FRACTAL STRUCTURE IN CURRENCY FUTURES PRICE DYNAMICS

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INTRODUCTION

Financial economists always strive for better understanding of the market dynamics of financial prices and seek improvement in modeling them. Although there have been many studies devoted to analyze high frequency dynamics of financial prices, only recently have low frequency dynamics received attention. An issue of concern is the behavior of financial prices over long versus short horizons—whether the seemingly random movements in financial prices over short horizons contain detectable structures over long horizons. In this study a specific form of dynamics called "fractal" [Mandelbrot (1977a, b)] is explored. A time series having fractal structure is characterized by long-term dependence and nonperiodic cycles.

Several studies have examined the cyclic long-term dependence property of financial prices, including stock prices [Aydogan and Booth

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(1988); Greene and Fielitz (1977)], gold prices [Booth, Kaen, and Koveos (1982a)], exchange rates [Booth, Kaen, and Koveos (1982b)], and commodity and stock index futures [Helms, Kaen, and Rosenman (1984); Milonas, Koveos, and Booth (1985)]. These studies use the classical rescaled range (R/S) analysis, first proposed by Hurst (1951) and later refined by Mandelbrot and Wallis (1969) and Wallis and Matalas (1970), among others. A problem with the classical R/S analysis is that the distribution of its regression-based test statistic is not well defined and the analysis is not robust to possible heteroskedasticity or short-term dependence in data. As a result, Lo (1991) proposed the use of a modified R/S procedure with improved robustness. The modified R/S procedure has been applied to study dynamic behavior of, e.g., stock prices [Lo (1991); Cheung, Lai, and Lai (1994)], gold prices [Cheung and Lai (1993)], and Eurodollar and T-bill futures prices [Lee and Mathur (1992)]. Both the classical and the modified R/S analyses serve to uncover nonperiodic long cycles in the corresponding data, and their test statistics are all derived from some standardized range measure of cumulative price movements—the difference between the largest and the smallest cumulative price movements over a sample period. Without modeling fractal structure explicitly, these R/S analyses provide indirect tests for fractal dynamics.

This study estimates fractional structure directly based on fractional time series models. Specifically, an interesting class of fractional models investigated by Geweke and Porter-Hudak (1983), Granger and Joyeux (1980), Hosking (1981), and Mandelbrot (1977b) is considered. The appeal of this class of fractional models is its ability to capture a wide range of long-term dependence with a single parameter. This parameter is sometimes referred to as the fractional parameter, which is amenable to estimation and statistical hypothesis testing. In the present study, a semi-nonparametric procedure devised by Geweke and Porter-Hudak (1983) is employed to estimate the fractional parameter. This semi-nonparametric procedure is useful because it is not sensitive to short-term dependence, nonnormal innovations, or variance nonstationarity. Monte Carlo results reported by Cheung (1993) support that the Geweke–Porter-Hudak method is robust to short-term dependence as well as variance shifts and conditional heteroskedastic effects. The robustness property with respect to nonstationarity in variance is especially attractive, given that financial prices have generally been found to display such nonstationarity. As noted by Milonas, Koveos, and Booth (1985), analyses of futures price dynamics are particularly prone to the problem of variance nonstationarity. Because of the maturity effect [Samuelson

(1965)], the variance of price changes can change systematically with the length of time to maturity of the futures contract. With the robustness of the Geweke–Porter-Hudak procedure, nonetheless, reliable evidence of fractal structure can be obtained and the problem of variance nonstationarity is minimized.

Uncovering fractal structure in currency futures prices is interesting in various respects. Studies by Diebold and Rudebusch (1989) and Shea (1991) find that economic fundamentals such as national output and interest rates contain fractal structure. It is thus interesting to examine whether fractal structure is also present in financial prices such as currency futures prices, reflecting the fractal dynamics in economic fundamentals. In addition, the relevance of fractal behavior can have further implications for time series modeling. When futures markets exhibit fractal dynamics, proper modeling of the dynamics using fractional processes can potentially improve forecasts over usual time series forecasting models. This is particularly relevant to long-term forecasts. Furthermore, if fractal structure is indeed present in currency futures, statistical inferences concerning currency futures pricing models based on standard testing procedures may not be valid. Moreover, theoretical and empirical models that allow for fractal price dynamics should be explored. Mandelbrot (1971) observes that in the presence of long-term dependence implied by fractal structure, the arrival of new market information cannot be fully arbitraged away and martingale models of asset prices cannot be obtained from arbitrage. Mandelbrot (1971) also shows that variability in the imperfectly arbitraged price may not be stationary and the return distribution is nonnormal.

SOME STUDIES ON NONLINEAR DYNAMICS

Analyses of nonlinear price dependence have enjoyed much attention recently. The apparently growing interest in nonlinear dynamics arises from the observation that the often wide and nonperiodic cyclical fluctuations of asset prices cannot be adequately captured by linear models. Usual findings of leptokurtosis in asset returns may be further indirect evidence of nonlinear dynamics. Scheinkman and LeBaron (1989) observe that conditional heteroskedastic processes can exhibit dependence similar to that of chaotic systems. Empirical evidence of chaotic dynamics or conditional heteroskedastic dependence in stock prices and exchange rates has been presented by Hsieh (1989, 1991), Mayfield and Mizrach (1992), and Scheinkman and LeBaron (1989). In addition, Frank and Stengos (1989) report evidence of conditional heteroskedastic effects and chaotic structures in both gold and silver prices. Blank (1991) and DeCoster, Labys, and Mitchell (1992) also find evidence of chaotic dynamics in futures markets for several commodities and the S&P 500 stock index.

The fractal dynamics examined in this study may be viewed as a specific form of nonlinear dynamics. Mandelbrot and Van Ness (1968, pp. 430–432) have discussed the nonlinearlity in the extrapolative and interpolative forecasts for fractional processes. Mandelbrot and Van Ness note that long samples of fractional series will show variable trends in extrapolation and interpolation [see also Mandelbrot (1977b, pp. 252)].[1] In addition, a series having fractal structure exhibits a special feature that does not occur for linear processes. A fractal series displays long cycles that are not periodic. The long cycles shown in different samples of the same fractal series have different wavelengths.

FRACTIONAL STATISTICAL ANALYSIS

Since Mandelbrot's (1977a, b) discussion of the fractal structure of fractional processes, further interesting properties of fractional processes have been explored by Granger and Joyeux (1980); Hosking (1981); and Sowell (1990). All these studies commonly examine members of a family of fractional processes called fractionally differenced processes. Because of their flexibility and simplicity in specification, these fractional processes are useful for modeling low-frequency dynamics.

Let $\{x_1, x_2, \ldots, x_T\}$ be a set of time series data. A general class of fractional processes is described by

$$B(L)(1 - L)^d x_t = D(L)v_t \tag{1}$$

where $B(L) = 1 - \beta_1 L - \ldots - \beta_p L^p$ and $D(L) = 1 + \delta_1 L + \ldots + \delta_q L^q$ are polynomials in the lag operator L, all roots of $B(L)$ and $D(L)$ are stable, and v_t is a white-noise disturbance term. When $p = 0 = q, x_t$ becomes a fractional noise process [Mandelbrot and Van Ness (1968)]. The fractional parameter, given by d, assumes noninteger values. The variance of the process x_t is finite when $d < 1/2$, but is infinite when $d \geq 1/2$ [Granger and Joyeux (1980)]. The fractional differencing operator, $(1 - L)^d$, implies the presence of infinite-order lag dependence with slowly declining weights since, through an approximating series

[1]Mandelbrot (1977b) and Mandelbrot and Van Ness (1968) discuss mainly in terms of fractional Brownian motions, which are continuous time variables. Their counterparts for discrete time variables are fractionally differenced processes considered by, e.g., Granger and Joyeux (1980), Hosking (1981), and Sowell (1990).

expansion, one obtains:

$$(1 - L)^d = \sum_{k=0}^{\infty} \Gamma(k - d)L^k/[\Gamma(k + 1)\Gamma(-d)] \qquad (2)$$

where $\Gamma(\cdot)$ is the standard Gamma function. This general fractional model includes the usual autoregressive moving-average (ARMA) model as a special case in which $d = 0$. Such extension to have noninteger values of d increases the flexibility in modeling long-term dynamics by allowing for a richer class of spectral behavior at low frequencies than that implied by ARMA models. This can be seen from the spectral density of x_t. Granger and Joyeux (1980) and Hosking (1981) show that the spectral density function of x_t, denoted by $f_x(\omega)$, is proportional to ω^{-2d} as ω becomes small. Thus, the fractional parameter crucially determines the low-frequency dynamics of the process. For $d > 0$, $f_x(\omega)$ is unbounded at frequency $\omega = 0$, rather than bounded as for ARMA processes with $d = 0$. Indeed, when $d > 0$, x_t is a long-memory process showing a slow autocorrelation decay at a hyperbolical rate [Hosking (1981)].[2] The larger the value of d is, the stronger the long-term dependence will be.

In this study, a spectral procedure developed by Geweke and Porter-Hudak (1983) is employed to estimate the fractional parameter. The Geweke–Porter-Hudak procedure provides a semi-nonparametric test for fractional processes that requires no explicit parameterization of the ARMA dynamics, which are generally not known a priori. Since this *statistical* procedure has not been widely applied in empirical work, a brief discussion of the basic setup of the procedure follows.[3]

The Geweke–Porter-Hudak test makes use of the fact that the low-frequency dynamics of a process are parameterized by the fractional parameter. The spectral density function of x_t is given by

$$f_x(\omega) = |1 - \exp(-i\omega)|^{-2d} f_u(\omega) = (2 \sin(\omega/2))^{-2d} f_u(\omega) \qquad (3)$$

where $u_t = B^{-1}(L)D(L)v_t$ is a stationary process and $f_u(\omega)$ is its spectral density at frequency ω [see, e.g., Hosking (1981)]. Taking logarithms of eq. (3) and evaluating at harmonic frequencies $\omega_j = 2\pi j/T$ $(j = 0, \ldots, T - 1)$, one obtains:

$$\ln(f_x(\omega_j)) = \ln(f_u(0)) - d \ln(4 \sin^2(\omega_j/2)) + \ln(f_u(\omega_j)/f_u(0)) \qquad (4)$$

[2]Hosking (1981) shows that, for $d > 0$, x_t has an autocorrelation function approximately equal to τ^{2d-1} as the lag τ becomes large.

[3]The Geweke and Porter-Hudak procedure has been applied by Diebold and Rudebusch (1989) and Shea (1991) to analyze fractal behavior in national output and interest rates.

For low-frequency ordinates ω_j near zero, the last term is negligible compared with the other terms. Adding $I(\omega_j)$, the periodogram at ordinate j, to both sides of eq. (4) yields

$$\ln(I(\omega_j)) = \ln(f_u(0)) - d\ln(4\sin^2(\omega_j/2)) + \ln(I(\omega_j)/f_x(\omega_j)) \quad (5)$$

This suggests estimating d using a simple spectral regression equation

$$\ln(I(\omega_j)) = \alpha_0 - \alpha_1 \ln(4\sin^2(\omega_j/2)) + \epsilon_t, \qquad j = 1, 2, \ldots, n \quad (6)$$

where ϵ_t, equal to $\ln(I(\omega_j)/f_x(\omega_j))$, is asymptotically i.i.d. across harmonic frequencies. The periodogram $I(\omega_j)$ is computed as the product of $2/T$ and the square of the exact finite Fourier transform of the series $\{x_1, x_2, \ldots, x_T\}$ at the respective harmonic ordinate. The number of low-frequency ordinates used for the regression n is an increasing function of T. Geweke and Porter-Hudak (1983) show that, for $n = T^{\mu}$ with $0 < \mu < 1$, the least squares estimate of α_1 provides a consistent estimate of d and hypothesis testing concerning the value of d can be based on the t-statistic of the slope coefficient. The theoretical asymptotic variance of ϵ_t is known to be equal to $\pi^2/6$, which can be imposed in estimation to raise efficiency.

DATA AND EMPIRICAL RESULTS

The data considered in this study are major currency futures prices for the British pound (BP), German mark (DM), Japanese yen (JY), and Swiss franc (SF). Each series consists of daily observations from January 4, 1982, through December 31, 1991, drawn from various issues of the *International Money Market Yearbook*. Currency futures contracts are traded for delivery at a fixed maturity data, namely the third Wednesday of March, June, September, or December. To increase the number of observations, daily settlement prices of the futures contracts closest to its maturity date are examined. The nearby futures price series thus constructed have 2527 observations for each currency. The use of a long data set is desirable since long-term dependence is of interest here. Two standard unit-root tests, the augmented Dickey–Fuller test and the Phillips–Perron test, are performed on all futures price series.[4] The results are summarized in Table I. They indicate that in no case can the null hypothesis of a unit root be rejected at any usual significance

[4]The two unit-root tests are asymptotically equivalent and differ only in terms of the way short-term dependence is adjusted. The Dickey–Fuller test uses autoregressive lags; whereas, the Phillips–Perron test relies on nonparametric adjustment derived from spectral analysis.

TABLE I
Tests for Unit-Root Nonstationarity

Unit-Root Test	*Currency Futures*			
	BP	*DM*	*JY*	*SF*
Level Series:				
ADF(2)	−2.6059	−2.1567	−1.7230	−2.2130
ADF(4)	−2.5404	−2.1289	−1.7109	−2.1376
ADF(6)	−2.4568	−2.0973	−1.7603	−2.0987
PP(2)	−2.6503	−2.1915	−1.7603	−2.2869
PP(4)	−2.6412	−2.1910	−1.7695	−2.2825
PP(6)	−2.6387	−2.1925	−1.7872	−2.2824
Differenced Series:				
ADF(2)	−29.7944	−28.6630	−28.2474	−28.9661
ADF(4)	−23.1936	−22.8409	−21.4645	−22.8114
ADF(6)	−19.1047	−18.5100	−17.8451	−18.9440
PP(2)	−49.5806	−52.8841	−60.6404	−51.7895
PP(4)	−49.5790	−52.8727	−50.6405	−51.7941
PP(6)	−49.5814	−52.8644	−50.6529	−51.7937

Notes: All series are in logarithms. The ADF (p) statistic gives the test statistic for the augmented Dickey–Fuller test with a lag parameter equal to p, and the PP(q) statistic gives the test statistic for the Phillips–Perron test with a lag parameter equal to q. Both tests examine the null hypothesis of a unit root in the considered series against the stationary alternative of no unit root. The null hypothesis will be rejected in favor of the stationary alternative when the test statistic is too small. A linear time trend is allowed in either test, and in no case is the time trend variable found to be statistically significant. Asymptotic critical values for both tests are given by −3.12 (10%) and −3.41 (5%).
For all the differenced series, the null hypothesis of unit-root nonstationarity can be rejected at the 5% significance level.

level. To remove the unit-root nonstationarity, which can affect proper statistical inferences, each data series is transformed into a return series by taking first differences in logarithms of the futures prices.

Some preliminary data analysis of the series of futures price changes is carried out, with particular attention paid to evidence concerning deviations from normality, autocorrelations, and conditional heteroskedasticity. Some descriptive statistics of the sample distribution of currency returns are provided in Table II. Specifically, the skewness and excess kurtosis of the return distributions are computed; the values are reported together with their corresponding p-values indicating the statistical significance. For a normal distribution, both skewness and excess kurtosis measures should be equal to zero. The results suggest the presence of significant departures from normality in all the currency futures series under examination. Note that the excess kurtosis coefficients are all positive, suggesting that the currency return distributions have a much flatter tail than the normal distribution. The significant deviations from normality can be a symptom of nonlinear dynamics.

TABLE II
Descriptive Statistics for Futures Price Changes

Statistics	*Currency Futures*			
	BP	*DM*	*JY*	*SF*
Maximum	0.0455	0.0519	0.0533	0.0554
Minimum	−0.0452	−0.0522	−0.0413	−0.0369
Median	0.0000	0.0000	−0.0001	0.0000
Mean	−0.0000	0.0001	0.0002	0.0001
Standard deviation	0.0077	0.0078	0.0069	0.0084
t-test for mean = 0	−0.1191	0.9422	1.5644	0.6109
[p-value]	[0.9052]	[0.3461]	[0.1177]	[0.5413]
Skewness	0.1967	0.2480	0.3797	0.3127
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0000][a]
Excess kurtosis	3.2655	4.5733	3.6117	2.0481
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0000][a]

Notes: The skewness is computed as the third sample moment standardized by the cube of the standard deviation. The excess kurtosis is the fourth sample moment divided by the square of the variance minus three. For a normal distribution both coefficients should be equal to zero. The numbers in brackets give the p-values of the respective test statistics.
[a]Significant at the 5% level.

Further analysis is conducted by fitting to the currency futures return data an autoregressive (AR(p)) model given by

$$x_t = c_0 + \sum_{j=1}^{p} c_p x_{t-j} + u_t \tag{7}$$

In this analysis, the lag parameter p is first determined using a model selection procedure based on the Schwarz information criterion. However, short-term dependence seems not significant in the data since a model with $p = 0$ is selected for all currency futures. For illustration purposes, nonetheless, an AR(3) model is fitted to the return series. The squared residual series is then tested for possible autoregressive conditional heteroskedastic (ARCH) effects using the standard TR^2 test. The test can be implemented as follows: Estimate an AR(r) process in terms of the squared residuals. The TR^2 statistic is computed as the product of the number of effective observations and the coefficient of multiple determination from the AR(r) regression. In addition, White's (1980) test for general heteroskedasticity is conducted. This test is derived from the method-of-moments approach. Unlike the TR^2 test for ARCH effects, the White (1980) test does not assume any specific structure of heteroskedasticity.

Table III contains the results concerning short-term dependence and heteroskedasticity. The Ljung–Box statistics suggest that the residu-

TABLE III
Short-Term Dependence and Heteroskedasticity in Futures Returns

Regressors and Statistics	*Currency Futures*			
	BP	*DM*	*JY*	*SF*
Constant	−0.0001	0.0001	0.0001	0.0000
	(−0.1009)	(0.9809)	(1.5831)	(0.6722)
x_{t-1}	0.0145	−0.0500	−0.0084	−0.0298
	(0.5624)	(−1.6105)	(−0.3561)	(−1.4188)
x_{t-2}	−0.0098	0.0036	0.0092	−0.0003
	(−0.4096)	(0.1580)	(0.4059)	(−0.0136)
x_{t-3}	−0.0242	0.0265	0.0226	0.0115
	(−1.1005)	(1.2789)	(0.9075)	(0.5379)
LB(6)	3.5164	5.5192	5.9481	3.7100
[p-value]	[0.7418]	[0.4791]	[0.4290]	[0.7158]
LB(12)	7.1815	8.0176	10.9020	5.4714
[p-value]	[0.8453]	[0.7838]	[0.5373]	[0.9404]
LB(18)	18.1037	18.0545	18.1394	17.1101
[p-value]	[0.4488]	[0.4521]	[0.4465]	[0.5155]
ARCH(6)	104.0446	133.1321	49.0242	47.2872
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0000][a]
ARCH(12)	127.6033	148.4920	64.8316	63.8797
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0000][a]
ARCH(18)	150.5968	152.9151	68.4037	83.7463
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0000][a]
White's Test	73.2506	161.4374	45.6626	16.2514
[p-value]	[0.0000][a]	[0.0000][a]	[0.0000][a]	[0.0927][b]

Notes: An AR(3) model is estimated for individual currency futures return series (x_t). The figures in parentheses are t-statistics, computed using standard errors obtained from White's (1980) heteroskedasticity-consistent covariance matrix estimator. The asymptotic distribution of the Ljung–Box statistic, LB(s), is $\chi^2(s)$ under the null hypothesis of no serial correlation in the residual. The ARCH(r) statistic, obtained as TR^2 from regressing the squared residual on a constant and its lagged values at r lags, is distributed asymptotically as $\chi^2(r)$ under the null hypothesis of no ARCH effects. The White (1980) test provides statistics for a TR^2 test for general heteroskedasticity. The numbers in brackets give the corresponding p-values of the LB, the ARCH, and the White test statistics.
[a]Significant at the 5% level.
[b]Significant at the 10% level.

als from all four regressions show no significant serial correlation, therefore supporting the adequacy of the respective autoregressive lag specification. The reported t-statistics for the autoregressions are computed from White's (1980) heteroskedasticity-consistent covariance matrix estimator. This allows proper inferences about the autoregressions to be made even in the presence of heteroskedasticity. As shown by the t-statistics, there are no significant autocorrelations in all futures return series. On the other hand, the ARCH test statistics strongly indicate the presence of substantial ARCH effects in all the four return series. The results from the White (1980) test also confirm the presence of significant heteroskedasticity in all the series under consideration.

The results in Table III suggest, in general, the presence of heteroskedasticity, though not short-term dependence, in the currency futures return data. In view of the results, it is desirable that a test for fractional processes should properly account for heteroskedasticity in the data; otherwise, reliable statistical inferences cannot be drawn. In this regard, these results provide support for the use of the Geweke–Porter-Hudak procedure given its robustness to variance nonstationarity.

In applying the Geweke–Porter-Hudak procedure, the number of low-frequency ordinates, n, used in the spectral regression is a choice variable. The choice necessarily involves judgment. Although a too large value of n will cause contamination of the d estimate due to medium- or high-frequency components, a too small value of n will lead to imprecise estimates due to limited degrees of freedom in estimation. To balance these two factors of consideration, a range of μ values is used for the sample size function, $n = T^{\mu}$. The results reported below are for $\mu = 0.60$, 0.65, and 0.70. This set of choice of μ values yields good test performance in the experiment.

Table IV contains the estimates for the fractional parameter from the Geweke–Porter-Hudak spectral regression. The d estimates are provided together with their t-statistics based on both empirical error variance estimates and the known theoretical error variance ($\pi^2/6$). In general, the d estimates appear robust with respect to the choice of the low-frequency ordinates. Moreover, these d estimates are positive for all the four currency return series considered, with the British pound series having the smallest fractional parameter estimate. The t-statistics are used to perform formal tests of the null hypothesis of a nonfractional process ($d = 0$) against the alternative of a long-memory fractional process ($d > 0$). The results indicate that, except for the British pound series, there is statistically significant evidence that the currency futures return series are well described by a long-memory fractional process. Qualitatively similar results can be obtained, independent of whether the t-statistics are computed using empirical error variance estimates or using the theoretical error variance. These results, on the whole, suggest that currency futures price dynamics contain fractal structure with long-term dependence.

CONCLUDING REMARKS

This study examines the relevance of fractal dynamics in major currency futures markets. Fractal dynamics are an interesting form of dynamics

TABLE IV
Estimates for the Fractional Parameter d

Low-Frequency Ordinates Used	*Currency Futures*			
	BP	*DM*	*JY*	*SF*
$n = T^{0.60}$	0.0318	0.0821	0.1225	0.1262
	(0.5242)	(1.3379)[a]	(1.8915)[b]	(2.2005)[b]
	{0.4825}	{1.2456}	{1.8577}[b]	{1.9148}[b]
$n = T^{0.65}$	0.0227	0.0839	0.1273	0.1313
	(0.4612)	(1.6333)[a]	(2.3630)[b]	(2.4424)[b]
	{0.4241}	{1.6713}[b]	{2.3830}[b]	{2.4578}[b]
$n = T^{0.70}$	0.0389	0.0683	0.1231	0.0763
	(0.9492)	(1.6560)[b]	(2.5347)[b]	(1.7534)[b]
	{0.8966}	{1.5749}[a]	{2.8384}[b]	{1.7584}[b]

Notes: The number of low-frequency ordinates included in the Geweke–Porter-Hudak spectral regression is given by $n = T^{\mu}$, where $T = 2{,}526$ is the sample size of the currency futures return series. The figures in ( ) and { } are the t-statistics for the d parameter computed based on empirical error variance estimates and the known theoretical error variance ($\pi^2/6$), respectively. The null hypothesis of a nonfractional process ($d = 0$) is tested against the long-memory alternative of a fractional process ($d > 0$).
[a]Significant at the 10% level.
[b]Significant at the 5% level.

characterized by irregular cyclical fluctuations and long-term dependence. This study contributes to the current literature on financial price dynamics by estimating directly the fractal structure in currency futures prices based on a time series model of fractional processes. A semi-nonparametric spectral method is employed to estimate the fractional model. Statistically significant evidence of fractal structure is found in three out of four currency futures return series considered.

The results call into question the adequacy of usual linear models, such as martingale models, to describe the behavior of currency futures prices, and it invites the development of pricing models that can account for fractal dynamics. Empirical problems can also arise in testing futures pricing models because standard statistical methods may not be appropriate in the presence of fractal dependence [see, e.g., Sowell (1990) and Yajima (1985)]. Moreover, the finding of fractal structure reopens the debate on pricing efficiency and market rationality.

Since the presence of fractal dynamics can have important theoretical and empirical implications, future research needs to examine the relevance of fractal dynamics in many other financial markets. Although the Geweke and Porter-Hudak procedure used to uncover fractal structure is not sensitive to variance nonstationarity, including nonlinear conditional heteroskedastic dependence, little is known about the possible effects of chaotic dynamics on the statistical procedure.

Future research on investigating the robustness of the Geweke and Porter-Hudak procedure to chaotic dynamics may be warranted.

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