行政院國家科學委員會專題研究計畫 成果報告

重隨機假設下之動態違約相關性描述
研究成果報告(精簡版)

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中 華 民 國 101 年 01 月 31 日

中 文 摘 要 ： 此研究計畫報告中,我們的目標在提出一能夠描述信用資產間動態違約相關性的模型,以利評價信用衍生商品.我們假設資產的違約機率是由一總體因子及一個別因子所驅動的動態過程.藉由假設總體因子為一卜瓦松分配,個別因子為一混合卜瓦松分配,我們可以以資產的歷史違約機率資料,或者信用衍生性商品的市場價格,來推估模型中的未知參數.我們提出的模型是 Hull and White (2008)模型的一個衍生.另外在校正模型參數的過程中,我們可以得到一個市場隱含的跳躍程度,這個隱含跳躍程度可以視為資產違約相關性的替代變數,並且可以用來做為市場造市,監控交易或是風險控管等用途.

中文關鍵詞： 擔保債務憑證,動態違約相關性,混合卜瓦松跳躍過程,動態存活機率

英 文 摘 要 ： In this report, we attempt to provide a model which can describe the dynamic nature of default correlation. We assume that default probability of asset follows a dynamic process which driven by a systematic and a idiosyncratic factors. Assuming that the systematic factor follows a Poisson process, and the idiosyncratic factor follows a Poisson mixture distribution, we can calibrate the unknown parameters from historical default probabilities or market prices of CDO. Our model is an extension of Hull and White (2008). Moreover, the implied jump size from calibration to market data can be considered as a measure of default correlation, and can be used for market making, trades monitoring, or risk management purpose.

英文關鍵詞： collateralized debt obligations,dynamic default correaltions, compound poisson jump process, dynamic survival probability

1. Introduction

In this report we attempt to provide a model which characterizes the dynamic nature of default correlations for a portfolio of credit entities. We assume that the sources of defaults are exogenously driven. Yet in contrast to the reduced-form approach (e.g. Duffie and Singleton (1999)) where defaults are defined solely as fatal bankruptcies of non-repetitive nature, here we allow them to be non-fatal repetitive credit events that are in direct accordance with the definitions of rating agencies such as Moody's and Fitch Ratings. By applying this model to the pricing and calibration of synthetic CDO (Collateralized Debts Obligations) tranches, we demonstrate the practicality of such characterization, and we show that our model is a generalized extension of Hull and White (2008).

Since the introduction of Li (2000) in applying the copulae functions to describe the default correlations for a pool of credit entities, the subsequent factor-copulae modeling approaches of Laurent and Gregory (2003), Andersen et al. (2003) and Hull and White (2004) have established themselves of industry standards for the pricing of multi-name credit derivatives. Yet these models are in fact static descriptions of default correlations.

Existing literature on the dynamic description of default correlations is extremely scarce. Totouom and Armstrong (2007) first attempted to model such dynamic nature by employing the time-dependent Archimedean Copulae. Their model possesses an exchangeable property where the correlation between two assets remains intact as the size of the reference pool expands. Empirical study of Das, Duffie, Kapadia and Saita (2007) suggests that the default probabilities of US firms during 1970 to 2004 are evidently under-estimated when observable macro-economic common factors are considered as the sole courses of defaults. Their findings suggest that default correlations generated by systematic factors based on macroeconomic conditions do not suffice in explaining the clustering effect of defaults . A default event of any credit entity can induces an increase in the default probability of others. The worsening credit quality of a single-name often results in a chain reaction of credit-deterioration among other credit enitities in a reference pool.

Along this line of thinking, Giesecke (2003) and in particular Hull and White (2008) that this research is most associated with, consider default events as non-fatal credit events which can take place simultaneously. While Poisson arrivals describe the occurrence of credit events, inhomogeneous jump sizes characterize the extent of detriments that the associated credit events are to the reference pool. Together, they dynamically determine the survival probabilities of the pool. A decline in the survival probability of the pool implicates a rise of its default probability and thus reflects an increase in the pool's cumulative expected loss. In such way this model provides a description of default correlation via a mechanism that dynamically generates survival probabilities. On the pricing CDO traches, the model is capable of incorporating the changes of market information in the dynamic generation of the underlying portfolio's loss distribution at any time point.

2. Model setting and assumptions

In contrast to Hull and White (2008), in the report we relax the assumption that there is only a single exogenous source in driving the accumulative hazard rate process to allow for a factor which generates idiosyncratic shocks. Moreover, whileHull and White (2008) assumes the default intensity to be a deterministic constant, which in turn neglects the possible effects of default contagion, our model provides an autonomous and random setting for the default intensities λ_1 and λ_2, and we allow them to be inter-temporally inter-acting. Such setting would reflect the reciprocal effect of the default intensities of among credit events when they are inter-temporally co-dependent

2.1 Survival process

Let $S(t)$ represent the cumulative process which generates survival probabilities at any time point t. And let $X(t) = -\ln S(t)$ such that X follows a jump process as follows:

$$d(-\ln S(t)) = dX(t) = \mu dt + dq_1 + dq_2 \tag{1}$$

where $P(dq_1 = H_1) = \lambda_1 dt$ and $P(dq_2 = H_2) = \lambda_2 dt$, H_1, H_2, λ_1, λ_2, $m > 0$. q_1 and q_2 are two independent non-homogeneous jump process driven by idiosyncratic factor and systematic factor separately. H_1 and H_2 are the non-homogeneous jump which reflect the extent of damage the occurred systemamtic/idiosyncratic credit evets are to the survival probability of the reference pool. In this research, we refer them as the damaging extents of credit events. In other words, H_1 is the damaging extent of an idiosyncratic credit event while H_2 is that of a systematic credit event.

From (1), if J_1 idiosyncratic jumps and J_2 systematic jumps occur, the dynamic cumulative survival probability of the reference pool can be expressed as the following formula:

$$S(t|J_1, J_2) = \exp\left(-M(t) - \sum_{j=1}^{J_1} H_{1j} - \sum_{j=1}^{J_2} H_{2j}\right) \tag{2}$$

where $M(t) = \int_0^t \mu(\tau) d\tau$ denotes the integration value of drift term against time.

J_1 represents the number of occurrence of idiosyncratic credit events, and its density is driven by λ_1, the frequency of the idiosyncratic credit event; J_2 represents the number of occurrence of systematic credit events, which is driven by the systematic credit event frequency λ_2. The increase in the credit event frequency λ_1, λ_2, the damage extent H_1, H_2, or integration of drift term $M(t)$, would all give rise to a decline in the dynamic survival probability of the reference pool. According to (2), the unconditional cumulative survival probability should be:

$$S(t)=\sum_{J_1,J_2} S\left(t\,|\,J_1,J_2\right)P\left(J_1,J_2,t\right) \tag{3}$$

2.2 Jump probability:

We assume that idiosyncratic credit events follows a mixed Poisson process, where λ_1 follows a two-parameter Gamma distribution denoted as $\Gamma(\alpha_1,\beta_1)$. For systematic credit events, we assume that its frequency λ_2 is a constant. Hull and White (2008) assume that the frequency of credit event is a constant λ_C, and the occurring number of credit events follows a Poisson process. The setting of taking the frequency as a constant makes the occurring number of credit events uncorrelated between each asset within non-overlapping time periods. Thus the model cannot describe the existence of the correlation of credit event probabilities between earlier and later stage appropriately.

Similar to Ruohonen (1987), the mixed Poisson for idiosyncratic credit events can be simplified as a negative binomial distribution. Thus, the probability density, which is the convolution of a negative binomial and a Poisson distribution, of total occurrence of idiosyncratic and system credit events is as follows:

$$P\left(J,t\right)=\sum_{k=0}^{J}\frac{\Gamma\left(k+\alpha_1\right)}{\Gamma\left(\alpha_1\right)k!}\left(\frac{\beta_1}{t+\beta_1}\right)^{\alpha_1}\left(\frac{t}{t+\beta_1}\right)^{k}\frac{\left(\lambda_2 t\right)^{J-k}e^{-\lambda_2 t}}{\left(J-k\right)!} \tag{4}$$

where $J=J_1+J_2$

2.3 Other assumption of distributions for idiosyncratic credit events frequencies

The two-parameter Gamma distribution for λ_1 is not the only choice. We can consider several distributions for λ_1. First, note that the distribution of idiosyncratic credit events can be regarded as a mixture of Poisson process, if the jump size is constant and $H=H_1=H_2$. We can obtain the moment generating function of the mixture by conditioning. For example, Villa and Escobar (2006) calculate the Gamma-Poisson mixture and show that it is a negative binomial distribution. The moment generating function of the Poisson distribution conditional on λ_1 is:

$$M_{J_1|\lambda_1}\left(u\right)=\exp\left\{\lambda_1\left[\exp(uH)-1\right]\right\},\quad u\in R \tag{5}$$

and we can obtain the moment generating function of the mixture of Poisson process by:

$$M_{J_1}(u)=E\left[M_{J_1|\lambda_1}(u)\right]=E\left[\exp\left\{\lambda_1\left[\exp(uH)-1\right]\right\}\right]=M_{\lambda_1}\left(\exp(uH)-1\right) \quad (6)$$

Next, the total jumps of idiosyncratic and systematic credit events can be viewed as an additive of two independent Poisson process. We can derive its moment generating function by multiplying two moment generating functions of idiosyncratic and systematic jumps:

$$M_J(u)=E\left(e^{uJ}\right)=E\left(e^{u(J_1+J_2)}\right)=E\left(e^{uJ_1}\right)E\left(e^{uJ_2}\right)=M_{J_1}(u)M_{J_2}(u) \quad (7)$$

where $J=J_1+J_2$ is the total occurrence of idiosyncratic and system credit events.

Consider the original case when idiosyncratic events follow the Gamma-Poisson mixture and systematic events follow the Poisson process with constant frequency. The moment generating functions of total occurrence is:

$$M_J(u)=M_{J_1}(u)M_{J_2}(u)=M_{\lambda_1}\left(\exp(uH)-1\right)M_{J_2}(u)$$

$$=\left(1-\frac{1}{\beta_1}\left(\exp(uH)-1\right)\right)^{-\alpha}\exp\left(\lambda_2\left(\exp(u)-1\right)\right)=M_{\lambda}\left(\exp(uH)-1\right) \quad (8)$$

where $\lambda=\lambda_1+\lambda_2$, and $M_\lambda(u)=M_{\lambda_1}(u)M_{\lambda_2}(u)$

From moment generating function, the expectation, variance and the third moment for the number of jumps occurring during one year can be derived as:

$$E(J)=\left(\frac{\alpha_1}{\beta_1}+\lambda_2\right)H \quad (9)$$

$$Var(J)=\left(\frac{\alpha_1}{\beta_1^2}+\frac{\alpha_1}{\beta_1}+\lambda_2\right)H^2 \quad (10)$$

$$E\left(\left(J-E(J)\right)^3\right)=\left(\frac{2\alpha_1}{\beta_1^3}+\frac{3\alpha_1}{\beta_1^2}+\frac{\alpha_1}{\beta_1}+\lambda_2\right)H^3 \quad (11)$$

Also we can derive the expectiation, variance and the third moment for intensity:

$$E(\lambda)=\frac{\alpha_1}{\beta_1}+\lambda_2 \quad (12)$$

$$Var(\lambda)=\frac{\alpha_1}{\beta_1^2} \quad (13)$$

$$E\left(\lambda-E(\lambda)\right)^3=2\frac{\alpha_1}{\beta_1^3} \quad (14)$$

With the same procedure, we can derive the expectation, variance and third moment for intensity of total jumps under different assumption of distributions. We list the results in table I, and will discuss parameters estimation in section 3.

Table I: The expectiation, variance and third moment under different assumption

$E[\lambda]$	$V[\lambda]$	$E\left[(\lambda - E[\lambda])^3\right]$
$\lambda_1 \sim \log - N(\mu, \sigma)$, λ_2 is a constant:		
$e^{\mu+\frac{1}{2}\sigma^2} + \lambda_2$	$e^{\sigma^2-1} e^{2\mu+\sigma^2}$	$\left(e^{\sigma^2}-1\right)^2 \left(e^{\sigma^2}+2\right)\left(e^{2\mu+\sigma^2}\right)^{1.5}$
$\lambda_1 \sim IG(\alpha_1, \beta_2)$, λ_2 is a constant:		
$\frac{\beta_1}{\alpha_1+1} + \lambda_2$	$\frac{\beta_1^2}{(\alpha_1-1)^2(\alpha_1-2)}$	$\frac{4\beta_1^3}{(\alpha-3)(\alpha-1)^3(\alpha-2)}$
$\lambda_1 \sim Pareto(\alpha_1, m_1)$, λ_2 is a constant:		
$\frac{\alpha_1 m_1}{(\alpha_1-1)} + \lambda_2$	$\frac{\alpha_1 m_1^2}{(\alpha_1-1)^2(\alpha_1-2)}$	$\frac{2(1+\alpha_1)\alpha_1 m^3}{(\alpha_1-1)^3(\alpha_1-2)(\alpha_1-3)}$

3. Numerical results

For parameters calibrating, we can first consider matching the moments with the historical occurring probability of credit events. Alternatively, we can calibrate all the parameters directly from market CDO tranche prices. We discuss these two methods in the next two sections.

3.1 Parameters estimation from historical downgrades and defaults

By observing the actual number of credit events from a pool of assets, the parameters can be calibrated with the moments of occurrences. In this article, we directly estimate the parameters with the actual occurring probability of credit events. The same method can be applied to other distribution assumptions. First, we derive the expectation, variance and the third moment as section 2.3. Next, by calculating the average $\overline{x_\lambda}$, variance s_λ^2 and the third moment s_λ^3 of actual occurring probability of credit events during a sample period, we can estimate the parameters $\overline{\alpha_1}$, $\overline{\beta_1}$, and $\overline{\lambda_2}$. We use the yearly probability of corporate bonds' downgrades and defaults from 1981 to 2010, reported by Standard & Poor's. We calculate the weighted average, variance and the third moment according to the number of issuers each year and the results are shown in table II:

$$w_i = \frac{N_i}{\sum_i N_i} \tag{21}$$

$$\overline{x_\lambda} = \sum_i w_i \lambda_i = \frac{\overline{\alpha_1}}{\overline{\beta_1}} + \overline{\lambda_2} \tag{22}$$

$$s_\lambda^2 = \sum_i w_i \left(\lambda_i - \overline{\lambda}\right)^2 = \frac{\overline{\alpha_1}}{\overline{\beta_1}^2} \tag{23}$$

$$s_\lambda^3 = \sum_i w_i \left(\lambda_i - \overline{\lambda}\right)^3 = \frac{2\overline{\alpha_1}}{\overline{\beta_1}^3} \tag{24}$$

Table II: Parameters estimation from historical downgrades and defaults data

Statistics summary

	$\overline{x_\lambda}$	s_λ^2	s_λ^3
1981~2007	12.8016%	0.1621%	0.0058%
1981~2010	13.4500%	0.1998%	0.0058%

Parameters estimations under different assumptions of distribution

$\lambda_1 \sim \Gamma(\alpha_1, \beta_2), \lambda_2$ is a consatant

	λ_2	α_1	β_1
1981~2007	0.0245	6.6042	63.8213
1981~2010	-0.0031	9.4846	68.8851

$\lambda_1 \sim \log\text{-}N(\mu, \sigma), \lambda_2$ is a consatant

	λ_2	μ	σ
1981~2007	-0.0305	-1.8729	0.2500
1981~2010	-0.0751	-1.5845	0.2109

$\lambda_1 \sim IG(\alpha_1, \beta_2), \lambda_2$ is a consatant

	λ_2	α_1	β_1
1981~2007	-0.0729	6.3065	30.3927
1981~2010	-0.1347	11.5527	41.9108

$\lambda_1 \sim Pareto(\alpha_1, m_1), \lambda_2$ is a consatant

	λ_2	α_1	m_1
1981~2007	0.0866	-0.4351	0.1367
1981~2010	0.0849	-0.4945	0.1500

Applying the probability data of actual downgrades and defaults for estimating the parameters may be restricted by the assumption that the cumulative survival probability is affected by systematic and idiosyncratic credit events. We may neglect the possible systematic or idiosyncratic factors such as the state of economy. Moreover, when calibrating, we should utilize the information from market prices.

3.2 Parameters estimation from market prices

In this section, we propose a procedure to directly calibrate the parameters from market prices under the assumption that λ_1 follows a two-parameter Gamma distribution. We will first search for a set of $(\mu, \alpha_1, \beta_1, \lambda_2)$ that minimize the square error between model index prices and market's given H. We use iTraxx Europe series 9 monthly fixing prices on Jan. 31th 2011 as an example. The tranche and index prices are presented in table IV. Note that for tranche 0-3% and 3-6%, the prices are the upfronts paid at the beginning with a fixed 500bps spread quarterly. For tranche 6-9%, the prices are the upfronts with a fixed 300bps spread. The prices of 0-3%, 3-6%, and 6-9% are quoted in terms of percentage of notional. For tranche 9-12% and 12-22%, the prices are the upfronts with a fixed 100bps spread and quoted in bps. Our goals for first step is to produce the model index prices matching the market index, which are 70bps for 5 years, 102bps for 7 years and 117bps for 10 years index. For example, we choose 0.05, 0.1, 0.3, 0.5 and 0.7 as initial H and matching the model prices with market index prices. The parameters estimated are given in Table III.

Table III: Parameters estimated from index prices on Jan. 31th 2011

Initial Jump Size	μ	α_1	β_1	λ_2
0.05	0.0095	1.4847	7.5137	0.0266
0.1	0.0063	1.0719	8.5179	0.0259
0.3	0.0049	1.0900	30.4737	0.0246
0.5	0.0043	1.0416	53.6187	0.0217
0.7	0.0043	0.9727	66.4006	0.0174

Next, for each tranche, given the optimal $(\mu, \alpha_1, \beta_1, \lambda_2)$, we can calibrate a set of $H_{tranche}$ to match each tranche prices. These $H_{tranche}$ are the jump sizes implied from market prices. We use 0.5 as initial *H* and show these implied jump sizes in table IV.

Table IV: iTraxx monthly fixing prices on Jan. 31th 2011

	Trache prices	Index Level	Implied jump size
5Y			Jan. 31th 2011
0-3%	22.17%	70	0.0028
3-6%	-3.77%	70	0.2132
6-9%	3.66%	70	0.1153
9-12%	69.13	70	0.1395
12-22%	30.06	70	0.2383

7Y			
0-3%	39.03%	102	0.0295
3-6%	3.19%	102	0.20
6-9%	0.88%	102	0.1376
9-12%	183.5	102	0.1348
12-22%	86	102	0.2310
10Y			
0-3%	49.56%	117	0.0489
3-6%	12.19%	117	0.2226
6-9%	8.13%	117	0.26
9-12%	270.4	117	0.1324
12-22%	121.5	117	0.2145

In summary, we use following procedure to calibrate the parameters:

1. Given *H*, find a set of optimal parameters $(\mu, \alpha_1, \beta_1, \lambda_2)$ to minimize the sum of square errors between model prices and iTraxx index prices for 5, 7 and 10 years.
2. For each tranche i, find $H_{tranche,i}$ to minimize the square error between model prices and iTraxx tranche prices.
3. Sum the square errors for all tranche, and repeat all steps until the sum of the square errors is minimized.

4. Conclusion

This research extends the Hull and White (2008) model under the assumption that the logarithm of survival process is driven by two sources of jumps. Our model introduces an autonomous and random setting for the default intensities, and we allow them to be inter-temporally inter-acting as to reflect the reciprocal effect of the default intensities of among credit events when they are inter-temporally co-dependent

We consider several distributional assumptions for both the systematic and idiosyncratic jump sources, and based on the historical rating histories of underlying credit entities we demonstrate how to calibrate the unknown parameters of this model. We provide detailed descriptions on the calibration procedures to the iTraxx quoted spreads. For each tranche, the implied jump size is retrieved such that the root mean square errors between the market quoted spreads and the model-generated ones are minimized . These implied jump sizes are a measure of default correlations and can be used for market making , trades monitoring, or risk management purposes.

國科會補助計畫衍生研發成果推廣資料表

日期:2012/01/31

國科會補助計畫	計畫名稱: 重隨機假設下之動態違約相關性描述
	計畫主持人: 江彌修
	計畫編號: 99-2410-H-004-066- 學門領域: 財務

無研發成果推廣資料

99 年度專題研究計畫研究成果彙整表

計畫主持人：江彌修			計畫編號：99-2410-H-004-066-				
計畫名稱：重隨機假設下之動態違約相關性描述							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	4	4	100%	人次	
		博士生	3	3	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	本專案計畫目前已積極編稿排版中，預計將發表於國外SSCI期刊

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

   ■達成目標

   □未達成目標（請說明，以 100 字為限）

   　　□實驗失敗

   　　□因故實驗中斷

   　　□其他原因

   說明：

2. 研究成果在學術期刊發表或申請專利等情形：

   論文：□已發表 □未發表之文稿 ■撰寫中 □無

   專利：□已獲得 □申請中 ■無

   技轉：□已技轉 □洽談中 ■無

   其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

   This research extends the Hull and White (2008) model under the assumption that the logarithm of survival process is driven by two sources of jumps. Our model introduces an autonomous and random setting for the default intensities, and we allow them to be inter-temporally inter-acting as to reflect the reciprocal effect of the default intensities of among credit events when they are inter-temporally co-dependent. We consider several distributional assumptions for both the systematic and idiosyncratic jump sources, and based on the historical rating histories of underlying credit entities we demonstrate how to calibrate the unknown parameters of this model. We provide detailed descriptions on the calibration procedures to the iTraxx quoted spreads. For each tranche, the implied jump size is retrieved such that the root mean square errors between the market quoted spreads and the model-generated ones are minimized . These implied jump sizes are a measure of default correlations and can be used for market making , trades monitoring, or risk management purposes.