity Investment website: https://www.fidelity.com/

The difference of market capital between the highest, the Emerging market fund, and the lowest, the American Growth Fund, is around 11.27%. From the market capital weights, it shows that the investment capital is moving toward the emerging markets. Historical yearly return of Latin American Fund, Emerging Market Fund, Asean Fund, and Australian Fund are higher than average, 31.3%, 29.36%, 22.27%, and 15.7% respectively. It is intuitive since emerging markets are on the rise for the past few years. On the other hand, the historical return of funds invested in US, Europe, and Japan are no more than 10%. The historical returns with big difference will produce extreme portfolio weights according to Michaud (1989). The implied equilibrium return of the funds are shown on the sixth column of Table 4-2, which are more flat with the range of 2.49%.

4.2 Specifying our views

As previously discussed, the Black-Litterman approach combines equilibrium returns with an explicit set of views. Expected returns can, in some sense, be interpreted as a complicated weighted average of the neutral (or equilibrium) returns and an investor's views. In this section, we will determine how much weight to put on the equilibrium returns and how to set the relative weights for each specific view.

Many scholars have found lots of evidence for the size effect in U.S. stock market, for example, Banz (1981), Reinganum (1981), Basu (1983), etc. The size effect indicates that the return of small size stocks is much higher than big ones, in that a negative relation between the company size and the stock return.

According to the empirical results, we propose our views:

1. European Small Companies Fund will outperform European Fund by 2%.

2. Japan Smaller Companies Fund will outperform Japan Fund by 1.5%.

Besides the size effect of target asset, Some believe that a large asset base erodes fund performance because of trading costs associated with liquidity or price impact. For example, Perold and Solomon (1991), and Lowenstein (1997) found that a small fund can easily put all of its money in its best ideas, and a lack of liquidity forces a large fund to have to invest in its not-so-good ideas and take larger positions per stock than is optimal, thereby eroding performance. Hence, we adopted the third view:

3. American Growth Fund will outperform American Fund by 1%.

Finally, we consider the growth and value effect and propose the fourth view:

4. American Fund, American Growth Fund, European Small Companies Fund, Japan Small Companies Fund, Latin American Fund, Australia Fund, and Emerging Market Fund will outperform European Fund and Japan Fund by 75 basis points.

According to the views above, our inputs are as followed:

$$P = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.16 & 0.07 & -0.44 & 0.15 & 0 & -0.56 & 0.12 & 0 & 0.13 & 0.11 & 0.26 \end{bmatrix} \quad (4.1)$$

$$Q = \begin{bmatrix} 0.02 \\ 0.015 \\ 0.01 \\ 0.0075 \end{bmatrix} \quad (4.2)$$

$$\Omega = \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & w_4 \end{bmatrix} \tag{4.3}$$

4.3 Intuitive results with the Black-Litterman Approach

We use Matlab to calculate the variance of the view and the Black-Litterman formula as Eq.(3.7). With all inputs available, it is easy to show the final result in Table 4-3.

The new weight vector in column 5 of Table 4-3 is based on the new combined return vector. One of the strongest features of the Black-Litterman model is illustrated in the final column of Table 4-3. Since we don not give any view on German Fund, its weight is the only one with no change. Besides, weights of the sets for which views were expressed change from their original market capitalization weights and the directions of the changes are intuitive. We plot the new weights and market capitalization weights on figure 4-1, from which it is clear to see the intuitive results. For example, view 3 presents the fund size effect and we propose American Growth Fund will outperform American Fund by 1%. The weight change of American Growth Fund and American Fund are 4.78% and -3.55%, respectively.

So far, we have showed the intuitiveness of the Black-Litterman approach to fund of funds allocation. The fund manager is now no longer obliged to specify views on all assets, but can specify views only if she holds one. The manner of specifying view is more natural because it is possible to represent the views in a relative manner. Also it is possible to express how certain one is about the view. These additions make the model more versatile.

Besides, the method combines the best of both the quantitative as well as the qualitative world. Two sources of information, which were previously considered to be disconnected, are now combined in the Black-Litterman formula. We can regard it from a global view. The new portfolio can be viewed as the sum of two portfolios, where Portfolio 1 is the original market capitalization-weighted portfolio, and Portfolio 2 is a series of long and short positions based on the views.

The Black-Litterman approach, however, does not solve all problems of mean-variance optimization. The optimizer is still the driving force behind the model. Although it has new estimates of the expected return and variance and should therefore be better behaved, inherently nothing has been changed in the optimization procedure.

4.4 Comparison of Markowitz Method and Black-Litterman Approach

We demonstrate the intuitive results of the Black-Litterman approach in section 4.3, and we carry out a simulation in this section. We want to see the difference of fund of fund allocation with traditional Markowitz' method[3] and Black-Litterman approach. In the first part, we compare the allocations with two different approaches. The second part demonstrates the comparison of rebalancing costs with the two approaches.

[3] Markowitz' method has been discussed and improved. The Black-Litterman approach can be regarded as an improved Markowitz' method as well. In this paper Markowitz' method represents the traditional optimization based on the historical return vector.

4.4.1 Recommended Portfolio Weights with Different Approaches

Following our views on the holding funds in section 4.2, we find the optimum weights for three portfolios on the basis of different weights. The optimal weights with Markowitz method based on historical returns and our views are presented in the second and third columns of Table 4-4, respectively. The recommend weights of market capitalization and the Black-Littermn approach as in the section 4.3 are presented in the fourth and fifth columns of Table 4-4, respectively.

We discussed in the Section 2.2.2, there are many problems when using Markowitz' method to achieve optimal weights. Not surprisingly, weights with Markowiz' method based on historical return and our views produce an extreme portfolio. Based on historical returns, the Markowitz model recommends the weights of 74.98%, 44.96%, and 24.69% on the European Fund, Australia Fund, and the emerging markets fund, respectively, but recommends -37.31%, -36.4%, -11.3% and -9.32% on the Germany Fund, European Small Companies Fund, Japan Smaller Companies Fund, and American Growth Fund, respectively. However, there are limits on long and short position in most countries. For instance, the long limit for a fund of funds should be 30% and no short position allowed in Taiwan. The extreme result with the Markowitz model may prevent investors from using the model. Besides the extreme results, a little change in the return vector causes big difference in the weights. For example, the recommended weights on European Fund and European Small Companies Fund are 74.98% and -36.4% originally. After the consideration of our views, the recommended weights on European Fund and European Small Companies Fund become -29.66% and 46.09%, which are almost opposite while we impose little change in the return vector.

Compared with the Markowitz model, the recommended weights with the Black-Litterman approach are not only intuitive but also more stable. For instance, we propose that European Small Companies Fund will outperform European Fund on the basis of the size effect, which results in increase of 0.18% on European Small Companies Fund and decrease of 1.7% on European Fund. We also propose that American Growth Fund will outperform American Fund, and which brings about the increase of 4.78% on American Growth Fund and decrease of 3.55% on American Fund. These results are intuitive. To make it clearer, we plot optimal weights in Figure 4-2 which shows the high sensitivity on inputs of Markowitz more clearly.

4.4.2 Simulation

Most related empirical papers use unconstrained weights to show the intuitiveness of the Black-Litterman model. However, there are a lot of restrictions in reality. For example, short selling is often restricted in many countries including Taiwan. To make our simulation close to the reality, we have to add the constraints on the weights:

$$U = w^T \prod -(\lambda / 2) w^T \Sigma w$$

$$\text{Subject to } \ 0 \le w \le 0.3 \qquad (4.4)$$

Utility of the fund manager is still the same as Eq.(3.1), except adding the constraint on the weight. In Taiwan, fund of fund should invest at least 5 funds and invest in each fund can not be over 30% of its total capitalization.

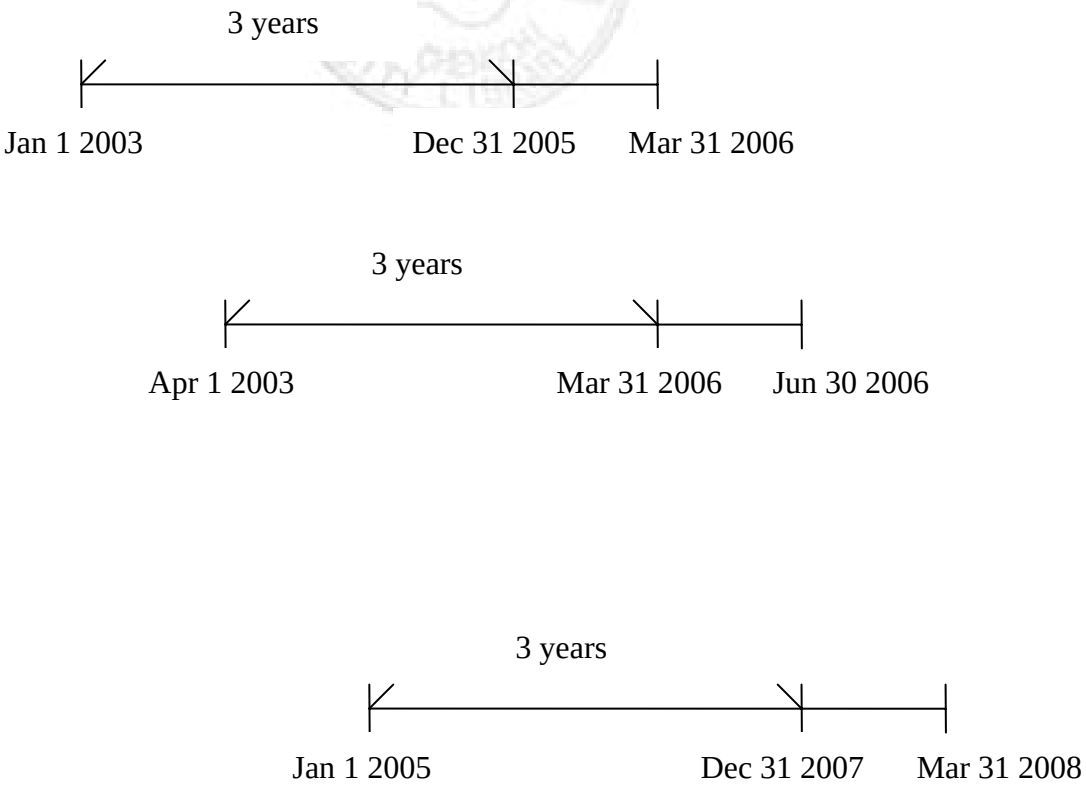
To make the rebalancing cost comparable, we first hold our views unchanged for each quarter. Secondly, we adopt rolling window analysis.

For single-window analysis, the program calculates one set of weights, using the

entire selected date range. Section 4.3 is an example of single window style analysis. For rolling window style analysis, we perform a whole sequence of style analyses. To this end, we specify 3 years as a small date range, starting from the beginning of our data. This window is then moved forward by an increment of 3 months, and the optimal weights are performed for each resulting window.

Figure 4-3 shows the rolling window analysis. We use the daily asset value of each fund in the period from January 1 2003 to December 31 2005 to calculate returns and covariance. We then use the statistics in the period to estimate the optimal weights of the first quarter in 2005. The optimal weights of the second quarter in 2005 are based on the data in the period from April 1 2002 to March 31 2005 and go forth like this process.

Figure 4-3. Rolling Window Analysis



The Markowitz optimal weights for each quarter from first quarter in 2006 to the first quarter in 2008 are represented in Table 4-5. Table 4-6 shows the Black-Litterman optimal weights for the same period. Figure 4-4 shows the comparison of rebalancing cost with the two models.

Not surprisingly, the rebalancing cost with Markowitz Model is higher than that with the Black-Litterman Model for each quarter. This result reflects the flaws of Markowitz model as we discuss before. A small change in the values of the input parameters can cause a large change in the composition of the portfolio with the Markowitz model, not to mention that the change in the inputs of the Markowitz model is not small. The simulation results also show corner solution. For example, there are at least three funds with zero weights and it is unreasonable. In particular, in the first quarter in 2007, there are only four funds with positive weights and three of them are almost close to 30% up limit.

Compared with the Markowitz model, this result reflects the stability of recommended weights with the Black-Litterman approach. There is no corner solution in the simulation for the Black-Litterman model. The changes in the portfolio weights for each quarter are not more than 10%. The highest rebalancing cost is around 23.3% in the fourth quarter in 2006. The average cost is around 11%, which is far below average rebalancing cost of 69% with the Markowitz method.

Investors care about the profitability more than the stability of the portfolio. However, the profitability of the Black-Litterman model may result from the good investment strategies. The attractiveness of the Black-Litterman is its combination of the

investment strategies with quantitative methods. Whether the investment strategies (specific views) make profit or not is not the focus of this paper.