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# Statistical Analysis with Soft Computation for Fuzzy Answering in Sampling Survey 

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Statistical Analysis with Soft Computation for Fuzzy Answering in Sampling Survey

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#### Abstract

In this paper we will provide the definition of fuzzy mode, related characteristics, and


 proofs. By discussing these characteristics, data analysis, and explanation for this issue, we apply the soft computing techniques on human thought measurement and public's idea with a more appropriate way. In the conclusion, future experiments and further suggestions are presented.Key words: fuzzy statistics, membership function, fuzzy answering, marketing survey,

1. Introduction. Fuzzy theory development since 1965 has gradually from the mathematical theory and engineering applications, extended to the statistical methodology of the social science

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community. From the study early, Verma (1997) suggesting fuzzy logic to construct offender profiles since the police officers received descriptions of suspects that were fuzzy in nature. Chen and Wang (1999) offered fuzzy statistical testing method to discuss the stability of Taiwan short-term money demand function. And according to Wu and Sun (2001), demonstrated the concepts of fuzzy statistic and applied it to social survey; Wu and Tseng (2002) showed fuzzy regression method of coefficient estimation to analyze Taiwan monitoring index of economic. In addition, Wu and Tseng (2002) applied fuzzy regression models to business cycle analysis. Besides, Ohdar and Ray (2002) proposed the fuzzy-based approach to measure and evaluate the performance of suppliers in the supply chain. Furthermore, Wu and Chang (2007) proposed new approaches on market research with fuzzy statistical analysis. Further, Metaxiotis, Psarras and Sanouilidis (2003) integrated fuzzy logic into a decision support system. Moreover, Wu and Hsu (2004) identified the model construction through qualitative simulation. What is more, Zhang, Li and Tam (2004) proposed the fuzzy discrete-event simulation to model the uncertain activity duration; Nguyen and Wu (2006) for an extensive treatment of the theory of fuzzy statistics. And Jeng,, Watada, Wu and Wu (2006) applied fuzzy forecasting techniques in DNA computing; Ho and Wu (2008) used integrated fuzzy statistics analysis in the valuation of intellectual capital. In addition, Ordoobadi (2008) used fuzzy logic to evaluate advanced technologies for decision

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makers and provided a model based on fuzzy logic for decision makers to help them with selection of appropriate suppliers in 2009. furthermore, Chen and Niou (2011) , Yeh (2011) Fuzzy relative weights of the analysis of fuzzy numbers, these studies are to obtain good results. There are more and more researches focus on the fuzzy statistical analysis and applications in the social science fields.
2. The human mind the measure and information presentation. Human thinking that is mainly from the subjective awareness of the natural and social phenomena. However, human thought and knowledge of language because of their own subjective consciousness, time, environment and judged things from a different angle with fuzziness. Generation of fuzzy theory, fuzzy measure is used in reference to the human way of thinking on the environment and classification principles. It's give a more robust description of the way to deal with the diverse and complex, ambiguous and uncertain phenomenon.
2.1. Fuzziness of human thinking. Human perception is fuzzy. For example, why was the definition of "cold"? How many degrees the temperature range is called the "cold"? Each answer is subjective. Even if I the same person, also because of the different seasons, the environment different from the previous scoping. Whether a person's feelings or mood can be used (1) or bad

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(0) entirely separate from it? Because human beings in the most natural state, the answer is almost are fuzzy. Therefore, we can already appreciate the need for fuzzy.
2.2. Membership functions. The membership function is foundation by fuzzy theory. It is derived from the characteristic function of the traditional set. The expression elements of fuzzy set membership grade, the range of 0-1. In the fuzzy set theory, Zadeh (1965) mentioned that, if an element belongs to a certain sets of the greater degree of its membership values closer to 1 , and vice versa is close to 0 .

The membership function of fuzzy theory the most basic concept. It can be divided into discretization and continuous of two types of membership function. Discretization of membership function is directly given the degree of membership of each element of the finite fuzzy sets, and the form of vector performance. Continuous membership function of several commonly used functions in the form [S-function Z-function - a function, triangular function, trapezoidal functions, Gaussian (index) function] to describe the fuzzy sets. The performance of the function definition is unlimited collection of fuzzy relationship between the element and its membership, the relationship between the limited elements of fuzzy set and its membership. About the various membership function types, refer to Zimmermann(2001, 344-355)。

## 3. How to compute sample mode for fuzzy data

In social science research, we want to know the ideas of the public about certain issues, and often will make use of the voting or sampling survey to assess. How the performance of the traditional

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voting behavior? (1) very satisfied (2) satisfied (3) ordinary (4) not satisfied (5)very dissatisfied. When the answer is a continuous way of the questionnaire is the first qualified ruler distance, and the respondents choose. For example, "What is your ideal salary?", "How long do you sleep?", " What is the marriageable age to you?" etc. Without the use of fuzzy theory, are not reasonable and accurate described in

### 3.1 Fuzzy mode with discrete data

Tradition statistics deals single answer or certain range of the answer through sampling survey, and unable to sufficiently reflect the complex thought of an individual. If people can use the membership function to express the degree of their feelings based on their own choices, the answer presented will be closer to real human thinking. Therefore, to collect the information based on the fuzzy mode should be the first step to take. Since a lot of times, the information itself embedded with uncertainty and ambiguity. It is nature for us to propose the fuzzy statistics, such as fuzzy mode and fuzzy median, to fit the modern requirement.

Traditional mode statistically refers to the views of the majority, but the view factor of the domain because each person is usually not clear. Example: Tomorrow, the stock market predict "More likely to rise, but may also be dropped" or the appearance describe "A little beauty, but not too beautiful", and this is very common. Fuzzy concept of statistical consensus is very important. Under that we were discrete and continuous two types of fuzzy mode do a study. Discrete fuzzy mode is defined as follows.

In this and next section we demonstrate the definitions for fuzzy mode and fuzzy median generalized from the traditional statistics. The discrete case is simpler than the continuous one's.

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## Definition 3.1 Fuzzy Mode (data with multiple values)

Let $U$ be the universal set (a discussion domain), $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of $k$-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2 \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$. For each sample $F S_{i}$, assign a linguistic variable $L_{j}$ a normalized membership $m_{i j}\left(\sum_{j=1}^{k} m_{i j}=1\right)$, let $S_{j}=\sum_{i=1}^{n} m_{i j}, j=1,2, \cdots, k$. Then, the maximum value of $S_{j}$ (with respect to $L_{j}$ ) is called the
fuzzy mode (FM) of this sample. That is $F M=\left\{L_{j} \mid S_{j}=\max _{1 \leq i \leq k} S_{i}\right\}$.

Note. A significant level $\alpha$ for fuzzy mode can be defined as follows: Let $U$ be the universe set (a discussion domain), $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of k-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2 \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$. For each sample $F S_{i}$, assign a linguistic variable $L_{j}$ a normalized membership $m_{i j}\left(\sum_{j=1}^{k} m_{i j}=1\right)$, let $S_{j}=\sum_{i=1}^{n} I_{i j}, j=1,2, \cdots, k$ $I_{i j}=1$ if $m_{i j} \geq \alpha, I_{i j}=0$ if $m_{i j}<\alpha, \alpha$ is the significant level. Then, the maximum value of $s_{j}$ (with respect to $L_{j}$ ) is called the fuzzy mode ( $F M$ ) of this sample. That is $F M=\left\{L_{j} \mid S_{j}=\max _{1 \leq i \leq k} S_{i}\right\} . \quad$ If there are more than two sets of $L_{j}$ that reach the conditions, we call that the fuzzy sample has multiple common agreement.

From the above definition, we could tell that there might be more than one fuzzy mode. Under this situation, we could conclude that this subject owns fuzzy modes or it has more than one common opinion.

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Example 3.1 Suppose eight voters are asked to choose a chairman from four candidates. Table 3.1 is the result from the votes with two different types of voting: traditional response and fuzzy response.

From left part of Table 3.1, we can find that three are there people choose B. Hence the mode
is B. But if we examine the right part of Table 2.2 for fuzzy response, we find that $B$ only gets the total memberships 2.1. Which is less than $\mathrm{C}=3.5$, the fuzzy mode. Hence we can see that the fuzzy response will illustrate people's thought more faithfully.

### 3.2 Continuous fuzzy mode.

There's Continuous fuzzy mode complex than the discrete fuzzy mode. In this study, only discuss the membership function was evenly distributed and single-peak symmetric two cases. Traditional mode discussion of continuous data, usually the data are equidistant distinguish. And then respondent chose it. Such as: "What is your monthly income? (1)less than 20,000 (2)20,000 to $40,000(3) 40,000$ to $60,000(4) 60,000$ to $80,000(5)$ more than 80,000 ". That's respondents to select the interval, is the traditional mode. However, when the respondents answer across the above two options, or less than 20,000 and more than 80,000 , we cannot determine why the real meaning of the respondents. We try to make the respondents were random fill out an interval, the use of fuzzy mode, expect to find a more reasonable explanation. We will be continuous and fuzzy public was evenly distributed and the number is defined as follows.

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## Definition 3.3 fuzzy mode (data with continuous membership)

Let $U$ be the universe set (a discussion domain), and $\left\{F S_{i}, i=1,2 \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$. For each sample $F S_{i}$ with its membership function $\left.\left\{\mu_{i}(X) \mid \int_{U} \mu_{i}(x) d x\right)=1\right\}$. Suppose we equally separated the total range of universe doman $U$ into $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$. Given the significance level $\alpha$, let $S_{j}=\sum_{i=1}^{n} L_{L_{j}} \mu_{i}(x) d x, j=1,2, \cdots, k$. Then, the subject or linguistic value of $L_{0}$ with respect to maximum value of $S_{j}$ is called the fuzzy mode (FM) of this sample. That is $F M=\left\{L_{0} \mid S_{0}=\max _{1 \leq j \leq k} S_{j}\right\}$. If there are more than two sets of $L_{0}$ reach the conditions, we call that the fuzzy sample has multiple common agreement.

Note that for each sample $F S_{i}$, if its sum of membership function is not equal to 1 , we have to make a normal transformation such that the sum of normalized memberships become 1, i.e $\sum_{j=1}^{k} m_{i j} x$

Set up $U$ for a domain, for the group n samples in the domain of $U$ and single peak symmetry. Let $\alpha$ be a significant membership standards, $a_{j}$ and $b_{j}$ are two root and $\mu_{j}^{-1}(\alpha)$, $b_{j}>a_{j}, j=1, \ldots, n$. We will $U$ divided into $k, k$, as practice and experience(Figure, 3.1), fuzzy mode as $F M=\left\{\mathrm{L}_{\mathrm{i}}=\left[P_{i}, P_{i+1}\right] \mid \forall j \neq i, A_{i}>A_{j}, i, j=1,2, \ldots, k\right\}$ Among $\left.A_{i}=\sum_{j=1}^{n} \int_{\max }^{\min \left(P_{i+1}, a_{j}\right)} \boldsymbol{b}_{j}\right)\left(\mu_{j}(X)-\alpha\right) d \mu(X)$, $i=1, \ldots, k$

$$
\approx \sum_{j=1}^{n}\left(\min \left(P_{i+1}, b_{j}\right)-\max \left(P_{i}, a_{j}\right) \times\left(\sup \left\{\mu_{j,\left[P_{i}, P_{i+1}\right]}(X)\right\}-\alpha\right), \quad i=1, \ldots, k\right.
$$

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3.3 Fuzzy characteristics of the mode. Statistical parameters can be used to express the characteristics of the population, but some properties it is difficult by conventional statistical parameter(Such as: expected value, median, mode) $\circ$ Especially in social science research, the need to express the views of the public, the traditional statistical parameters will be more not applicable. The paper, we consider the uncertainty of human thinking, based on fuzzy set theory, fuzzy definition of plural analysis, expectations for human thought and better expression.

## Properties Related with the Fuzzy Mode

Though features of a population can be illustrated by certain statistical parameters, there still many characteristics which are difficult to be measured based on traditional parameters such as expectation, medium, and mode. Especially, when we are going to investigate the public opinions in the social science research, traditional parameters seems not enough for application. Therefore, we consider the uncertainty of human thinking and present the definitions and analysis of fuzzy mode based on fuzzy theory so that the human thinking can be more precisely measured.

The following properties aim to illustrate the useful applications for the presented definition of fuzzy mode and discuss some valuable properties. We will also compare these two types of modes with the traditional ones.

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## Property 3.1

Let $U$ be the universe set, $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of $k$-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2, \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$ (for data with multiple values). If the maximum membership $m_{i j}$ in each $F S_{i}$ is larger than the significant level $\alpha$ and located at $L_{j}$, then the fuzzy mode is consist with the traditional mode.

Proof: If the maximum membership $m_{i j}$ in each sample $F S_{i}$ is located on $L_{j}$, then the traditional mode is $L_{j}$. On the other hand, since the maximum membership given by each fuzzy sample $F S_{i}$ is larger than the significant level $\alpha$ and located at $L_{j}$. According to Definition 3.1,

$$
F M=\left\{L_{j} \mid S_{j}=\max _{1 \leq \leq \leq} S_{i}\right\}
$$

Thus, $L_{j}$ is also the fuzzy mode for the fuzzy sample $\left\{F S_{i}, i=1,2, \cdots, n\right\}$.

## Property 3.2

Let $U$ be the universe set (a discussion domain), $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of $k$-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2, \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$ (for data with multiple values). If there exist a membership $m_{i j}$ in each $F S_{i}$ with values $m_{i j}>0.5$, then fuzzy mode consists with the traditional mode for any significant level $\alpha \geq 0.5$.

Proof: Since $m_{i j}\left(\sum_{j=1}^{k} m_{i j}=1\right)$ is the membership of fuzzy sample $F S_{i}$. If there exists exact one

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membership $m_{i j}$ of each fuzzy sample $F S_{i}$ is larger than 0.5 , the rest $m_{i k}, k \neq j$ must be smaller than 0.5 . Under the significant level $\alpha \geq 0.5$. The fuzzy sample data will reduce to the traditional sample data type after the following transformation

$$
I_{i j}=\left\{\begin{array}{lll}
1 & \text { if } & m_{i j} \geq \alpha \\
0 & \text { if } & m_{i j}<\alpha
\end{array} .\right.
$$

Hence the fuzzy mode will consist with the traditional mode.

## Property 3.3

Let $U$ be the universe set, $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of $k$-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2, \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$ (for data with multiple values). If there are samples whose maximum membership falls on two or more of $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ values. Then we can not compute the traditional mode without discarding these samples. However, by choosing an appropriate significant level $\alpha$, we can compute it by

$$
S_{j}=\sum_{i=1}^{n} I_{i j} \text { and } F M=\left\{L_{j} \mid S_{j}=\max _{1 \leq i \leq k} S_{i}\right\}
$$

to get the fuzzy mode.

Note: When the sample has the same membership of preference with two or more options, the binary logic (0 or 1 ) cannot express the information so we cannot find the mode. If we apply the fuzzy logic concept, the degree of preference can be easily expressed and have the fuzzy mode according to the definition 3.2.

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## Property 3.4

We can adjust the values of $S_{j}=\sum_{i=1}^{n} I_{i j}$ and then change the location of fuzzy mode by choosing the different significant level $\alpha$.

Proof: For $L=\left\{L_{1}, L_{2}, \cdots, L_{k}\right\}$ be a set of $k$-linguistic variables on $U$, and $\left\{F S_{i}, i=1,2, \cdots, n\right\}$ be a sequence of random fuzzy sample on $U$ (for data with multiple values). For each fuzzy sample $F S_{i}$, change the different significant level $\alpha$ will result in different values of $S_{j}=\sum_{i=1}^{n} I_{i j}$.

And finally it will influence the decision of $F M$.
Note: If the significant level $\alpha$ is chosen too large, the value of fuzzy mode will be low. If we lower the significant level $\alpha$, the value of fuzzy mode will increase. So, the degree of significant level $\alpha$ is an essential factor that influenced information of fuzzy mode. The past experience can help us to choose an appropriate significant level $\alpha$ according to the human thought or social utility.

## 4. Empirical Studies

### 4.1 The favorite exercise location in the college

In order to find those are the favorite exercise location for National Chi Nan University. We select 10 students as a sampling survey to find the favorite sports in the college. Six competitive items $A=$ Athletic Field, $B=$ Stadium, $C=$ Outdoor Basketball Court, $D=$ Swimming Pool, $E=$ Golf

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course are displayed.

The experimenters are expected to answer the question with three kinds of answering: only one choice, ordering and membership for the questionnaire. The result is illustrated on Table 4.1. From Table 4.1 we can see the traditional mode for favorite exercise location is $\mathrm{C}=$ outdoor basketball courts (5 votes). In terms of ordering answer is still the Stadium highest. While based on to membership answering, we can find that $A=$ Athletic Field has the maximum membership. (Membership function 4.3). Therefore, based on the alternative answer, the Stadium will be no votes may be mistaken for non-favorite of sports facilities to produce false positives in field of health.

### 4.2 The degree of satisfaction for physical education curriculum

In order to know those are the degree of satisfactions for favorite exercise location for National Chi Nan University. We select 10 students as a sampling survey. Table 4.2 shows: it's each received five votes by ordinary and satisfaction. In the membership function answer (fuzzy mode): Satisfaction (4.4) the highest score, followed by very satisfied (1.9), and then ordinary (1.7) very dissatisfied (0). The result shows that the showing in the traditional mode number of votes, no identification is the degree of student satisfaction. It is revealing the accurate data

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presentation that fuzzy mode. Showing the physical education curriculum is still a considerable degree of satisfaction of students.

### 4.3 Consensus for a public criterion

Traditional public opinion surveys usually perform the one person with one vote policy to select the object they want to vote. Often many people have difficulty to give the reply when two of the choice have same feeling tendency. There might be $A$ and $B$ at the same time people wish to vote instead of choosing a single one. Hence, we use statistics: fuzzy mode, to find the common agreement. Since unification and independence topic on Taiwan and China is the most popular debate topic, we randomly ask 10 congressmen in Taiwan about their view of Taiwan's current political tendencies.

We designed two kinds of questionnaires, traditional and fuzzy of the respondents to answer. In the fuzzy survey: We ask the respondents according to their own degree of reunification and independence between different decimal numbers between 0-1, to give a degree of membership, while a traditional questionnaire ask the respondents to determine tick one option, see Table 4.3.

From Table 4.3, we can see that item (4), "independence", is the mode for the traditional sampling survey. While if we let the voters to record their answer via membership function, we

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can see that the fuzzy mode will be item 3, Maintain the Status Quo, since it has the maximum sum of membership, which is 3.5 .

### 4.4 Admission fee for Sun Moon Lake

Tourism Bureau at Nan-Tou wants to investigate up a reasonable admission fee for the
famous sight-seeing place Sun Moon Lake. They randomly selected eight village chiefs from

Nan-Tou County to ask them, how much do they feel the admission fee of Sun Moon Lake is?

The respondents' answer are recorded with the interval data. Since the total range of the sample is $0-100$, we equally divide the range into 5 parts: below $20,20-40,40-60,60-80,80-100$, units is

U\$ dollars. Table 4.4 shows the result of sampling survey as well as the statistics. 4.5

## Penalty Compensating for Da-Du river pollution

A community at Taiwan wants to set up the penalty compensating for Ta-Du river pollution.

They ask 12 impartial individuals to evaluate the amount of the penalty. The respondents' answers are recorded with fuzzy data via various type: the interval type, triangle or trapezoid types. The triangle data is demonstrated as the form [a=left point of the triangle, $b=$ center point, $\mathrm{c}=$ the right point of the triangle $]$. The trapezoid data is demonstrated as the form [a=left point of the triangle, $b=l e f t$ center point, $\mathrm{c}=$ the right center point the right point of the triangle, $d=$ right

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point]. Since the range runs over from 0 to 200 (U\$ Thousand), we separate the range into seven parts. Table 4.5 illustrates the result of the surveys.

From table 4.5 we can see the fuzzy mode for penalty for a local river pollution is $9 \sim 12$.
5. Conclusions. In this study, the fuzzy questionnaire is to overcome the aforementioned two questions, the results of this questionnaire is not only reasonable and clearly expressed information presented message.

This research presents the definition and the application of the fuzzy modes and hopes to more closely express the human thinking. From the analysis of the experiments, we can apply the analysis of the fuzzy mode to get the fuzzy information which is more reasonable than the traditional modes. Since the fuzzy modes uses more reasonable way to find the public consensus, it is really important for us to know the fuzzy mode. For the educational and psychological research of the fuzzy mode, we can have the following conclusions:

1. Fuzzy modes present the public consensus of the target group. When we try to use linguistic to evaluate or vote for public consensus, fuzzy modes can be always applied for the possible results. In the areas of education and psychology, fuzzy modes can be applied to the assessment, evaluation of text, and design of lecture hours. Also, no matter the evaluation or decision making, it follows certain standards and processes to make the decision. The category and degree of the standard follows the public consensus. Fuzzy modes can also get

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the degree of that standard.
2. The fuzzy modes presented by this research can also be applied to the employment of teachers, the evaluation of the class, and other educational experiments.

The further research will include the following suggestions:

1. Either a discrete fuzzy data or a continuous fuzzy data, its operation and the analysis of fuzzy statistical process is very complex. If we can develop computer software for the user, it will help us to the process the empirical studies more efficiently.
2. We can ask for psychological and linguistic experts to further the kind of research: If the action and thought reacts at the same. This is the weakness of this research and can be the reference for further study.
3. The Delphi technique is to look for the consensus of experts and it is always applied by researchers. But, it may be difficult to get the consensus of experts during the process of the research. During this study, we still meet some insurmountable problems. Such as, Continuous fuzzy mode, only to discuss the type of membership function are evenly distributed and single-peak symmetric both. For other functional forms, such as S-functions, Z-function, the trapezoidal function with Gaussian functions, and so, there was no more to explore. In addition, Continuous and fuzzy mode for symmetric single-peak sample in the calculation is too complex. Although, this have study has raised about the assessment formula, but if the application of computer-aided will be precisely. How to select the appropriate level of significance $\alpha$ value, have not been able to raise a convenient and reasonable criteria. In the basic of descriptive statistics, they can still fuzzy expectations, fuzzy median and fuzzy variance, and depth study. The basic descriptive statistics, they can still further study of the

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fuzzy expected value, the fuzzy median and fuzzy variance. This is study inadequate to provide future researchers.

## REGFERENCES

[1] A. Verma (1997), Construction of offender profiles using fuzzy logic, An International Journal of Police Strategies \& Management, vol.20, no.2, pp.408-418.
[2] B, Wu, and C. Sun (2001), Interval-valued statistics, fuzzy logic, and their use in computational semantics. Journal of Intelligent and Fuzzy Systems, vol.11, pp.1-7.
[3] B. Wu and K. Chang (2007), On testing hypothesis o fuzzy mean. Journal of Industrial and Applied Mathematics, vol. 24, no.2, pp.171-183.
[4] B. Wu and N. Tseng (2002), A new approach to fuzzy regression models with application to business cycle analysis. Fuzzy Sets and System, vol.130, pp.33-42.
[5] B. Wu and Y-Y. Hsu (2004), A New approach of bivariate fuzzy time series: with applications to the stock index forecasting, International Journal of Uncertainty, Fuzziness and Knowledge-based Systems, vol.11, no.6, pp.671-690.
[6] B. Wu and Y-Y. Hsu (2004), The use of kernel set and sample memberships in the identification of nonlinear time series. Soft Computing Journal, vol.8, pp.207-216.

## ACCEPTED MANUSCRIPT

[7] C. T. Yeh (2011), Weighted semi-trapezoidal approximations of fuzzy numbers. Fuzzy Sets and Systems, vol.165, pp.61-80.
[8] D. J-F. Jeng, J. Watada, B. Wu and J-Y. Wu (2006), Fuzzy Forecasting with DNA Computing, Lecture Notes in Computer Science, vol.4287, pp.324-336.
[9] H. -J. Zimmermann (2001), Fuzzy Sets and Its Applications. USA: Kluwer Academic Publisher.
[10] H. T. Nguyen and B. Wu (2006), Fundamentals of statistics with fuzzy data, Springer-Verlag: Heidelberg.
[11] H. Zhang, H. Li, and C. M. Tam (2004), Fuzzy discrete-event simulation for modeling uncertain activity duration. Engineering, Construction and Architectural Management, vol.11, no.6, pp.426-437.
[12] K. Metaxiotis, J. Psarras and E. Sanouilidis (2003), Integrating fuzzy logic into decision support systems: current research and future prospects, Information Management \& Computer Security, vol.11, no.2, pp.53-59.
[13] L. A. Zadeh (1965), Fuzzy Sets. Information and Control, vol.8, pp.338-353.
[14] S. M. Ordoobadi (2008), Fuzzy logic and evaluation of advanced technologies. Industrial Management \& Data Systems, vol.108, no.7, pp.928-946.

## ACCEPTED MANUSCRIPT

[15] S._R. Ohdar, and P. K. Ray (2002), A Fuzzy Logic and Genetic Algorithm Based Supplier

Performance Evaluation Methodology for an Effective. The Second International

Conference on Electronic Business Taipei, Taiwan, December, pp.10-13.
[16] S-M. Chen and S-J. Niou (2011), Fuzzy multiple attributes group decision-making based on fuzzy induced OWA operators, Expert Systems with Applications, vol.38, 4097-4108.
[17] S-M. Ho, and B. Wu (2008), Evaluating Intellectual Capital with Integrated Fuzzy Statistical Analysis : A Case Study on the CD POB, International journal of information and management, vol.19, no.1-2, pp.21-40.
[18] T. Chen, and M. Wang (1999), Forecasting methods using fuzzy concepts, Fuzzy Sets and System, vol.105, pp.339-352.

Table 3.1 Response comparison for the eight voters

| Candidate Voter | traditional response |  |  |  | fuzzy response |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | A | B | C | D |
| 1 |  | $\checkmark$ |  |  |  | 0.7 | 0.3 |  |
| 2 | $\checkmark$ |  |  |  | 0.5 |  | 0.4 | 0.1 |
| 3 |  |  |  | $\checkmark$ |  |  | 0.3 | 0.7 |
| 4 |  |  | $\checkmark$ |  | 0.4 |  | 0.6 |  |
| 5 |  | $\checkmark$ |  |  |  | 0.6 | 0.4 |  |
| 6 |  |  |  | $\checkmark$ | 0.4 |  | 0.4 | 0.6 |
| 7 |  | $\checkmark$ |  |  |  | 0.8 | 0.2 |  |
| 8 |  |  | $\checkmark$ |  |  |  | 0.8 | 0.2 |
| Total | 1 | 3 | 2 | 2 | 1.3 | 2.1 | 3.5 | 1.6 |

## ACCEPTED MANUSCRIPT

Table 4.1 favorite sports locations

| voters | Selected one |  |  |  |  | Ordering |  |  |  |  | Memberships Answer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E | A | B | C | D | E | A | B | C | D | E |
| 1 | $\checkmark$ |  |  |  |  | 5 | 1 | 2 | 3 | 4 |  | 0.5 | 0.2 | 0.2 | 0.1 |
| 2 |  |  | $\checkmark$ |  |  | 3 | 1 | 5 | 2 | 4 | 0.1 | 0.5 |  | 0.4 |  |
| 3 |  |  | $\checkmark$ |  |  | 4 | 2 | 5 | 1 | 3 | 0.2 | 0.2 | 0.1 | 0.3 | 0.2 |
| 4 |  |  |  | $\checkmark$ |  | 3 | 1 | 4 | 5 | 2 | 0.1 | 0.4 | 0.1 | 0.1 | 0.3 |
| 5 |  |  |  | $\checkmark$ |  | 3 | 1 | 4 | 5 | 2 | 0.1 | 0.4 | 0.1 | 0.1 | 0.3 |
| 6 |  |  | $\checkmark$ |  |  | 4 | 1 | 5 | 3 | 2 | 0.1 | 0.5 |  | 0.1 | 0.2 |
| 7 |  |  |  | $\checkmark$ |  | 2 | 1 | 4 | 5 | 3 | 0.3 | 0.5 | 0.1 | 0.1 | 0.1 |
| 8 | $\checkmark$ |  |  |  |  | 5 | 1 | 4 | 2 | 3 |  | 0.4 | 0.1 | 0.3 | 0.2 |
| 9 |  |  | $\checkmark$ |  |  | 2 | 1 | 5 | 3 | 4 | 0.2 | 0.4 | 0.1 | 0.2 | 0.1 |
| 10 |  |  | $\checkmark$ |  |  | 2 | 1 | 5 | 3 | 4 | 0.3 | 0.5 |  | 0.2 | 0.1 |
| Total | 2 | 0 | 5 | 3 | 0 | 33 | 11 | 43 | 32 | 31 | 1.4 | 4.3 | 0.8 | 2 | 1.6 |

## ACCEPTED MANUSCRIPT

Table 4.2 The satisfaction of the physical education curriculum

| voters | Traditional mode answer |  |  |  |  | Membership function answer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| 1 |  |  | $\checkmark$ |  |  |  | 0.5 |  | 0.5 |  |
| 2 |  |  |  | $\checkmark$ |  |  |  |  | 0.4 | 0.6 |
| 3 |  |  |  | $\checkmark$ |  |  |  |  | 0.5 | 0.5 |
| 4 |  |  |  | $\checkmark$ |  |  |  | 0.2 |  | 0.8 |
| 5 |  |  |  | $\checkmark$ |  |  |  |  | 1 |  |
| 6 |  |  | $\checkmark$ |  |  |  |  | 1 |  |  |
| 7 |  |  |  | $\checkmark$ |  |  |  | 0.5 | 0.5 |  |
| 8 |  |  | $\checkmark$ |  |  |  | 0.5 |  | 0.5 |  |
| 9 |  |  | $\checkmark$ |  |  |  | 0.5 |  | 0.5 |  |
| 10 |  |  | $\checkmark$ |  |  |  | 0.5 |  | 0.5 |  |
| Total | 0 | 0 | 5 | 5 | 0 | 0 | 2 | 1.7 | 4.4 | 1.9 |

Note: 1= Very Dissatisfied 2= Not Satisfied, 3= Ordinary, 4=Satisfaction, 5= Very Satisfied.

Table 4.3 Assessment unification and independence issues

| voters | Traditional answer |  |  |  |  | Membership function answer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| 1 |  |  |  | $\checkmark$ |  |  |  | 0.4 | 0.6 |  |
| 2 |  |  |  | $\checkmark$ |  |  |  | 0.2 | 0.8 |  |
| 3 |  | $\checkmark$ |  |  |  |  | 0.6 | 0.4 |  |  |
| 4 |  |  | $\checkmark$ |  |  |  | 0.2 | 0.8 |  |  |
| 5 |  |  | $\checkmark$ |  |  |  | 0.3 | 0.7 |  |  |
| 6 |  |  |  | $\checkmark$ |  |  |  |  | 0.8 | 0.2 |
| 7 |  |  |  | $\checkmark$ |  |  |  |  | 1 |  |
| 8 |  |  |  |  | $\checkmark$ |  |  |  |  | 1 |
| 9 |  |  | $\checkmark$ |  |  |  | 0.2 | 0.8 |  |  |
| 10 |  | $\checkmark$ |  |  |  |  | 0.6 | 0.4 |  |  |
| Total | 0 | 2 | 3 | 4 | 1 | 0 | 1.9 | 3.5 | 2.3 | 1.2 |

Note: $\mathbf{P D}=$ Political Tendencies $\mathbf{1 =}$ Radical Reunification, 2=Reunification, 3=Maintain the

Status Quo, 4= Independent, 5= Radical Independent. Item3,

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Table 4.4 Investigating of admission fee for a new open playground

|  | Below 20 | $20-40$ | $40-60$ | $60-80$ | $80-100$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S1=[10,30] | 0.5 | 0.5 |  |  |  |
| S2=[20,30] |  | 1 |  |  |  |
| S3=[15,35] | 0.25 | 0.75 |  |  |  |
| S4=[30,60] |  | 0.33 | 0.66 |  |  |
| S5=[20,80] |  | 0.33 | 0.33 | 0.33 |  |
| S6=[45,95] |  |  | 0.3 | 0.4 | 0.3 |
| S7=[50,90] |  |  | 0.25 | 0.5 | 0.25 |
| S8=[75,100] |  |  |  | 0.2 | 0.8 |
| Total | 0.75 | 2.85 | 1.54 | 1.43 | 1.35 |

From Table 4.4 we can get the fuzzy mode is $20-40$. This is just an experimental example. In the real cases, we can partition our total range into $3,4,5,6,7$ etc. sections dependent on the necessary

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Table 4.5 Penalty of river pollution

|  | $0 \sim 3$ | $3 \sim 6$ | $6 \sim 9$ | $9 \sim 12$ | $12 \sim 15$ | $15 \sim 18$ | $18 \sim 21$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S1=[0,2,4,6] | 0.5 | 0.5 |  |  |  |  |  |
| S2=[5,6,7,9] |  | 0.2 | 0.8 |  |  |  |  |
| S3=[3,6,9,12] |  | 0.3 | 0.6 | 0.3 |  |  |  |
| S4=[6,6,99] |  | 1 |  |  |  |  |  |
| S5=[5,6,9,12] |  |  | 0.08 | 0.44 | 0.44 |  |  |
| S6=[6,6,21,21] |  |  | 0.2 | 0.2 | 0.2 | 0.20 | 0.2 |
| S7=[9,9,12,15] |  |  |  | 0.67 | 0.33 |  |  |
| S8=[9,12,15,18] |  |  |  | 0.3 | 0.6 | 0.3 |  |
| S9=[11,12,13,15] |  |  |  | 0.2 | 0.8 |  |  |
| S10=[9,12,18,21] |  |  |  | 0.17 | 0.33 | 0.33 | 0.17 |
| S11=[12,12,15,18] |  |  |  |  | 0.75 | 0.15 |  |
| S12=[12,15,18,21] | 0.5 | 1 | 2.68 | 2.28 | 3.75 | 1.58 | 0.6 |
| Total |  |  |  |  |  | 0.3 |  |



Figure, 3.1 Continuous and single-peak symmetric fuzzy mode

