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保險死亡率期限結構之共同因子分析與配適(第2年) 研究成果報告(完整版)

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中文摘要：本研究建議採用六因子 Nelson-Siegel 模型預測全世界 23 個國家的死亡率期限結構的動態行為。這些 23 個樣本國家的涵蓋範圍相當廣泛，包括美國，英國，德國，日本，澳洲等幾大洲的先進與新興國家。這些國家的死亡率期限結構的特性相差也非常大，因此對建構適當的預測模型而言，是一件艱鉅的任務。本研究所建議的預測模型在不論樣本內的配適程度或是樣本外的預測準確度方面，都表現得相當優異。尤其在與傳統的死亡率預測模型，Lee-Carter 模型，相比較下，更是展現出顯著且優異的預測能力。本研究所進行的預測規模是保險學界難得一見的，也因此更能凸顯出本研究所採用的六因子 NS 模型的優勢和實務應用價值。最後，我們更透過追蹤資料版本的 Diebold-Mariano Forecasting Test 檢定六因子 NS 模型和 Lee-Carter 模型的相對預測能力，最終檢定結果更進一步確認六因子 NS 模型的優異預測能力。

中文關鍵詞：死亡率，配適，預測，六因子 Nelson-Siegel 模型

英文摘要：Motivated by the analogy between yield curves and mortality rate curves, we introduce a curve-fitting model to reduce the dimension of the age parameters and allow additional degrees of freedom in the forecasting of the period effect. Large-scale comparisons are carried out against the benchmark Lee-Carter model. The results reveal robust improvements in forecasting accuracy across 23 sampled countries. Our approach dominates in 96 percent of male cases and 88 percent of female cases, with an average RMSE improvement of 14.8 percent and 8.4 percent respectively. The panel Diebold-Mariano test indicates that these improvements are significant.

英文關鍵詞：Mortality rate, Fitting, Forecasting, Nelson-Siegel model

A Curve-fitting Approach to the Modeling and Projection of Mortality Rate Curves for Worldwide Cases

ABSTRACT

Motivated by the analogy between yield curves and mortality rate curves, we introduce a curve-fitting model to reduce the dimension of the age parameters and allow additional degrees of freedom in the forecasting of the period effect. Large-scale comparisons are carried out against the benchmark Lee-Carter model. The results reveal robust improvements in forecasting accuracy across 23 sampled countries. Our approach dominates in 96 percent of male cases and 88 percent of female cases, with an average RMSE improvement of 14.8 percent and 8.4 percent respectively. The panel Diebold-Mariano test indicates that these improvements are significant.

Keywords: Mortality rate, Fitting, Forecasting, Nelson-Siegel model

JEL Classification: C53, G22, J11

1. INTRODUCTION

Given that the future mortality has major impacts on life insurance products, pensions and retirement policies, erroneous mortality rate forecasting will clearly undermine both the solvency of insurance companies and the sustainability of the retirement benefits of workers. A model capable of producing accurate mortality curve forecasting is therefore of significant importance.

The forecasting of the mortality curve requires the consideration of a two-dimensional problem involving both age and time. The most widely-adopted model, proposed by Lee and Carter (1992), used one principal component to describe the period effect, along with a fixed sensitivity parameter for each age; it therefore represents a single-factor model. Despite its impressive empirical performance, the model has been criticized by many (for example, Girosi and King, 2007), as being potentially oversimplified. Furthermore, since the age effect parameters are obtained through principal component method, the age effect is discrete, and unrestricted; that is to say, the sensitivities for two adjacent ages can be shown to be completely different.

In the present study, we propose a six-factor model for the forecasting of the adult mortality curve, which, despite using fewer parameters, produces more accurate forecasts than the Lee-Carter model. The strategy adopted is one of simplifying the age-effect modeling using a parsimonious curve-fitting function; if the curve-fitting function can effectively describe the age effect with fewer parameters, then we can take the degrees of freedom saved from the age-effect modeling and include them in the period-effect modeling, thereby increasing the number of factors. In particular, we introduce an extended Nelson-Siegel model (1987), which is a dynamic model with six factors. Diebold and Li (2006) demonstrated that as compared to seven other benchmark models, the dynamic Nelson-Siegel family model was capable of producing accurate interest rate term structure forecasting.

A total of 23 countries were selected from the Human Mortality Database to test our approach, with both male and female populations. The sample countries spreading over Europe, North America and the Asia-Pacific region had varying degrees of mortality characteristics and economic status. Since the sampling period for all countries began in 1950, our tests covering the post-World War II period can be seen as comprehensive.

Our approach provides good performance in both in-sample fitting and out-of-sample forecasting; indeed, our model was found to prevail in all of the comparisons of in-sample fitting performance for all 46 of the gender-specific populations, with an average 20 percent improvement in both 'root mean squared error' (RMSE) and 'mean absolute error' (MAE). The model was further found to provide better fits for male mortality and female mortality, with respective average improvements of 26 percent and 15 percent.

Based upon a series of comparisons over 1-, 3-, 5-, 10- and 15-year forecasting horizons, our six-factor model demonstrates superior forecasting performance. It was found

to prevail in 96 percent of the 115 comparisons of male mortality forecasting, with an average RMSE improvement of 14.8 percent, and similar comparison of female mortality forecasting was found to prevail in 88 percent of the 115 comparisons, with an average RMSE improvement of 8.4 percent. Therefore, our model produces more accurate forecasts than the Lee-Carter model for all horizons.

A panel version of the Diebold-Mariano (DM) test was subsequently applied to worldwide data in order to determine whether our model could indeed produce statistically more accurate forecasts. The panel DM test, proposed by Pesaran, Schuermann and Smith (2009), indicated that our model is indeed more accurate; the test statistics revealed that for almost all the ages – with the exceptions of both ends of the mortality curve – our forecasting was significantly better.

The remainder of this paper is organized as follows. The models and estimation procedures for in-sample fitting and out-of-sample forecasting are presented in Section 2, along with a description of the data. The analysis of our empirical results from in-sample fitting are presented in Section 3, with Section 4 providing analysis on the results of the out-of-sample forecasting. Finally, the conclusions are drawn in Section 5.

2. THE FORECASTING MODELS

2.1 The Six-Factor Model

We specify a six-factor model for mortality curve forecasting based upon a series of polynomial and exponentially decayed terms (also known as Laguerre polynomials). The central death rate of age x at year t is denoted by $m(x,t)$; thus, our mortality model takes the following form:

$$\begin{aligned} \log m(x,t) = & \beta_{1t} + \beta_{2t} \left(\frac{1-e^{-\lambda_1 x}}{\lambda_1 x} \right) + \beta_{3t} \left(\frac{1-e^{-\lambda_2 x}}{\lambda_2 x} \right) + \beta_{4t} \left(\frac{1-e^{-\lambda_1 x}}{\lambda_1 x} - e^{-\lambda_1 x} \right) \\ & + \beta_{5t} \left(\frac{1-e^{-\lambda_2 x}}{\lambda_2 x} - e^{-\lambda_2 x} \right) + \beta_{6t} \left(\frac{1-e^{-\lambda_1 x}}{\lambda_1 x} - e^{-2\lambda_1 x} \right). \end{aligned} \quad (1)$$

The approximating capability of Equation (1) comes from the exponential components which vary with both age x (or maturity in the interest rate analogy), and decay parameters λ_1 and λ_2 . The exponential components in Equation (1), also referred to as the ‘factor loadings’, are used to capture the shape of the mortality curve, with the factors, β_{it} , $i = 1, \dots, 6$, being used to capture the period effect. Each of the exponential components can be seen as reflecting the degree of sensitivity of the mortality curve to a particular factor; hence, the term ‘factor loadings’. These components, referred to by the age region in which the factor loading has the highest value (i.e., where it is most sensitive to the factor movement), are illustrated in Figure 1 as a function of age.

<Figure 1 is inserted about here>

The ‘young-age’ component has the highest value in the youngest age group, which can be seen from the term $(1 - e^{-\lambda_1 x})/(\lambda_1 x)$, where it initially shows the greatest value in the youngest age group, and then starts to decay monotonically at a speed controlled by the decay parameter λ_1 . The ‘middle-age’ factor loading $(1 - e^{-\lambda_1 x})/(\lambda_1 x) - e^{-\lambda_1 x}$ is a hump-shaped function which reaches its maximum value during middle age, where the location of the maximum is again determined by the value of λ_1 . The ‘elder-age’ factor loading is a constant of all ages and is therefore found to be dominant amongst the elder age groups. An additional component $(1 - e^{-\lambda_1 x})/(\lambda_1 x) - e^{-2\lambda_1 x}$ is also presented in order to capture the small, but frequent, variations amongst the younger age groups.

For the purpose of flexibility, two pairs of factor loadings are specified within the model, each with different decay parameters, λ_1 and λ_2 . By adjusting the value of the decay parameter (λ_i), the factor loadings reflect the different behavior patterns of the age effect, as shown in Figures 2 and 3. In the ‘young-age’ factor loading, the decay parameter determines the speed of decay in terms of sensitivity to the period effect. As shown in Figure 2, a larger value of λ_i indicates a more rapid decay in the factor loading value with an increase in age. In Figure 3, the decay parameter also controls the speed of the decay in the middle-age factor loading, with a larger λ_i implying a larger hump in the middle-age factor loading.

<Figures 2 and 3 are inserted about here>

The period effect is explained by the factors $\beta_t = [\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}, \beta_{6t}]'$, which are the drivers determining the evolution of the mortality rate curves across different periods. As noted by Diebold and Li (2006), these factors can be seen as dynamic latent factors governed by time-series models; in specific terms, each of the factors represents the forces governing the evolution of the mortality curve, with the factor loadings representing the sensitivity of the mortality curve to such forces.

We can forecast these factors by selecting an appropriate time-series model and then obtain a forecast of the mortality curve by recombining the forecasted factors with their associated factor loadings. If each factor is assumed to be Gaussian, then the mortality rate becomes a Gaussian random variable, in which case, the density forecast can be obtained.

2.2 The Lee-Carter Model

The original version of the Lee-Carter model takes the following form:

$$\log m(x, t) = a_x + b_x k_t + \varepsilon_{x,t} . \quad (2)$$

where the central death rate for each age x at time t , $m(x, t)$, is associated with a base parameter a_x and an additional parameter, b_x , which together describe the age effect; the a_x and b_x parameters are also referred to as the ‘age-specific constant’, essentially because they relate only to age, and also because once they are estimated, they are fixed. The k_t variable alone captures the period effect.

Although the Lee-Carter model has been widely adopted in numerous prior studies, and with some empirical success, it does have some weaknesses which are essentially based upon

its simplicity. Criticism is often directed at the single factor (k_t) setting of the model, which raises two major concerns; firstly, a single source of time variation implies that all shocks to mortality rates are perfectly correlated over time; and secondly, the invariant b_x under the single source of time variation restricts changes in the mortality rates to the relative magnitudes of b_x (Giroi and King, 2007). An additional concern is the distinct lack of smoothness in the mortality curve produced by the Lee-Carter model due to the fact that b_x is unconstrained.

Our approach provides improvements upon the Lee-Carter benchmark model from two aspects. These theoretical advantages are illustrated in Equations (3) to (6), where both the Lee-Carter model and our six-factor model (with the age subscripts suppressed) are expressed as:

$$LC: \log m(t) = a + bk_t + \varepsilon_t \quad (3)$$

$$k_t = \theta + k_{t-1} + v_t \quad (4)$$

$$6F: \log m(t) = X\beta_t + \mu_t \quad (5)$$

$$\beta_t = \delta + \beta_{t-1} + \eta_t \quad (6)$$

where $\log m(t)$ denotes the log mortality curve in year t ; a and b are the vectors of a_x and b_x in the Lee-Carter model, θ is a scalar drift for k_t , X is the matrix of the factor loadings, δ is the drift vector for β_t , and β_t represents the factors in year t .

According to Giroi and King (2007), both models may be further rewritten as:

$$LC: \log m(t) = \log m(t-1) + \theta b + (bv_t + \varepsilon_t - \varepsilon_{t-1}) \quad (7)$$

$$6F: \log m(t) = \log m(t-1) + X\delta + (X\eta_t + \mu_t - \mu_{t-1}) \quad (8)$$

The error terms are essentially restrictions on the covariance matrices implied by both models. Mortality in the Lee-Carter model is subject to only two shocks: bv_t that is perfectly correlated across all ages and ε_t being uncorrelated across different ages. Conversely, our six-factor model exhibits multiple sources of shocks from the different factors across various ages. The more flexible correlation structure of Equation (8) is obtained by simultaneously considering multiple evolutions in mortality trends, as opposed to single evolution.

3. IN-SAMPLE ESTIMATIONS

3.1 Parameter Estimation

We estimate our six-factor model using a two-stage procedure, with the first stage involving the determination of the decay parameters, $\lambda = [\lambda_1, \lambda_2]$, by minimizing the quadratic estimation error in the log mortality curve. The second stage involves the use of the OLS method to estimate the β_t factors, based upon the λ determined in the first stage. The first stage optimization problem can be written as:

$$\begin{aligned} \min_{\lambda} & \quad \sum_{x,t}^{X,T} (\log m(x,t) - \log \hat{m}(x,t, \beta_t(\lambda)))^2 \\ \text{s. t.} & \quad \lambda_1 - \lambda_2 \geq c, c \in \mathbb{R} \end{aligned} \quad (9)$$

where X and T are the last age and period in the sample. Once the optimized values of $\lambda^* = [\lambda_1^*, \lambda_2^*]$ have been determined, Equation (1) immediately becomes linear in β_t ; that is, plug in λ^* into factor loading X in Equation (5) to have:

$$\log \hat{m}(x,t, \beta_t(\lambda^*)) = X^* \beta_t. \quad (10)$$

We can then obtain β_t in the second stage using the standard OLS technique and enter the OLS estimate back into the first stage to calculate the estimation error.

Prior to commencing with the first stage estimation, we must specify a set of constraints for the ranges of λ to avoid problems that may arise from inappropriate values of λ . A poorly specified λ would result in a failure to identify the components and multicollinearity amongst the components (Bolder and Streliski, 1999; de Pooter, 2007).

To avoid the potential numerical problem referred to above, we follow Gilli, Grosse and Schumann (2010), in which it was suggested that a restriction should be placed upon the value of λ based upon the correlations between the factor loadings; we therefore set the boundary of λ at $[0.0291, 0.0414]$. Given the value of the decay parameter, we can calculate the correlation between the two factor loadings. We restrict the correlation between the ‘young-age’ and ‘middle-age’ factor loading to $[-0.7, 0.7]$. The limit is chosen to be below the level of ‘highly correlated’ from standard advice of statistic textbook.

A second constraint is then imposed on the minimum distance between λ_1 and λ_2 ; this constraint is $\lambda_1 - \lambda_2 > 0.0037$. The minimum distance constraint for λ controls the distance between the two factor loadings, an approach proposed by de Pooter (2007) as a means of further alleviating the potential risk of multicollinearity between the factor loadings. In the extreme case where the two decay parameters are equal, the six-factor model would degenerate to a four-factor one and result in numerically inaccurate parameter estimates.

In order to avoid such a case, we set a minimum distance of five years between the ages where the maximum loading appears for two middle-age factors; this implied minimum difference between λ_1 and λ_2 is set at 0.0037 by a simple grid search within the implied boundary of λ . Given this boundary for λ , the maximum value for middle-age factor loadings would appear between the ages of 43 and 62, thereby providing a good fit for our purpose of describing the middle-age mortality rate.¹

3.2 Data Description and Accuracy Measure

¹ The location of the maximum loading provides a good reflection of the location of the curl on the mortality rate curve. The bend points for two middle-age factor loadings should not be too close, otherwise multicollinearity will be found to exist between the middle-age factor loadings. Although the selection of five years as the minimum distance was an *ad hoc* choice, further tests were carried out using three and seven years in order to provide a test for robustness, with the results showing that the forecasting performance remained intact with these alternative choices.

Our empirical tests on the two competing models were performed using the central death rates for ages from 20 to 100 obtained from the Human Mortality Database (2011). Samples were obtained for every country for which mortality data during the post-1950 period were available, including both male and female populations within our sample. The sample period for each country was determined by the last mortality rate available within the database.² The 23 countries in our sample are listed in Table 1.

<Table 1 is inserted about here>

Two accuracy measures, root mean square error (RMSE) and mean absolute error (MAE), are used in this study to evaluate model fitness. RMSE imposes heavier penalties on greater errors by squaring the errors, whilst MAE focuses on the average of the absolute errors. The measures for each fitting period, t , are computed as:

$$\text{RMSE}(t) = \sqrt{\frac{1}{N} \sum_{x=x_1}^{x_N} (\log m(x, t) - \log \hat{m}(x, t))^2} \quad (11)$$

$$\text{MAE}(t) = \frac{1}{N} \sum_{x=x_1}^{x_N} |\log m(x, t) - \log \hat{m}(x, t)| \quad (12)$$

where $m(x, t)$ is the observed (actual) central death rate of age $x = x_1, \dots, x_N$ in year t , and $\hat{m}(x, t)$ is the fitted or forecasted value obtained from the model, with a greater RMSE or MAE indicating greater estimation errors. We then calculate the improvement ratios for the above measures defined as the RMSE of our model over the RMSE of the Lee-Carter benchmark model, thereby providing a direct comparison of in-sample fitting and out-of-sample forecasting performance, as follows:

$$\text{RMSE improvement ratio} = 1 - \frac{\sqrt{\sum_{t=1}^T \sum_{x=x_1}^{x_N} (\log m(x, t) - \log \hat{m}^{6F}(x, t))^2}}{\sqrt{\sum_{t=1}^T \sum_{x=x_0}^{x_N} (\log m(x, t) - \log \hat{m}^{LC}(x, t))^2}} \quad (13)$$

$$\text{MAE improvement ratio} = 1 - \frac{\sum_{t=1}^T \sum_{x=x_1}^{x_N} |\log m(x, t) - \log \hat{m}^{6F}(x, t)|}{\sum_{t=1}^T \sum_{x=x_1}^{x_N} |\log m(x, t) - \log \hat{m}^{LC}(x, t)|} \quad (14)$$

where T denotes the data period, x_1 denotes the initial age, and x_N denotes the last age within the data. A positive ratio would indicate that our six-factor model had produced a more accurate forecast.

3.3 Fitting Results

The results shown in Table 2 reveal that our model fits worldwide mortality curves better than the Lee-Carter model, with average (median) improvement ratios across the sampled countries of 20.3 percent (16.4 percent) for RMSE, and 20.2 percent (17.9 percent) for

² The sample for Iceland was ultimately excluded from this study because it contained too many zero mortality rates which resulted in numerical inconsistencies within the estimations of both the Lee-Carter model and our six-factor model.

MAE . The maximum improvement in RMSE is found for female mortality in Japan, at 56.4 percent, whilst the minimum improvement is found for female mortality in the Netherlands, at 2.5 percent; similar results are also reported for MAE. These comparison results suggest that our model has superior fitting performance in all of the sampled countries.

<Table 2 is inserted about here>

As shown in Table 3, in terms of both RMSE and MAE, our six-factor model is found to produce a better fit for female mortality across all 23 countries. The average RMSE (MAE) for the six-factor model was found to be 9.17 percent (6.34 percent), whilst that for the Lee-Carter model was found to be 10.76 percent (7.64 percent). Table 3 also indicates the efficacy of our six-factor model with regard to male mortality. For instance, our model prevails in all male populations in terms of RMSE, with an average of 7.12 percent versus the 9.60 percent of the Lee-Carter model. The resulting improvement ratio for RMSE is reaching 26 percent, almost twice as much as that found for the female mortality curves (15 percent). Improvement ratios in excess of 40 percent are discernible for Bulgaria, Japan, Portugal, Spain and the United Kingdom. As regards performance for different gender samples, both the six-factor model and the Lee-Carter model demonstrate better fitting performance on male data than on female data.

<Table 3 is inserted about here>

The samples were then categorized into six age groups: 20-34 years, 35-44 years, 45-64 years, 65-74 years, 75-84 years and 85-100 years, with Table 4 reporting the results. The figures shown in the 'Average' row in Table 4 demonstrate that across the different age groups, our model invariably provides better fitting performance than the Lee-Carter model worldwide. Furthermore, our model dominates 86 percent of the cases in terms of RMSE and MAE.

<Table 4 is inserted about here>

With regard to the fitting to each age group, the average improvement ratio is found to be 23.7 percent and 24.2 percent, in terms of RMSE and MAE for the 20-34 age group. Our model provides more accurate estimates for every country in the age group of 20-34. Within the 35-44 group, the estimate of our model is more accurate in 21 countries in terms of both RMSE and MAE; similar comparisons on the age 45-64 group suggest that our model is more accurate in 17 countries. The average improvement ratios for MAE were 18.00 percent in the 65-74 age group, 33.70 percent in the 75-84 age group, and 28.10 percent in the 85-100 age group. There are 15 countries favoring our model in the 65-74 age group, 22 countries in the 75-84 age group, and 20 countries in the 85-100 age group. Similar advantages of our model relative to the Lee-Carter model can be found even we switch the accuracy measure to RMSE. Our model has a clear comparative advantage in the older age groups, which is important since these groups are considered to be the primary source of longevity risk.

For further robustness check, the sampling periods were subsequently split into six decade groups: the 1950s, 1960s, 1970s, 1980s, 1990s and 2000s, with Table 5 showing that

the fitting accuracy of our model is consistently better across each of these periods. Throughout all six periods, our model outperform in 14 of the 23 countries, with the most significant performances being discernible for Japan, Hungary, the United Kingdom and Spain. Among all decade groups, the best relative performance is found in the 1950s. We speculate that in the 1950s, the mortality curves of some countries were still subject to the effects of World War II, and hence, irregular patterns are demonstrated. The flexibility of our model appears to be sufficient in capturing the irregular patterns during that particular period. The results from the are similar for each period, with our six-factor model demonstrating improvements of around 18 percent in the RMSE ratios. In the periods from 1960s to the 1990s, our model is found to have both a lower RMSE and a lower MAE for almost all countries. In the 2000s, our model reveals an average improvement ratio of 14.2 percent.

<Table 5 is inserted about here>

4. OUT-OF-SAMPLE FORECASTING

4.1 Forecasting Procedure

The forecasting procedure beginning with the application of the two-stage estimation of the parameter, λ , and the factor, β_t , as described in the previous section. After obtaining the forecasted values of these factors, they are re-entered into Equation (1), along with the associated λ , in order to project the mortality rate curves.

We adopt the random walk with drift (RWD) model for the individual factors for two reasons. Firstly, although the usual augmented Dickey-Fuller test and Box-Jenkin procedure should be applied to the factors in order to select the best model, in practice, we sometimes find that the use of different time-series models for each of the factors may lead to inconsistent forecasting. We therefore select the RWD model to simplify the procedure and achieve greater consistency.³

Secondly, in order to compare the effects of the modeling trade-off between the age and period effects, we need to control the time-series model at the very least so as to maintain comparable grounds. The mortality trend, k_t , in the Lee-Carter model has produced empirically accurate forecasting in the past, an example of which is provided by Tuljapurkar et al. (2000). Furthermore, it is well known that mortality rates show declining trends across countries and age groups during the post-war period, which also suggests that the RWD model may be an appropriate choice.

The window used in the first-stage estimation is set at 30 years in order to ensure that an adequate number of samples is reserved for model construction. This also ensures that sufficient numbers of unused observation are available for use in the forecasting tests. The

³ A similar approach was also adopted by Lee and Carter (1992); according to their findings, “a similar model with an ar(1) term added was marginally superior, but we preferred the (0,1,0) model on grounds of parsimony” (pp.669).

first estimation window runs from 1950 to 1979, with the window then rolling forward by a one-year step, such that the second estimation period runs from 1951 to 1980, and so on; 1-, 3-, 5-, 10- and 15-year factor forecasts are then generated for each of these rolling windows. We repeat these forecasting procedures for at least 27 rounds for the 1-year forecast horizon, 25 rounds for the 3-year forecasts, 23 rounds for the 5-year forecasts, and so on, depending upon the overall duration of the data available for each of the countries.

4.2 Forecasting Results

Our initial assessment of relative forecasting performance is undertaken from the aspect of gender. As we can see from the comparisons of the male mortality samples in Panel A of Table 6, our six-factor model dominates in almost all of the RMSE improvement ratios. Out of the 115 comparisons (23 countries x 5 forecasting periods), our model was found to outperform the Lee-Carter model in 110 rounds (i.e., 96 percent of the time). Given the broad range of our sample, the average improving magnitude across the various forecasting periods is found to be quite significant; the average improvement ratio of 1-, 3-, 5-, 10- and 15-year forecasting horizons are 19.9 percent, 20.2 percent, 17.1 percent, 10.0 percent and 6.80 percent, with an overall average improvement ratio of 14.8 percent.

<Table 6 is inserted about here>

The improvement ratios in RMSE for female mortality are reported in Panel B of Table 6, from which we can see that our model dominates in 88 percent of all cases. The average improvement ratios of 1-, 3-, 5-, 10- and 15-year forecasting horizons are 7.5 percent, 9.5 percent, 9.4 percent, 8.1 percent and 7.3 percent, with an overall average improvement ratio of 8.4 percent. It is clear that our model provides more accurate forecasting of both male and female mortality rates, with the improvements over the Lee-Carter model being found to be more significant for male mortality rates. The improvements in MAE for different horizons largely resemble those already found with regard to RMSE.

The last rows of Table 6 show that the greatest improvement ratios are found in the 3-year forecasting horizons. The improvements then decline with the forecasting horizon, which may be attributable to the small number of factors used to depict the overall dynamics of the entire mortality rate curves. Erratic changes may be discernible with a lengthening of the forecasting horizon; In addition, sudden structural changes may arise to which the RWD drift is unable to respond as noted by Coelho and Nunes (2011). Nevertheless, as long as life insurers carry out periodic dynamic hedging on the mortality rate risk, this inherent ‘weakness’ of our model will disappear.

4.3 The Panel Diebold-Mariano Test

Notwithstanding the evidence already presented, we cannot reject the ‘equal accuracy’ hypothesis without carrying out a statistical test. We conduct a panel Diebold-Mariano (DM) test which was developed by Pesaran, Schuermann and Smith (2009) to examine same- age time-series panel data across different countries – to determine whether the improvements

of our model over the Lee-Carter model is statistically significant.⁴ Following Diebold and Mariano (1995), the loss differential in the mortality rates is defined as:

$$z(x, t) = [e^{6F}(x, t)]^2 - [e^{LC}(x, t)]^2 \quad (15)$$

where $e^{6F}(x, t)$ and $e^{LC}(x, t)$ are the respective one-year ahead forecast error terms in our six factor model and the Lee-Carter model; x refers to age (in years); and $t = 1, \dots, n$ refers to the calendar year of the forecast. We pick out every other five age starting from age 20 to test the accuracy of mortality forecast. The panel DM test statistic was derived by Pesaran et al. (2009) as follows:

$$DM = \frac{\bar{z}}{\sqrt{V(\bar{z})}} \sim N(0,1) \quad (16)$$

where

$$\bar{z} = \frac{1}{m} \sum_{i=1}^m \bar{z}(i) \quad (17)$$

$$\bar{z}(i) = \frac{1}{n} \sum_{t=1}^n z(i, t) \quad (18)$$

$$V(\bar{z}) = \frac{1}{m} \sum_{i=1}^m \frac{\hat{\sigma}^2(i)}{n(i)} \quad (19)$$

$$\hat{\sigma}^2(i) = \sum_{t=1}^n \frac{(z(i, t) - \bar{z}(i))^2}{n(i) - 1} \quad (20)$$

and where the number of countries is denoted as $i = 1, \dots, 23$. A negative DM test statistic would indicate the better forecasting of our model over the Lee-Carter model.⁵

The results are reported in Table 8. We can see that almost all of the DM statistics are negative and significant at the 95 percent confidence level, thereby suggesting that as compared to the Lee-Carter model, our six-factor model produces more accurate forecasting for different age groups and across different countries.

<Table 8 is inserted about here>

Table 8 also suggests that our model is significantly better at profiling the younger aged mortality. From age 25 to age 65, the DM statistics of both male and female population became significantly negative and rejected the null hypothesis except age 50 in female population. For age above 85, the DM statistics show that our projection is more accurate

⁴ Also available to us is the possibility of examining the forecasting errors within each country and then carrying out the original DM test across different ages; however, the problem with this approach is that the forecasted series has only limited length, and any statistical tests using small samples will not be convincing. Thus, the panel DM test, involving comparisons of the forecasting accuracy of the two models for individual age mortality rates, using our comprehensive panel dataset, is more appealing.

⁵ Pesaran et al. (2009) noted that there was potential serial autocorrelation in the loss differential, a problem which could be dealt with using the HAC estimator. Another concern is that the DM statistic may no longer be asymptotic to standard normal distribution if the cross-section dependence is significant and the number of forecasts is small; we therefore go on to compute Newey-West variance with a maximum lag equal to 1, and find that the results are similar.

for male mortality. A possible explanation for the better fitting on the younger aged mortality is that the oldest mortality estimates in our model tend to converge to the elder-age factor (β_0), with the estimated mortality rates at the very end being prone to changes in the elder-age in our model. Furthermore, with a decline in the effects of the other factors, the elder-age factor alone cannot accurately describe the end-point mortality rate.

5. CONCLUSIONS

We analyze the applicability of two different mortality modeling strategies to worldwide cases in the present study: a curve-fitting approach and the widely adopted Lee-Carter model. The major contributions of our approach are the parsimonious age effect, the feasibility of interpolation, and an analytical tractable function form. Mortality curve forecasting in our approach is built upon linear, additive factors, where each of the factors is projected by a random walk with drift model.

We carried out a large-scale comparison using mortality data on 46 gender-specific populations. The results indicate that our approach significantly outperforms the standard benchmark approach, in both in-sample fitting and out-of-sample forecasting. We subsequently carried out a panel Diebold-Mariano (DM) test to determine statistical significance of the improvement of our model over the Lee-Carter model. The results validated the improvements found in most of the sample populations. We can therefore conclude that our model is capable of producing more accurate forecasting of the adult mortality curve than the Lee-Carter model.

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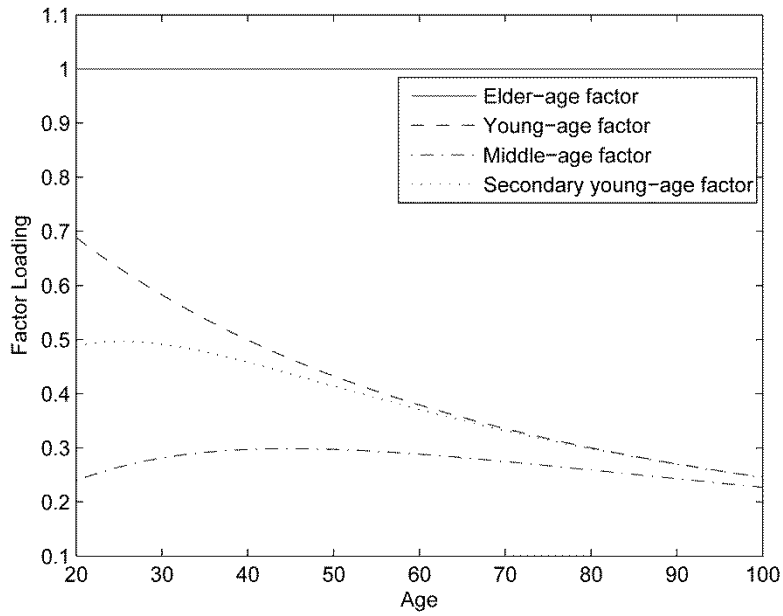


Figure 1 Factor loadings for different age components

Note: This figure illustrates the four distinct components in the six-factor model, with each of the components (factors) being referred to as the age region at which the sensitivity (factor loading) is the highest.

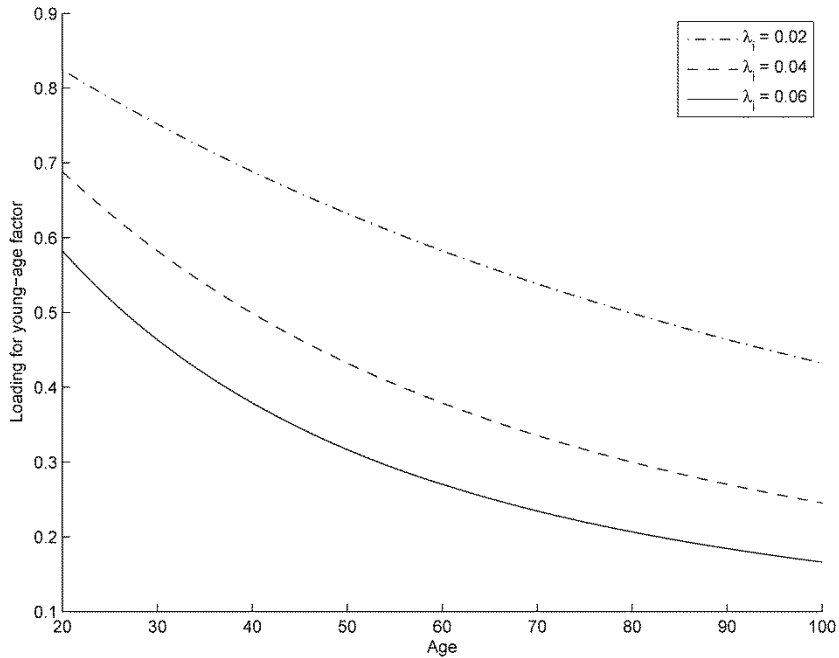


Figure 2 Changes in young-age factor loading with decay parameter λ_i , $i = 1, 2$

Note: This figure illustrates the variations in the young-age factor loading values at different ages, given three different decay parameter values, with the weighting of each age relative to the young-age factor being determined by the value of the factor loading. The decay parameter controls the speed at which the value of the factor loading decreases with age.

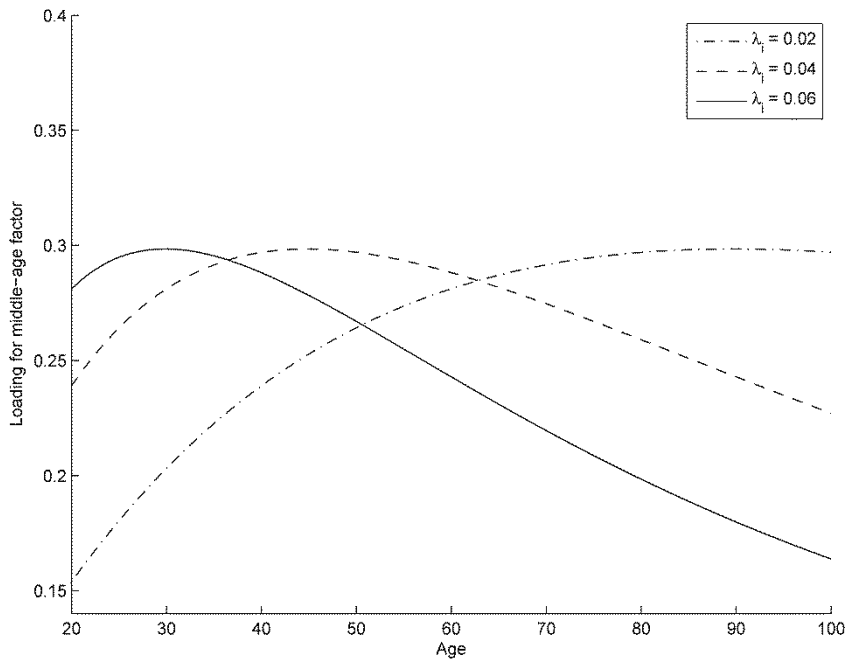


Figure 3 Changes in middle-age factor loading with decay parameter λ_i , $i = 1,2$

Note: This figure illustrates the variations in the middle-age factor loading values at different ages, given three different decay parameter values, with the weighting of each age relative to the middle-age factor being determined by the value of the factor loading. The decay parameter controls the location of the 'hump' in the factor loading, the point at which nearby ages are most sensitive to the middle-age factor.

Table 1 Sample countries from the Human Mortality Database (n = 23)

Geographical Region (No. of Countries)	Country	Sample Period
Asia-Pacific (3)	Australia	1950-2007
	New Zealand	1950-2008
	Japan	1950-2009
Europe (18)	Hungary; Ireland; Spain	1950-2006
	France; Italy; Switzerland	1950-2007
	Austria; Denmark; Netherlands; Norway; Sweden;	1950-2008
	Belgium; Bulgaria; Czech Republic; Finland; Portugal; Slovakia; United Kingdom	1950-2009
	Canada; United States	1950-2007

Table 2 Descriptive statistics on improvement ratios of the six-factor model versus the Lee-Carter model across 46 gender-country populations

Variable	No. of Samples	Mean (%)	Median (%)	S.D. (%)	Max. (%)	Min. (%)
Improvement in RMSE	46	20.34	16.41	13.43	56.41	2.53
Improvement in MAE	46	20.23	17.87	12.96	56.17	2.66

Table 3 *In-sample fitting performance, by genders*

Country	Female					
	RMSE			MAE		
	Six-factor Model	Lee-Carter Model	Improve ment	Six-factor Model	Lee-Carter Model	Improve ment
Australia	7.03	8.06	12.80	5.04	5.80	13.10
Austria	10.00	10.76	7.10	6.60	7.29	9.50
Belgium	9.41	9.99	5.80	6.38	6.88	7.30
Bulgaria	9.09	11.37	20.10	6.48	8.81	26.40
Canada	6.20	6.52	4.90	4.64	4.87	4.70
Czech Republic	8.80	10.16	13.40	5.80	7.14	18.80
Denmark	12.93	14.25	9.30	8.35	9.94	16.00
Finland	12.66	15.16	16.50	8.10	10.49	22.80
France	5.95	6.77	12.10	4.54	4.91	7.50
Hungary	7.61	10.44	27.10	5.12	7.81	34.40
Ireland	16.48	17.42	5.40	11.40	12.19	6.50
Italy	5.30	7.36	28.00	3.98	5.45	27.00
Japan	5.05	11.58	56.40	3.95	9.01	56.20
Netherlands	7.51	7.71	2.60	5.24	5.38	2.60
New Zealand	14.94	15.82	5.60	10.28	10.78	4.60
Norway	11.75	13.53	13.20	7.68	8.93	14.00
Portugal	8.81	11.01	20.00	6.33	8.16	22.40
Slovakia	12.65	13.81	8.40	8.11	9.42	13.90
Spain	7.41	10.2	27.40	5.50	7.75	29.00
Sweden	10.52	10.98	4.20	7.14	7.42	3.80
Switzerland	11.76	13.05	9.90	8.19	8.76	6.50
United States	3.83	4.30	10.90	3.09	3.42	9.60
United Kingdom	5.11	7.25	29.50	3.79	5.22	27.40
Average	9.17	10.76	14.80	6.34	7.64	17.00

Country	Male					
	RMSE			MAE		
	Six-factor Model	Lee-Carter Model	Improveme nt	Six-factor Model	Lee-Carter Model	Improvem ent
Australia	5.49	7.66	28.30	4.09	5.63	27.40
Austria	7.84	8.24	4.90	5.95	6.15	3.30
Belgium	7.43	8.43	11.90	5.53	6.36	13.10
Bulgaria	7.41	12.27	39.60	5.55	7.91	29.80
Canada	5.09	6.06	16.00	3.90	4.67	16.50
Czech Republic	7.07	9.03	21.70	5.34	6.61	19.20
Denmark	9.40	11.04	14.90	6.60	7.97	17.20
Finland	8.32	10.28	19.10	6.07	7.51	19.20
France	5.77	6.90	16.40	4.58	4.95	7.50
Hungary	6.53	13.34	51.00	4.85	8.95	45.80
Ireland	12.67	14.52	12.70	9.14	10.22	10.60
Italy	5.66	8.70	34.90	4.51	6.44	30.00
Japan	4.70	7.90	40.50	3.58	6.19	42.20
Netherlands	5.96	7.86	24.20	4.40	6.12	28.10
New Zealand	10.99	12.38	11.20	7.86	8.77	10.40
Norway	8.36	10.27	18.60	5.88	7.19	18.20
Portugal	6.42	10.78	40.40	4.72	7.85	39.90
Slovakia	8.62	12.84	32.90	6.41	9.90	35.30
Spain	5.99	10.58	43.40	4.45	7.69	42.10
Sweden	7.64	8.52	10.30	5.42	6.05	10.40
Switzerland	8.31	10.50	20.90	6.13	7.43	17.50
United States	3.81	5.50	30.70	3.05	4.07	25.10
United Kingdom	4.35	7.30	40.40	3.35	5.39	37.80
Average	7.12	9.60	25.80	5.28	6.96	24.20

Table 4 *In-sample fitting performance, by age groups*

(%)	<i>Age 20-34</i>						<i>Age 35-44</i>						<i>Age 45-64</i>					
	RMSE			MAE			RMSE			MAE			RMSE			MAE		
	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.
Australia	9.6	12.8	25.1	7.3	10.2	28.4	7.7	9.7	21.1	5.8	7.5	23.5	5.5	6.0	9.3	4.3	4.7	8.6
Austria	14.6	16.3	10.5	11.0	12.3	10.6	10.8	11.1	3.5	8.2	8.8	6.5	7.8	7.7	-1.0	6.1	6.0	-0.3
Belgium	14.1	15.7	10.4	10.9	12.1	10.4	10.0	10.2	2.4	7.7	8.0	3.7	6.8	7.1	4.0	5.3	5.6	6.2
Bulgaria	12.4	19.0	35.0	9.5	13.4	29.3	9.5	12.1	21.4	7.3	9.4	22.1	6.8	7.5	9.8	5.2	5.8	9.3
Canada	8.3	9.4	11.0	6.5	7.4	12.3	6.7	7.3	8.4	5.3	5.8	8.9	4.9	5.2	5.4	3.9	4.1	5.9
Czech Republic	13.1	15.9	17.6	9.9	12.4	19.6	9.8	10.9	9.7	7.4	8.4	12.4	6.1	7.6	19.0	4.8	5.9	19.0
Denmark	20.4	22.1	7.8	15.4	16.9	8.8	13.5	14.6	7.3	10.4	11.3	8.1	8.0	9.0	11.0	6.3	7.1	11.6
Finland	18.8	22.6	17.0	14.0	17.1	18.2	12.8	13.7	6.4	9.8	10.5	6.4	8.3	9.3	11.2	6.3	7.2	12.2
France	7.7	11.7	34.1	6.0	9.2	34.1	6.2	6.4	3.1	4.8	5.0	4.0	5.7	5.3	-7.0	4.6	4.1	-11.1
Hungary	11.6	20.0	42.1	8.9	15.1	40.7	8.1	13.0	37.8	6.2	10.0	37.4	5.8	7.3	21.7	4.5	5.6	20.5
Ireland	22.9	27.6	17.2	17.7	21.4	17.3	18.7	18.4	-1.7	14.7	14.4	-2.1	13.5	12.9	-4.2	10.3	10.3	-0.3
Italy	7.5	13.6	45.3	5.9	10.8	45.4	6.3	7.3	13.9	4.8	5.7	15.4	4.9	6.3	21.2	4.0	4.9	18.9
Japan	6.2	13.4	53.8	4.9	10.8	55.1	5.3	6.9	23.5	4.1	5.5	25.8	5.1	5.0	-1.4	4.0	3.9	-3.3
Netherlands	10.9	12.5	12.9	8.3	9.8	15.4	8.2	7.9	-4.6	6.3	6.1	-4.6	5.6	6.0	6.0	4.4	4.7	5.4
New Zealand	22.0	25.0	12.0	17.2	19.6	12.4	16.6	17.2	3.9	13.0	13.6	4.2	10.8	10.6	-2.2	8.4	8.2	-2.9
Norway	17.7	21.9	19.2	13.3	16.7	20.4	12.8	13.3	4.0	9.7	10.1	4.6	7.8	8.1	3.3	6.0	6.3	5.0
Portugal	11.6	18.0	35.3	9.0	14.5	38.1	9.0	10.5	14.9	7.0	8.4	17.2	6.9	8.1	14.8	5.3	6.4	16.2
Slovakia	18.9	20.5	7.8	14.2	15.4	8.3	12.9	17.0	24.4	9.7	13.7	28.9	8.1	12.3	34.1	6.3	9.7	34.9
Spain	8.1	17.0	52.5	6.5	13.3	51.3	7.5	8.5	11.7	5.8	6.6	12.1	6.5	7.6	15.2	4.9	5.9	16.9
Sweden	15.7	17.3	9.0	12.1	13.3	9.0	11.4	11.8	2.8	8.8	9.1	3.6	7.1	6.9	-2.3	5.5	5.4	-1.7
Switzerland	16.8	21.8	22.8	12.9	17.1	24.9	12.7	12.7	0.2	9.9	10.0	1.6	8.1	8.3	1.9	6.3	6.5	3.4
United States	4.3	7.2	39.9	3.4	5.6	38.9	3.7	6.0	38.5	2.9	4.6	37.2	3.6	4.3	14.6	2.9	3.3	12.3
United Kingdom	7.0	12.0	41.8	5.4	9.3	41.5	5.5	7.5	27.5	4.3	6.0	28.6	4.3	6.3	32.5	3.4	5.0	32.9
Average	13.0	17.1	23.7	10.0	13.2	24.2	9.8	11.0	11.2	7.6	8.6	12.4	6.9	7.6	9.6	5.3	5.9	9.9

Table 4 (Cont.)

(%) Country	Age 65-74						Age 75-84						Age 85-100					
	RMSE			MAE			RMSE			MAE			RMSE			MAE		
	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.
Australia	4.8	4.8	-0.6	3.7	3.8	3.1	3.9	4.0	3.3	3.0	3.3	6.5	4.2	5.6	24.8	3.1	4.4	29.2
Austria	6.1	6.1	0.2	4.7	4.9	3.7	4.2	5.2	20.0	3.3	4.2	20.8	5.3	5.0	-5.8	3.8	3.8	0.0
Belgium	5.8	6.6	11.5	4.5	5.2	13.2	3.8	5.4	28.4	3.0	4.2	28.1	5.4	5.5	2.2	3.9	4.3	9.4
Bulgaria	6.3	8.8	29.2	4.8	7.0	31.4	6.2	8.8	29.2	4.7	7.1	33.3	6.5	10.5	38.1	4.5	7.9	42.9
Canada	4.5	4.3	-5.1	3.6	3.4	-6.8	3.4	4.2	17.9	2.7	3.4	18.2	4.4	5.3	17.7	3.4	4.3	19.8
Czech Republic	5.3	6.9	22.6	4.2	5.2	19.7	4.2	6.6	35.9	3.3	5.2	36.8	5.1	5.4	5.5	3.6	4.1	11.5
Denmark	5.7	7.4	23.0	4.4	5.9	24.8	4.3	7.4	41.8	3.5	6.0	42.5	5.6	7.8	29.0	4.1	6.2	32.7
Finland	6.0	9.1	33.3	4.7	7.1	34.1	5.0	8.5	41.3	3.8	6.4	40.7	4.9	6.9	29.0	3.4	5.5	38.6
France	5.8	4.9	-19.7	4.8	3.8	-25.7	3.7	4.9	24.5	2.9	3.8	24.4	4.9	4.2	-17.7	3.9	3.3	-18.3
Hungary	4.7	6.9	32.2	3.6	5.5	35.1	4.3	9.3	53.9	3.2	7.5	57.2	4.6	9.6	52.5	3.2	7.0	54.5
Ireland	10.3	11.1	7.5	7.8	9.0	13.3	9.3	8.2	-13.5	7.0	6.5	-7.2	6.1	6.8	9.9	4.2	5.2	20.0
Italy	5.4	6.1	12.1	4.5	5.0	11.2	3.7	6.1	39.4	2.9	5.0	41.8	4.3	5.1	14.5	3.4	4.0	16.6
Japan	4.7	9.2	48.4	3.9	7.2	45.9	2.9	13.2	77.8	2.4	11.2	78.6	3.9	10.4	62.1	3.0	8.5	64.9
Netherlands	4.6	6.2	25.3	3.6	4.9	25.6	3.1	4.8	35.5	2.4	3.9	37.0	4.5	6.2	27.5	3.4	4.9	30.5
New Zealand	7.8	7.1	-9.5	6.1	5.6	-8.4	7.0	6.7	-4.0	5.6	5.4	-3.7	5.5	7.0	21.4	3.9	5.6	29.6
Norway	5.7	6.8	16.7	4.5	5.4	16.6	3.9	6.4	39.5	3.1	4.9	37.3	5.0	5.9	14.5	3.7	4.6	20.0
Portugal	6.4	8.9	28.3	4.8	6.7	27.6	5.8	8.6	32.0	4.3	6.5	33.4	4.3	7.2	40.7	2.8	5.5	48.6
Slovakia	6.3	8.2	23.1	4.9	6.6	24.7	6.0	8.4	28.8	4.6	6.7	32.1	5.2	7.3	27.7	3.6	5.5	34.6
Spain	6.4	9.3	31.4	4.9	7.7	36.3	5.8	9.6	39.4	4.0	7.9	49.9	5.9	6.9	13.5	3.8	5.3	28.9
Sweden	5.1	5.0	-1.4	4.1	4.0	-2.3	3.8	4.5	15.4	3.0	3.5	14.5	5.0	5.8	13.4	3.7	4.5	17.8
Switzerland	6.4	6.1	-3.6	5.1	5.0	-3.4	4.7	6.1	23.5	3.7	4.8	22.6	6.1	5.6	-9.1	4.6	4.4	-4.6
United States	3.8	3.5	-8.2	3.1	2.9	-9.4	3.4	3.0	-10.9	2.7	2.4	-13.8	3.9	4.1	4.4	3.2	3.4	4.4
United Kingdom	3.8	5.8	34.1	3.1	4.6	33.0	3.1	4.0	22.0	2.4	3.1	22.8	3.5	4.2	16.5	2.7	3.4	19.4
Average	5.7	6.9	17.2	4.5	5.5	18.0	4.6	6.7	31.4	3.5	5.3	33.7	5.0	6.4	23.0	3.6	5.0	28.1

Table 5 *In-sample fitting performance, by sample periods*

(%) Country	1980s						1990s						2000s					
	RMSE			MAE			RMSE			MAE			RMSE			MAE		
	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.	6F	LC	Imp.
Australia	7.5	9.0	16.9	5.6	6.6	14.8	6.4	7.9	18.8	4.8	5.8	17.5	6.9	8.7	21.0	5.0	6.3	19.5
Austria	8.2	8.5	3.7	5.9	6.1	3.9	9.9	9.8	-0.3	6.8	6.6	-4.0	11.8	10.8	-9.7	8.7	7.1	-21.8
Belgium	7.5	8.6	12.7	5.2	6.3	18.5	8.9	9.4	4.6	6.0	6.5	7.1	10.8	9.7	-11.7	8.2	6.9	-18.7
Bulgaria	8.2	9.1	9.7	5.8	6.9	15.7	8.1	10.1	19.1	5.9	7.5	21.3	9.6	11.9	19.9	6.9	9.0	23.6
Canada	5.5	6.5	15.8	4.1	5.1	18.2	5.4	6.3	14.1	4.1	4.7	13.1	5.4	6.1	10.4	4.1	4.6	11.1
Czech Republic	7.9	11.0	28.5	5.5	8.1	32.8	8.1	8.9	9.1	5.6	6.2	8.8	9.7	9.7	0.2	7.0	6.7	-4.2
Denmark	11.4	13.1	13.2	7.7	9.7	20.7	10.9	12.3	11.5	7.4	8.5	12.2	13.4	13.8	2.5	8.5	9.0	5.1
Finland	10.4	12.6	17.8	6.8	9.1	25.6	11.4	12.5	9.1	7.4	8.6	13.9	13.5	14.5	7.4	9.0	9.9	8.8
France	6.4	5.9	-8.7	5.2	4.4	-18.5	5.9	8.3	29.7	4.7	6.0	21.9	7.4	7.2	-2.6	6.0	5.2	-14.9
Hungary	6.2	9.3	33.4	4.5	7.0	35.9	7.1	11.5	38.2	4.8	8.2	42.0	9.7	18.1	46.6	6.9	12.9	46.6
Ireland	14.1	16.5	14.4	9.6	11.6	17.0	13.5	14.9	9.2	8.8	10.2	13.0	13.5	17.7	23.7	8.9	13.3	32.9
Italy	5.9	8.7	32.7	4.7	6.8	30.6	6.2	9.7	36.3	4.8	6.7	29.3	5.8	6.1	4.7	4.6	4.7	3.4
Japan	5.8	8.3	30.7	4.6	6.6	30.2	5.7	5.6	-2.3	4.7	4.4	-6.5	5.8	12.8	54.7	4.7	10.5	55.1
Netherlands	7.4	8.7	14.5	5.2	6.6	21.1	7.0	7.5	7.1	5.0	5.4	8.5	8.9	8.3	-7.7	6.3	5.8	-10.1
New Zealand	12.3	13.8	10.9	8.5	9.7	12.1	12.4	13.2	6.0	9.0	9.3	3.5	13.7	14.5	5.2	9.5	9.9	4.1
Norway	9.1	11.0	16.5	6.1	7.5	18.4	12.1	13.1	7.2	7.8	8.6	9.1	13.5	13.9	3.0	9.5	9.2	-2.4
Portugal	7.5	9.1	18.2	5.5	6.6	16.7	7.3	11.2	35.3	5.0	8.3	40.4	8.6	10.7	19.5	6.0	7.8	23.1
Slovakia	10.5	13.6	22.8	7.2	10.3	30.3	11.4	13.3	14.6	7.1	8.9	19.8	11.8	14.8	20.2	7.8	11.0	29.1
Spain	5.6	8.5	34.2	4.3	6.3	31.5	6.6	13.9	52.7	5.0	10.2	50.6	6.0	8.0	25.5	4.5	6.3	28.8
Sweden	9.1	9.7	6.6	6.3	6.8	7.6	9.9	10.4	4.5	6.7	6.5	-1.7	11.6	11.4	-1.5	8.2	7.4	-11.5
Switzerland	10.3	11.7	11.9	7.5	7.8	4.6	10.5	13.5	22.0	7.6	9.4	19.6	12.9	13.4	3.4	9.4	8.5	-10.0
United States	4.2	5.4	23.2	3.4	4.2	17.7	3.7	6.1	38.3	3.0	4.2	28.8	3.8	5.6	31.7	3.1	4.5	31.5
United Kingdom	5.0	7.9	36.7	3.9	6.0	34.4	4.2	6.0	30.5	3.1	4.7	34.0	4.9	7.4	34.1	3.7	5.8	35.6
Average	8.1	9.8	18.0	5.8	7.2	19.8	8.4	10.2	18.1	5.9	7.2	18.4	9.5	11.1	14.2	6.8	7.9	14.2

Table 6 RMSE improvement ratios, by forecasting horizon and country

Country	Forecasting Horizons				
	1 year	3 years	5 years	10 years	15 years
Panel A: Male Samples (%)					
Australia	24.0	23.5	18.9	16.5	9.7
Austria	1.0	4.7	7.6	9.2	9.5
Belgium	5.7	10.4	11.8	15.0	12.6
Bulgaria	11.5	12.5	10.9	3.2	5.7
Canada	27.2	25.8	19.5	12.3	12.2
Czech Republic	8.4	14.3	16.5	13.1	7.9
Denmark	12.8	17.5	18.3	13.0	5.6
Finland	8.1	9.8	10.5	11.6	6.8
France	22.1	25.7	22.6	10.1	4.6
Hungary	38.5	32.6	23.2	8.2	8.6
Ireland	15.2	16.6	12.4	10.2	10.3
Italy	41.1	36.3	29.2	14.9	12.0
Japan	10.3	17.9	18.1	10.5	5.9
Netherlands	17.9	20.5	19.4	16.4	12.6
New Zealand	12.4	17.4	16.6	14.4	10.5
Norway	10.4	8.5	6.1	5.8	5.0
Portugal	33.1	32.4	23.9	5.5	3.8
Slovakia	12.3	13.1	16.0	14.5	7.1
Spain	49.7	38.7	24.5	-2.1	-9.9
Sweden	8.1	10.0	8.0	4.8	6.7
Switzerland	22.2	23.6	21.3	9.9	-2.5
United States	33.0	19.1	9.8	-3.8	-0.4
United Kingdom	31.9	33.0	28.1	16.7	12.9
Average	19.9	20.2	17.1	10.0	6.8

Country	Forecasting Horizons				
	1 year	3 years	5 years	10 years	15 years
Panel B: Female Samples (%)					
Australia	5.2	9.4	11.6	15.2	11.8
Austria	-0.8	-0.1	1.1	3.0	5.8
Belgium	-5.2	-4.3	-2.8	5.6	5.7
Bulgaria	9.1	10.8	11.2	3.8	4.3
Canada	5.5	5.8	4.2	9.8	12.4
Czech Republic	2.0	4.8	2.7	4.9	4.8
Denmark	7.1	10.8	12.2	13.2	14.4
Finland	3.1	5.1	5.2	3.7	3.0
France	-0.1	7.6	7.4	2.8	0.5
Hungary	23.4	28.3	26.9	18.7	12.5
Ireland	6.9	5.9	9.4	10.4	9.1
Italy	17.9	18.8	14.9	5.6	7.4
Japan	17.7	23.7	21.4	13.8	8.1
Netherlands	-6.7	-2.6	-2.2	2.0	-0.8
New Zealand	6.5	8.4	11.2	14.5	16.0
Norway	3.4	0.8	1.4	0.8	1.3
Portugal	8.8	9.4	10.4	7.8	11.0
Slovakia	2.5	4.3	3.7	5.9	4.0
Spain	31.0	24.8	18.0	5.4	8.8
Sweden	-0.4	-1.4	1.6	1.9	4.0
Switzerland	2.7	7.2	9.1	3.5	-1.1
United States	17.9	17.2	12.6	10.1	9.5
United Kingdom	15.6	22.9	26.1	23.0	15.5
Average	7.5	9.5	9.4	8.1	7.3

Table 7 MAE improvement ratios, by forecasting horizon and country

Country	Forecasting Horizons				
	1 year	3 years	5 years	10 years	15 years
Panel A: Male Samples (%)					
Australia	22.1	23.3	19.9	16.4	11.2
Austria	1.1	5.7	8.5	10.2	10.0
Belgium	6.7	14.6	14.7	15.2	11.6
Bulgaria	11.2	12.7	11.6	6.1	9.1
Canada	29.7	30.1	23.4	14.5	13.5
Czech Republic	9.1	17.3	17.9	14.7	9.4
Denmark	13.8	19.4	20.0	12.1	3.1
Finland	12.6	15.9	16.0	13.4	8.5
France	14.3	22.1	21.8	10.4	9.2
Hungary	35.6	33.7	25.4	14.2	14.0
Ireland	16.5	17.6	12.5	10.6	10.5
Italy	35.7	34.5	27.9	17.0	15.1
Japan	9.5	18.0	18.1	11.6	6.8
Netherlands	23.4	26.1	22.9	16.4	12.3
New Zealand	10.6	15.9	16.9	15.8	12.1
Norway	11.6	10.1	7.2	5.8	4.1
Portugal	27.4	27.3	21.3	10.4	7.2
Slovakia	13.5	15.8	18.9	15.7	8.7
Spain	42.8	35.3	23.9	-3.6	-3.9
Sweden	5.0	9.1	7.7	5.3	6.8
Switzerland	14.6	17.9	16.8	8.4	3.9
United States	29.6	22.1	16.5	7.3	5.8
United Kingdom	34.4	35.4	30.0	19.4	15.9
Average	22.1	23.3	19.9	16.4	11.2

Country	Forecasting Horizons				
	1 year	3 years	5 years	10 years	15 years
Panel B: Female Samples (%)					
Australia	3.7	8.2	12.2	14.6	12.5
Austria	-4.2	-0.1	3.7	5.3	6.8
Belgium	-9.0	-5.4	-2.6	5.3	3.7
Bulgaria	10.5	11.4	12.7	4.7	5.6
Canada	6.5	9.3	8.7	13.7	16.8
Czech Republic	2.9	6.7	6.9	6.0	4.7
Denmark	6.0	13.5	14.7	12.0	10.4
Finland	8.5	9.4	10.3	6.5	4.0
France	-10.8	-1.8	0.4	-0.4	-1.6
Hungary	26.9	31.8	31.6	25.8	18.9
Ireland	9.5	9.4	11.3	12.2	10.2
Italy	14.1	16.9	15.4	12.1	14.2
Japan	17.3	26.8	27.9	19.4	12.0
Netherlands	-10.2	-5.1	-2.0	0.2	-2.7
New Zealand	4.7	7.5	9.8	14.7	15.6
Norway	1.8	1.3	0.9	-1.3	1.4
Portugal	8.3	8.3	10.5	11.8	15.2
Slovakia	3.2	5.7	7.8	9.8	7.3
Spain	29.2	25.9	20.7	7.8	12.0
Sweden	-6.3	-4.5	0.4	4.0	8.4
Switzerland	-4.1	0.5	2.3	3.6	-4.3
United States	17.8	18.1	13.9	13.7	11.3
United Kingdom	14.6	21.5	23.5	19.3	12.5
Average	6.1	9.4	10.5	9.3	8.5

Table 8 Panel Diebold-Mariano (DM) statistics^a

Loss differential: $z_{x,t} = [e_{x,t}^{6F}]^2 - [e_{x,t}^{LC}]^2$		
Age	Male ^b	Female ^b
20	-0.87	3.60
25	-6.74 **	-3.86 **
30	-12.21 **	-4.53 **
35	-10.49 **	-4.67 **
40	-8.30 **	-3.89 **
45	-8.78 **	-3.47 **
50	-7.34 **	1.06
55	-8.33 **	-2.50 **
60	-11.61 **	-6.37 **
65	-8.30 **	-4.16 **
70	-0.93	2.23
75	-5.08 **	-2.15 *
80	-8.95 **	-6.80 **
85	-3.70 **	1.35
90	0.16	4.35
95	-3.06 **	-5.65 **
100	13.17	15.09

Notes:

^a DM statistic with a negative value indicates that the six-factor model is favored.

^b ** indicates significance at the 1% level and * indicates significance at the 5% level; the respective 1% and 5% critical values are -2.326 and -1.645.

國科會補助計畫衍生研發成果推廣資料表

日期:2012/10/21

國科會補助計畫	計畫名稱: 保險死亡率期限結構之共同因子分析與配適
	計畫主持人: 郭維裕
	計畫編號: 99-2410-H-004-069-MY2 學門領域: 財務
無研發成果推廣資料	

99 年度專題研究計畫研究成果彙整表

計畫主持人：郭維裕		計畫編號：99-2410-H-004-069-MY2				計畫名稱：保險死亡率期限結構之共同因子分析與配適	
成果項目		量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）	
		實際已達成數（被接受或已發表）	預期總達成數（含實際已達成數）	本計畫實際貢獻百分比			
國內	論文著作	期刊論文	0	1	100%	篇	
		研究報告/技術報告	1	1	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力 （本國籍）	碩士生	0	0	100%	人次	
		博士生	1	1	100%		
		博士後研究員	1	1	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		章/本
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力 （外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

<p>其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)</p>	本研究計畫目前並無其他成果
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	成果項目	量化	名稱或內容性質簡述
科 教 處 計 畫 加 填 項 目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與(閱聽)人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

達成目標

未達成目標（請說明，以 100 字為限）

實驗失敗

因故實驗中斷

其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文： 已發表 未發表之文稿 撰寫中 無

專利： 已獲得 申請中 無

技轉： 已技轉 洽談中 無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

本研究計畫最後決定採用非常複雜的六因子 Nelson-Siegel 模型針對世界二十幾個國家的死亡率期限結構進行預測，結果發現：樣本內配適以及樣本外預測結果都相當不錯。死亡率一直都是保險公司的重要風險因子之一，是否能準確的估計甚至預測死亡率的動態行為對保險公司提列準備金的多寡有舉足輕重的影響，也因此本研究所建議的預測模型未來將有助於保險公司準確得預測未來的死亡率，以便更有效率地提列準備金，同時增進其收益率。除了實務的應用價值外，本研究成果亦具備相當程度的學術價值。本研究首度提出將傳統的利率期限結構預測模型應用於預測保險契約死亡率的期限結構上，這是一個創新的想法。未來應可嘗試將兩個領域進行更密切的結合，而有助於保險相關研究領域的發展。