國立政治大學統計學系

碩士學位論文

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新的加權平均損失管制圖

A New Weighted Average Loss Control Chart

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ABSTRACT

In recent years, a few researchers had proposed different types of single charts that jointly monitor the process mean and the variation. In this project, we use the weighted average loss (WL) to construct WL control charts for monitoring the process mean and variance simultaneously while the target value may be different from the in-control mean. This statistic WL applied a weighted factor to adjust the weights of the loss due to the square of the deviation of the process mean from the target and the variance change. So the WL charts are more effective than unadjusted loss function charts. We not only construct the fixed parameters (FP) WL chart but also the adaptive WL charts which included variable sampling interval (VSI) WL chart, variable sample size and sampling interval (VSSI) WL chart and variable parameters (VP) WL chart. We calculate the average run length (ARL) for FP WL chart and using Markov chain approach to calculate the average time to signal (ATS) for adaptive WL charts to measure the performance and compare each other. From the comparison, we find the adaptive WL charts are more effective than the FP WL chart. We also proposed the optimal adaptive WL charts using an optimization technique to minimize ATS_1 (ARL_1) when the process was out-of-control. In addition, in order to detect the small shifts of the process mean and variance effectively, we construct the WL charts using the EWMA scheme. The proposed charts are based on only one statistic and are more effective than the $\bar{X}-S$ chart. And the WL charts are easy to understand and apply than using two charts for detecting the mean and variance shifts simultaneously.

Keywords: Statistical process control; Weighted average loss; Adaptive control chart; Markov chain; Optimization technique; EWMA scheme

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CHAPTER 1. INTRODUCTION

In statistical process control, control charts are used to monitor processes and detect the assignable cause(s) which changed the process mean and variance. Shewhart (1924) first developed the control charts to monitor the process. The Shewhart control charts have been widely used for industrial processes ever since. An $\bar{X}$ chart is usually used to monitor the process mean and an S (or R) chart is used to monitor the variation of a process. However Shewhart charts are insensitive in detecting small shifts, in 1954, Page (1954) first introduced the cumulative sum (CUSUM) chart to combat this drawback. Then in 1959 Roberts (1959) introduced the exponentially weighted moving average (EWMA) control chart to detect small shifts in the process mean.

When dealing with a quality characteristic of a variable, it is usually necessary to monitor both the mean value and its variability simultaneously. Two control charts are usually required to monitor the process mean and the process variability respectively. Shewhart's $\bar{X}-S$ or $\bar{X}-R$ control charts are the common choices, in which $\bar{X}$ chart monitors the process mean while S (or R) chart monitors the variation. However, this approach is laborious and time-consuming. Hence, it is advantageous to consider using a single chart based on only one statistic to monitor the process mean and the variation simultaneously.

In recent years, a few researchers had proposed different types of single charts that jointly monitor the process mean and the variation. They are of Shewhart-types, EWMA-types and CUSUM-types. Chen and Cheng (1998) proposed the Max chart which has effectively combined the $\bar{X}$ chart and S chart into one single chart. The main advantage of the Max chart is that one can monitor both the process mean and the process variance by looking at one chart. Chao and Cheng (1996) developed the

Semicircle (SC) chart. This chart combines the detection of the mean shift and the variability change into one chart, and is simple to use and easy to understand. The advantage of the SC chart is that it is easy to attribute an out-of-control signal to the cause of the mean shift or/and the variability change. However, the SC chart is insensitive to small changes within a process. In order to improve the SC chart, Chen *et al*. (2004) used the statistic of the SC chart to construct a new chart, EWMA-SC chart. This chart not only could monitor both the process mean and the process variance simultaneously, but also it is sensitive to small changes in the mean shift or/and the variability. However, the EWMA-SC chart is slow in detecting the most common assignable causes that had shifted the process mean but did not change the process variance. In order to improve this chart's deficiency, Costa and Rahim (2006) introduced a new statistic to construct an EWMA chart that gave better overall performance. Khoo, Wu, Chen and Yeong (2010) proposed a single EWMA $\bar{X}-R$ chart by combining the $\bar{X}$ and R charts which monitors the mean and variance simultaneously and identifies the source of the signal(s).

Loss function is used broadly in industry to measure the cost due to poor quality. Several researchers have developed the loss chart based on the concept of the loss function. From Cyrus (1997), the average loss L is proportional to the sum of squares of the deviations of the quality characteristic from the in-control process mean. Wu and Tian (2006) proposed the weighted loss function chart based on the average weighted loss function which uses a weighted factor to adjust the weight of the loss due to the square of the mean shift and the variance change. This single chart is able to monitor the mean shift and the variance increase simultaneously and is more powerful than the $\bar{X}-S$ chart. However, they assumed that the process mean was target.

The charts mentioned above are monitoring a process with a fixed sampling interval, sample size and control limits. Their effectiveness for detecting process shifts in mean and variance becomes less for today's manufacturing environment. A method to improve the problem is to construct the adaptive control charts. Reynolds *et al.* (1988) and Chengalur *et al.* (1989) first studied the use of $\bar{X}$ charts with variable sampling intervals (VSI) to detect mean shifts. Prabhu *et al.* (1994) studied the propertied of the $\bar{X}$ chart with both the sample sizes and the sampling intervals variable (VSSI). Costa (1999a) proposed a variable parameters (VP) $\bar{X}$ chart which all design parameters were variable, including sample size, sampling intervals and control limit coefficient. Costa and De Magalhaes (2007) investigated the performance of X-bar and R charts with VSSIs. Yang and Su (2007) addressed the adaptive control schemes for monitoring mean and variance in two dependent process steps. Ko and Yeh (2010) studied the effect of VSI economic chart with minimum average loss. Yang and Chen (2010) proposed VSI mean and variance control charts to monitor dependent process steps.

Some researches of using a single chart to monitor the mean and variance simultaneously with the adaptive control schemes had been studied in recent years. Wu, Tian and Zhang (2005) proposed a control chart based on the average weighted loss function (WLF) with VSSI scheme. Zhang and Wu (2006) use the average weighted loss function to construct a new chart with CUSUM scheme and with variable sampling intervals. Wu, Zhang and Wang (2007) proposed a VSSI WLF chart which is a CUSUM chart based on the weighted loss function with VSSI scheme. Yang and Lin (2009) proposed a New VP Loss function chart allowing process mean not to be the target.

In this project, we propose a new chart -- the weighted average loss (WL) chart to monitor the two-sided mean shifts and the variance changes while the target value may be different from the in-control mean. The WL chart's monitoring statistic WL is derived from Taguchi's (1986) loss function. This statistic applied a weighted factor to adjust the weights of the loss due to the square of the deviation of the process mean from the target and the variance change. We described the derivation in chapter 2. The difference of the statistics between the WL chart and Wu's weighted loss function chart is that the in-control mean may not equal to the target in the WL chart but equal to target in Wu's.

The fixed parameters (FP) WL chart monitors a process with fixed sampling interval, sample size and false alarm rate. Based on the WL, we also developed the adaptive charts (VSI WL chart, VSSI WL chart and VP WL chars). The VSI WL chart uses variable sampling intervals. The VSSI WL chart uses variable sampling intervals and sample sizes. The VP WL chart uses variable sampling intervals, sample sizes and false alarm rates. We also proposed the optimal adaptive WL charts using an optimization technique. In addition, in order to detect the small shifts of the process mean and variance effectively, we also construct the WL charts using the EWMA scheme further.

CHAPTER 2. THE DISTRIBUTION OF THE WEIGHTED AVERAGE LOSS

2.1 Taguchi Loss Function and its Expectation and Estimator

Taguchi (1986) claimed that "quality is the loss a product causes to society after being shipped, other than any losses caused by its intrinsic functions". Furthermore Taguchi quantifies the deviations of a product characteristic from the target in terms of monetary units by using a quadratic loss function defined as

$$L(X) = K(X - T)^2 \tag{1}$$

where X is the quality characteristic, T is the target value for X and K is the quality loss coefficient. We can derive the expectation of the loss function to explain the average loss per unit product from equation (1), that is

$$E(L) = E\left[K(X - T)^2\right]$$

$$= K\left[\sigma^2 + (\mu - T)^2\right]$$

where μ and σ^2 are the mean and variance of X.

2.2 The Estimator of the Expectation of the Loss Function and Weighted Average Loss

Since the mean and the variance of a process are usually unknown, so is the average loss function. Hence we have to estimate it and the estimator is proportional to the sum of squares of the deviations of the quality characteristic X from the target, i.e.

$$\hat{E}(L) = \frac{\sum_{i=1}^{n}(X_i - T)^2}{n} = \frac{n-1}{n}S^2 + (\bar{X} - T)^2 . \tag{2}$$

It is clear that the estimator comprises the sample variance and the square deviation of the sample mean from the target.

We now would modify the estimator of the average loss function (eq. (2)) to a weighted average loss (WL), that is

$$WL = aS^2 + (1-a)(\bar{X} - T)^2 \tag{3}$$

where the weighting factor "*a*" ($0 \le a \le 1$) is used to adjust the weight of the loss due to the variance shift and the square of the mean shift from the target. We will use the weighted average loss (WL) to construct the WL control chart. The chart could monitor the mean shift from the process target and the variance change simultaneously.

2.3 The Approximate Distribution of Weighted Average Loss

In order to construct control chart based on WL, we need to find the distribution of the WL statistic. Since we can't get the exact expression of this distribution, we would apply Patnaik's (1949) and Moschopoulos and Canada's (1984) results to get the approximate distribution of WL. Patnaik (1949) illustrated the transformation of a non-central chi-square distribution to a central chi-square distribution. Moschopoulos and Canada (1984) proposed a method to get the approximate cumulative probability function (c.d.f.) for the linear combination of the central chi-square random variables (see Appendix 1).

Assume X follows a normal distribution with mean μ and variance σ^2, that is $X \sim N(\mu, \sigma^2)$. The weighted average loss (WL) (eq. (3)) is

$$WL = aS^2 + (1-a)(\bar{X} - T)^2$$

where $0 \le a \le 1$.

Since

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1} \tag{4}$$

where $\chi^2_{(n-1)}$ is a central chi-square distribution with (n–1) degrees of freedom,

$$\frac{n(\bar{X} - T)^2}{\sigma^2} \sim \chi^2_{1,\tau} \tag{5}$$

where $\chi^2_{1,\tau}$ is a non-central chi-square distribution with 1 degree of freedom and a non-centrality parameter τ, $\tau = \frac{n(\mu - T)^2}{\sigma^2}$.

From equations (4) and (5),

$$S^2 \sim \frac{\sigma^2}{(n-1)} \chi^2_{(n-1)} \tag{6}$$

$$(\bar{X} - T)^2 \sim \frac{\sigma^2}{n} \chi^2_{1,\tau} \tag{7}$$

And then

$$WL = aS^2 + (1-a)(\bar{X} - T)^2 \sim a\frac{\sigma^2}{(n-1)} \chi^2_{(n-1)} + (1-a)\frac{\sigma^2}{n} \chi^2_{1,\tau}$$

which is a linear combination of a central chi-square distribution and a non-central chi-square distribution. However, its exact distribution is untractable. We would try to find its approximate distribution.

The following steps show how to find the approximate c.d.f. of WL:

Step 1. Use Patnaik's method to transform a non-central chi-square distribution into a central chi-square distribution.

Transform equation (5) to a central chi-square distribution using Patnaik's method (1949):

$$\frac{n(\bar{X} - T)^2}{\sigma^2 \rho} = \frac{\chi^2_{1,\tau}}{\rho} \sim \chi^2_{(v)}$$

where $\chi^2_{(v)}$ is a central chi-square distribution with v degree of freedom, and

$\rho = 1 + \frac{\tau}{1+\tau}$ and $v = 1 + \frac{\tau^2}{1+2\tau}$.

The distribution of $(\bar{X} - T)^2$ is derived as

$$(\bar{X} - T)^2 \sim \frac{\sigma^2 \rho}{n} \chi^2_{(v)} \tag{8}$$

From equations (6) and (8), we have

$$WL = aS^2 + (1-a)(\bar{X} - T)^2 \sim a\frac{\sigma^2}{(n-1)} \chi^2_{(n-1)} + (1-a)\frac{\sigma^2 \rho}{n} \chi^2_{(v)}$$

which is a linear combination of central chi-square random variables.

Step 2. Use the method of Moschopoulos and Canada (1984) to obtain the approximate distribution of WL.

The distribution of WL statistic is a linear combination of central chi-squares:

$$WL \sim a\frac{\sigma^2}{(n-1)}\chi^2_{(n-1)} + (1-a)\frac{\sigma^2\rho}{n}\chi^2_{(v)} \tag{9}$$

Let $c_1 = a\frac{\sigma^2}{(n-1)}$, $c_2 = (1-a)\frac{\sigma^2\rho}{n}$, $v_1 = n-1$ and $v_2 = v$.

Using the method of Moschopoulos and Canada (1984) (See Appendix 1), WL is approximate the distribution of Q with parameters n and τ. That is,

$$WL \sim Q_{n,\tau}. \tag{10}$$

The c.d.f. of $Q_{n,\tau}$ (Moschopoulos and Canada (1984)) is

$$F_{n,\tau}(x) = P(WL \le x) = b_2\sum_{j=0}^{\infty} a_j \int_0^w g_j(y)dy \tag{11}$$

where $g_j(y) = \frac{y^{s+j-1}e^{-y/2c_1}}{(2c_1)^{s+j}\Gamma(s+j)}$ is the p.d.f. of Gamma distribution, $\Gamma(s+j, 2c_1)$, with the shape parameter $s+j$ and scale parameter $2c_1$, and $s = m_1 + m_2$,

$b_2 = (\frac{c_1}{c_2})^{m_2}$, $a_j = A(c_2, j)$ with $A(c_i, j) = \frac{(m_i)_j(1-c_1/c_i)^j}{j!}$ for j = 0, 1, 2…, $m_1 = \frac{v_1}{2}$,

$m_2 = \frac{v_2}{2}$ and $(m_i)_r = m_i(m_i+1)\cdots\cdots(m_i+r-1)$.

Now we will give the in-control process and out-of-control process distributions of X, and derive the c.d.f. of the statistic WL.

For the In-control process:

$$X \sim N(\mu_0, \sigma_0^2)$$

Let the process target be *T*, and

$$T - \mu_0 = \delta_3\sigma_0,\ \delta_3 > 0. \tag{12}$$

The relationship of T and μ_0 is shown in figure 1 below:

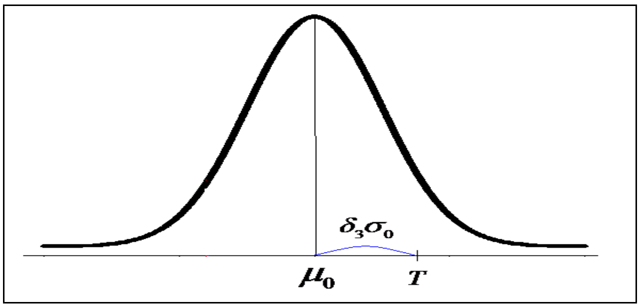

Figure 1. The Relationship between the Target and μ_0

Following (9), we get the distribution of WL below:

$$WL = aS^2 + (1-a)(\bar{X}-T)^2$$
$$\sim a\frac{{\sigma_0}^2}{(n-1)}{\chi^2}_{(n-1)} + (1-a)\frac{{\sigma_0}^2\rho_0}{n}{\chi^2}_{(v_0)} \quad (13)$$

where $\rho_0 = 1+\frac{\tau_0}{1+\tau_0}$, $v_0 = 1+\frac{\tau_0^2}{1+2\tau_0}$ and $\tau_0 = \frac{n(\mu_0 - T)^2}{\sigma_0^2} = n\delta_3^2$.

Hence,

$$WL \sim Q_{n,\tau_0}.$$

The c.d.f. of C_{n,τ_0} is

$$F_{n,\tau_0}(x) = P(WL < x \mid WL \sim Q_{n,\tau_0}) \quad (14)$$

For the out-of-control process:

$$X \sim N(\mu_1, \sigma_1^2)$$

where $\mu_1 = \mu_0 - \delta_1\sigma_0$, $\delta_1 \neq 0$ and $\sigma_1^2 = \delta_2^2\sigma_0^2$, $\delta_2 \neq 1$.

The distribution of WL is then

$$WL = aS^2 + (1-a)(\bar{X}-T)^2$$

$$\sim a\frac{\sigma_1^{\ 2}}{(n-1)}\chi^2_{\ (n-1)}+(1-a)\frac{\sigma_1^{\ 2}\rho_1}{n}\chi^2_{\ (v_1)} \qquad (15)$$

where $\rho_1=1+\frac{\tau^*}{1+\tau^*}$, $v_1=1+\frac{\tau^{*2}}{1+2\tau^*}$ and $\tau^*=\frac{n(\mu_0-\delta_1\sigma_0-T)^2}{\delta_2^2\sigma_0^2}=n\left(\frac{\delta_1+\delta_3}{\delta_2}\right)^2$.

Hence,

$$WL \mathrel{\dot\sim} Q^*_{n,\tau^*}. \qquad (16)$$

The c.d.f. of Q^*_{n,τ^*} is

$$F^*_{n,\tau^*}(x)=P(WL<x\,|\,WL \mathrel{\dot\sim} Q^*_{n,\tau^*}) \qquad (17)$$

CHAPTER 3. DESIGN AND ARL ANALYSIS OF THE FP, VSI, VSSI AND VP WL CHARTS

3.1 Design of the FP WL Chart

The FP (fixed parameters) WL chart uses fixed sampling interval h_0, sample size n_0 and false alarm rate α_0. From (14), the upper control limit (UCL) and lower control limit (LCL) of the FP WL chart are:

$$UCL = F^{-1}_{n_0,\tau_0}(1-\frac{\alpha_0}{2}),$$

$$LCL = F^{-1}_{n_0,\tau_0}(\frac{\alpha_0}{2}).$$

The performance of a control chart can be measured by the average run length (ARL), which is the average number of samples required to get the first signal. The out-of-control ARL_1 is used to measure the detection effectiveness of the control chart and the in-control ARL_0 measures the false alarm rate. When the process is out-of-control, the ARL_1 is calculated by:

$$1-\beta = 1-P(LCL < WL < UCL \mid WL \sim Q^*_{n_0,\tau^*})$$

$$= 1-F^*_{n_0,\tau^*}(F^{-1}_{n_0,\tau_0}(1-\frac{\alpha_0}{2})) - F^*_{n_0,\tau^*}(F^{-1}_{n_0,\tau_0}(\frac{\alpha_0}{2})).$$

Then

$$ARL_1 = \frac{1}{1-\beta}$$

Ideally ARL_1 should be as small as possible.

3.2 Design of the VP WL Control Chart

The VP (variable parameters) WL chart uses variable sampling intervals h_q, sample sizes n_q and false alarm rates α_q, q = 1, 2. It has the warning limits (UWL_q and LWL_q) and the control limits (UCL_q and LCL_q) that divide the chart into three regions: central region, warning region and action region, see Figure 2.

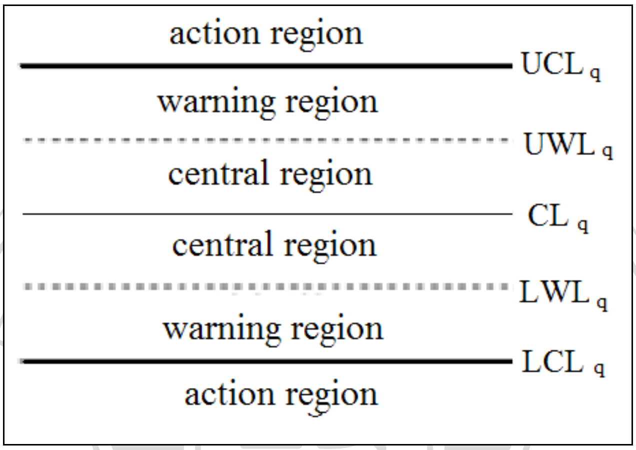

Figure 2. The Structure of VP WL Chart

Let h_0, n_0 and α_0 are the sampling interval, sample size and false alarm rate for FP WL chart. The variable sampling intervals h_q, sample sizes n_q and false alarm rate α_q, q = 1, 2, are adopted where $0 < h_2 < h_0 < h_1 < \infty$, $0 < n_1 < n_0 < n_2 < \infty$, $0 < \alpha_1 \le \alpha_0 \le \alpha_2 < \infty$. From (14), the UCL_q and LCL_q of the VP chart are

$$UCL_q = F^{-1}_{n_q,\tau_q}(1-\frac{\alpha_q}{2})$$

$$LCL_q = F^{-1}_{n_q,\tau_q}(\frac{\alpha_q}{2})$$

where $\tau_q = n_q\delta_3^2$,q=1,2.

If the statistic WL falls into the central region, the next sampling interval should be long, sample size should be small and false alarm rate should be small, (h_1, n_1, α_1),with corresponding control limits (UCL_1, LCL_1) and warning limits (UWL_1, LWL_1)

for the next sample. If WL falls into the warning region, the next sampling interval should be short, sample size should be large and false alarm rate should be large, (h_2, n_2, α_2), with corresponding control limits (UCL_2, LCL_2) and warning limits (UWL_1, LWL_1). If WL falls into the action region, an out-of-control signal would have occurred.

In order to compare the performance of VP and FP chart, we should demand the same average sampling interval, sample size and false alarm rate when the process is in-control process. That is,

$$p_1 h_1 + (1 - p_1) h_2 = h_0 \tag{18}$$

$$p_1 n_1 + (1 - p_1) n_2 = n_0 \tag{19}$$

$$\alpha_1 h_1 + (1 - p_1) \alpha_2 = \alpha_0 \tag{20}$$

where

$$p_1 = P(LWL_q < WL < UWL_q \mid LCL_q < WL < UCL_q, WL \sim Q_{n_q,\tau_q}),\ q = 1, 2.$$

which is the probability of the WL falls in the central region under the in-control process.

The derivation of the warning limits is as follows:

Specify $n_0, n_1, n_2, h_0, h_2, \alpha_0$ and α_1, we could calculate the value of p_1 from (19):

$$p_1 = \frac{n_0 - n_2}{n_1 - n_2}$$

and the values of the h_1 and α_2:

From (18) and (19),

$$h_1 = \frac{h_0(n_2 - n_1) - h_2(n_0 - n_1)}{n_2 - n_0}.$$

From (19) and (20),

$$\alpha_2 = \frac{\alpha_0(n_1 - n_2) - \alpha_1(n_0 - n_2)}{n_1 - n_0}.$$

Let the cumulative probability of the center line (CL) be p^*, that is

$$F_{n_q,\tau_q}(CL_q) = p^*, \text{ q = 1, 2} \tag{21}$$

and let the probability of WL falling between LWL_q and CL_q equal to the probability of WL falling between CL_q and UWL_q, i.e.

$$P(LWL_q < WL < CL_q \mid WL \sim Q_{n_q,\tau_q}) = P(CL_q < WL < UWL_q \mid WL \sim Q_{n_q,\tau_q}) \tag{22}$$

Hence

$$p_1 = P(LWL_q < WL < UWL_q \mid LCL_q < WL < UCL_q, WL \sim Q_{n_q,\tau_q})$$

$$= \frac{P(LWL_q < WL < UWL_q \mid WL \sim Q_{n_q,\tau_q})}{P(LCL_q < WL < UCL_q \mid WL \sim Q_{n_q,\tau_q})}$$

That is,

$$P(LWL_q < WL < UWL_q \mid WL \sim Q_{n_q,\tau_q}) = P(LCL_q < WL < UCL_q \mid WL \sim Q_{n_q,\tau_q}) \cdot p_1$$

From (22),

$$P(LWL_q < WL < CL_q \mid WL \sim Q_{n_q,\tau_q}) + P(CL_q < WL < UWL_q \mid WL \sim Q_{n_q,\tau_q}) = (1-\alpha_q) \cdot p_1$$

Then,

$$P(LWL_q < WL < CL_q \mid WL \sim Q_{n_q,\tau_q}) = (1-\alpha_q) \cdot \frac{p_1}{2} \tag{23}$$

$$P(CL_q < WL < UWL_q \mid WL \sim Q_{n_q,\tau_q}) = (1-\alpha_q) \cdot \frac{p_1}{2} \tag{24}$$

Simplify (23),

$$F_{n_q,\tau_q}(CL_q) - F_{n_q,\tau_q}(LWL_q) = (1-\alpha_q) \cdot \frac{p_1}{2}$$

Hence, the LWL could be derived as

$$LWL_q = F^{-1}_{n_q,\tau_q}\left(F_{n_q,\tau_q}(CL_q) - (1-\alpha_q) \cdot \frac{p_1}{2}\right)$$

From eq. (21), $F_{n_q,\tau_q}(CL_q) = p^*$,

$$LWL_q = F^{-1}_{n_q,\tau_q}\left(p^* - (1-\alpha_q) \cdot \frac{p_1}{2}\right)$$

Similarly, we could derive the UWL_q

$$UWL_q = F^{-1}_{n_q,\tau_q}(p^* + (1-\alpha_q)\cdot\frac{p_1}{2})$$

The performance of a control chart could be measured by the average time to signal (ATS_1), which is the average time required to signal a process shift after it has occurred. The ATS_0 is the average time until a false alarm occurred.

Apply the Markov chain approach to derive the ATS_1, and all possible process states are defined in Table 1. Table 1 shows the 3 possible process states based on Markov property. States 1 and 2 are transient states since they may transit from one to other states, while state 3 is an absorbing state because it cannot transit to any other states. After the states are defined, the transition probability from state i to j with sampling intervals h_q, sample sizes n_q and false alarm rate α_q, $p^*_{ij}(h_q, n_q, \alpha_q)$, i, j=1, 2, 3, could be calculated.

Table 1. Definition of Process States for the VP WL Chart

State	Region	Alarm	Next Sampling Interval, Size and False Alarm Rate (h_q, n_q, α_q)	State Status
1	Central	No	(h_1,n_1,α_1)	Transient
2	Warning	No	(h_2,n_2,α_2)	Transient
3	Action	Yes	Stop	Absorbing

Transition probability：

$$p^*_{11}(h_1, n_1, \alpha_1) = P(LWL_1 < WL < UWL_1 \mid WL \sim Q^*_{n_1,\tau_1^*}) = F^*_{n_1,\tau_1^*}(UWL_1) - F^*_{n_1,\tau_1^*}(LWL_1)$$

$$p^*_{12}(h_1, n_1, \alpha_1) = P(LCL_1 < WL < LWL_1 \;\; or \;\; UWL_1 < WL < UCL_1 \mid WL \sim Q^*_{n_1,\tau_1^*})$$

$$= (F^*_{n_1,\tau_1^*}(LWL_1) - F^*_{n_1,\tau_1^*}(LCL_1)) + (F^*_{n_1,\tau_1^*}(UCL_1) - F^*_{n_1,\tau_1^*}(UWL_1))$$

$$p^*_{13}(h_1, n_1, \alpha_1) = 1 - P(LCL_1 < WL < UCL_1 \mid WL \sim Q^*_{n_1,\tau_1^*}) = 1 - (F^*_{n_1,\tau_1^*}(UCL_1) - F^*_{n_1,\tau_1^*}(LCL_1))$$

$$p^*_{21}(h_2, n_2, \alpha_2) = P(LWL_2 < WL < UWL_2 \mid WL \sim Q^*_{n_2,\tau_2^*}) = F^*_{n_2,\tau_2^*}(UWL_2) - F^*_{n_2,\tau_2^*}(LWL_2)$$

$$p_{22}^*(h_2,n_2,\alpha_2)=P(LCL_2<WL<LWL_2 \ \ or \ \ UWL_2<WL<UCL_2 \mid WL \sim Q_{n_2,\tau_2^*}^*)$$

$$=(F_{n_2,\tau_2^*}^*(LWL_2)-F_{n_2,\tau_2^*}^*(LCL_2))+(F_{n_2,\tau_2^*}^*(UCL_2)-F_{n_2,\tau_2^*}^*(UWL_2))$$

$$p_{23}^*(h_2,n_2,\alpha_2)=1-P(LCL_2<WL<UCL_2 \mid WL \sim Q_{n_2,\tau_2^*}^*)=1-(F_{n_2,\tau_2^*}^*(UCL_2)-F_{n_2,\tau_2^*}^*(LCL_2))$$

$$p_{31}^*=p_{32}^*=0,\ \ p_{33}^*=1$$

The transition probability from state i to j can be expressed by a square matrix

$$\mathbf{P}^*=\begin{bmatrix} p_{11}^* & p_{12}^* & p_{13}^* \\ p_{21}^* & p_{22}^* & p_{23}^* \\ 0 & 0 & 1 \end{bmatrix},$$

The transition probability matrix which contains the transient probability from transient state i to transient state j

$$\mathbf{Q}^*=\begin{bmatrix} p_{11}^* & p_{12}^* \\ p_{21}^* & p_{22}^* \end{bmatrix}.$$

The ATS_1 is derived as

$$ATS_1=\mathbf{r}^{*\prime}(\mathbf{I}-\mathbf{Q}^*)^{-1}\mathbf{h},$$

where $\mathbf{I}$ is the identity matrix of order 2, $\mathbf{h}'=(h_1,h_2)$ is a (1x2) vector of sampling time for transient state i, i = 1, 2, $\mathbf{r}^{*\prime}=(r_1^*,r_2^*)$ is a (1x2) vector with the steady-state starting probability, r_i^*, i=1, 2, for transient state i. The r_i^* can be obtained by solving the equation

$$\mathbf{r}^{*\prime}\mathbf{Q}^*=\mathbf{r}^{*\prime} \ \ \text{and} \ \ \sum_{i=1}^{2} r_i^*=1$$

That is,

$$r_1^*=\frac{1-\dfrac{p_{11}^*}{p_{11}^*+p_{12}^*}}{1-\dfrac{p_{11}^*}{p_{11}^*+p_{12}^*}+\dfrac{p_{21}^*}{p_{21}^*+p_{22}^*}} \quad \text{and} \quad r_2^*=\frac{\dfrac{p_{21}^*}{p_{21}^*+p_{22}^*}}{1-\dfrac{p_{11}^*}{p_{11}^*+p_{12}^*}+\dfrac{p_{21}^*}{p_{21}^*+p_{22}^*}}.$$

We also use the average number of observations to signal (ANOS) to measure the performance of a control chart, which ANOS is the average number of observations required to signal a process shift after it has occurred. It is derived as

$$ANOS = \mathbf{r}^{*\prime}(\mathbf{I}-\mathbf{Q}^{*})^{-1}\mathbf{n}\,, \tag{25}$$

where $\mathbf{n}' = (n_1, n_2)$ is a (1x2) vector of sample size for transient state i, i=1,2.

When $\alpha_1 = \alpha_0 = \alpha_2$, the VP design can reduce to the VSSI design. The derivation of the control limits and warning limits and the calculation of the ATS_1 and ANOS are the same as the VP chart.

When $\alpha_1 = \alpha_2 = \alpha_0$ and $n_1 = n_2 = n_0$, the VP design can reduce to the VSI design where $p_1 = \dfrac{h_0 - h_2}{h_1 - h_2}$ is calculated from (18). The derivation of the control limits and warning limits and the calculation of the ATS_1 are the same as the VP chart.

3.3 ATS_1 Analysis and ATS_1 Comparison among the FP, Specified VSI, VSSI and VP WL Charts

(1) ARL_1 Analysis of FP WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or $ARL_0 = 370.37$) and $a = 0.6$. Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2) and δ_3 = (0, 1, 2). The 27 combinations of δ_1, δ_2 and δ_3 are arranged by Orthogonal Arrays (O.A.) $L_{27}(3^{13})$. The ARL_1s of the 27combinations of the FP chart are listed in Table 2.

Table 2. ARL_1 for FP WL Chart

No.	δ_1	δ_2	δ_3	ARL_1	ANOS	No.	δ_1	δ_2	δ_3	ARL_1	ANOS
1	0.5	1.1	0	109.54	547.70	16	1.2	2	0	1.88	9.41
2	0.5	1.1	1	38.67	193.35	17	1.2	2	1	1.59	7.96
3	0.5	1.1	2	29.36	146.81	18	1.2	2	2	1.63	8.12
4	0.5	1.5	0	7.79	38.93	19	1.5	1.1	0	9.86	49.32
5	0.5	1.5	1	5.90	29.51	20	1.5	1.1	1	2.70	13.51
6	0.5	1.5	2	6.20	31.02	21	1.5	1.1	2	2.04	10.19
7	0.5	2	0	2.36	11.80	22	1.5	1.5	0	3.01	15.07
8	0.5	2	1	2.31	11.55	23	1.5	1.5	1	1.79	8.97
9	0.5	2	2	2.66	13.32	24	1.5	1.5	2	1.64	8.18
10	1.2	1.1	0	21.67	108.35	25	1.5	2	0	1.67	8.34
11	1.2	1.1	1	5.10	25.51	26	1.5	2	1	1.39	6.97
12	1.2	1.1	2	3.60	17.99	27	1.5	2	2	1.39	6.95
13	1.2	1.5	0	4.14	20.72						
14	1.2	1.5	1	2.42	12.09						
15	1.2	1.5	2	2.21	11.05						

In order to compare the ANOS among FP, VSSI and VP charts later, we have to calculate the ANOS of FP chart by letting $n_1 = n_2 = n_0$ and $\alpha_1 = \alpha_2 = \alpha_0$ in the equation (25) of the VP design.

To investigate the effect of δ_1, δ_2 and δ_3 on the ARL_1, the response table and response diagram show the results of the sensitivity analysis (Table 3 and Figure 3).

Table 3. FP WL Chart– Response Table of δ_1, δ_2, δ_3 and $\overline{ARL_1}$

δ_1	$\overline{ARL_1}$	δ_2	$\overline{ARL_1}$	δ_3	$\overline{ARL_1}$
0.5	22.76	1.1	24.73	0	17.99
1.2	4.92	1.5	3.90	1	6.88
1.5	2.83	2	1.88	2	5.64

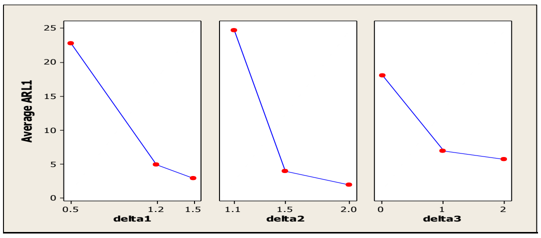

Figure 3. FP WL Chart– Response Diagram of δ_1, δ_2, δ_3 and $\overline{\mathrm{ARL}_1}$

From Table 3 and Figure 3, we found the average ARL_1

a. is decreasing when δ_1 increasing,

b. is decreasing when δ_2 increasing,

c. is decreasing when δ_3 increasing δ_3.

(2) $\mathbf{ATS_1}$ Analysis of VSI WL Chart and $\mathbf{ATS_1}$ Comparison between FP WL Chart and Specified VSI WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or ARL_0 = 370.37) and $a = 0.6$. Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2), δ_3 = (0, 1, 2), (h_2, h_1) = (0.2, 2), (0.5,1.5), (0.8, 1.2) and $p^* = (0.4, 0.5, 0.6)$. The 27 combinations are arranged by $L_{27}(3^{13})$. The ATS_1s of the 27combinations of the VSI chart are listed in Table 4.

Table 4. ATS_1 for VSI WL Chart and ATS_1 saved % Compared to FP WL Chart

					Specified VSI (h_1, h_2)		FP	ATS_1
No	δ_1	δ_2	δ_3	p^*	(h_2,h_1)	ATS_1	ARL_1	saved %
1	0.5	1.1	0	0.6	(0.2, 2)	108.58	109.54	0.87%
2	0.5	1.1	1	0.5	(0.5,1.5)	32.49	38.67	15.99%
3	0.5	1.1	2	0.4	(0.8, 1.2)	26.25	29.36	10.60%
4	0.5	1.5	0	0.6	(0.2, 2)	5.56	7.79	28.64%
5	0.5	1.5	1	0.5	(0.5,1.5)	4.24	5.90	28.18%
6	0.5	1.5	2	0.4	(0.8, 1.2)	5.42	6.20	12.63%
7	0.5	2	0	0.6	(0.2, 2)	1.24	2.36	47.28%
8	0.5	2	1	0.5	(0.5,1.5)	1.52	2.31	34.41%
9	0.5	2	2	0.4	(0.8, 1.2)	2.30	2.66	13.87%
10	1.2	1.1	0	0.5	(0.5,1.5)	16.31	21.67	24.75%
11	1.2	1.1	1	0.4	(0.8, 1.2)	4.16	5.10	18.42%
12	1.2	1.1	2	0.6	(0.2, 2)	1.27	3.60	64.81%
13	1.2	1.5	0	0.5	(0.5,1.5)	2.71	4.14	34.66%
14	1.2	1.5	1	0.4	(0.8, 1.2)	1.98	2.42	17.97%
15	1.2	1.5	2	0.6	(0.2, 2)	0.90	2.21	59.42%
16	1.2	2	0	0.5	(0.5,1.5)	1.14	1.88	39.28%
17	1.2	2	1	0.4	(0.8, 1.2)	1.31	1.59	17.71%
18	1.2	2	2	0.6	(0.2, 2)	0.71	1.63	56.19%
19	1.5	1.1	0	0.4	(0.8, 1.2)	8.23	9.86	16.56%
20	1.5	1.1	1	0.6	(0.2, 2)	0.77	2.70	71.45%
21	1.5	1.1	2	0.5	(0.5,1.5)	1.05	2.04	48.46%
22	1.5	1.5	0	0.4	(0.8, 1.2)	2.49	3.01	17.33%
23	1.5	1.5	1	0.6	(0.2, 2)	0.59	1.79	67.19%
24	1.5	1.5	2	0.5	(0.5,1.5)	0.89	1.64	45.77%
25	1.5	2	0	0.4	(0.8, 1.2)	1.37	1.67	17.78%
26	1.5	2	1	0.6	(0.2, 2)	0.50	1.39	64.31%
27	1.5	2	2	0.5	(0.5,1.5)	0.80	1.39	42.85%

Note: ATS_1 saved % = $\left(FP\text{-}ARL_1 - VSI\text{-}ATS_1\right) / FP\text{-}ARL_1$ %

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1, the response table and response diagram show the results of the sensitivity analysis (Table 5 and Figure 4).

Table 5. VSI WL Chart– Response Table of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1}$

δ_1	$\overline{ATS_1}$	δ_2	$\overline{ATS_1}$	δ_3	$\overline{ATS_1}$
0.5	22.76	1.1	24.73	0	17.99
1.2	4.92	1.5	3.90	1	6.88
1.5	2.83	2	1.88	2	5.64

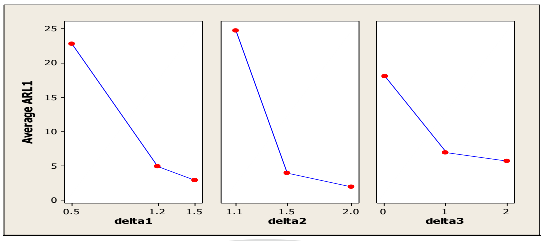

Figure 4. VSI WL Chart– Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1}$

From Table 4, Table 5 and Figure 4, we found

a. ATS_1 saved % from 0.87% to 71.45%

b. the $\overline{\text{ATS}_1}$ is decreasing when δ_1 increasing.

c. the $\overline{\text{ATS}_1}$ is decreasing when δ_2 increasing.

d. the $\overline{\text{ATS}_1}$ is decreasing when δ_3 increasing.

(3) ATS_1 Analysis of VSSI WL Chart and ATS_1 Comparison between FP WL Chart and specified VSSI WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or $ARL_0 = 370.37$), $p^* = 0.5$ and $a = 0.6$. Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2), δ_3 = (0, 1, 2), h_2 = (0.1, 0.5, 0.9) and (n_1, n_2): (2, 6), (3, 10), (4, 20). The 27 combinations are arranged by $L_{27}(3^{13})$. The ATS_1s of the 27combinations of the VSSI chart are listed in Table 6.

Table 6. ATS_1 and ANOS for VSSI WL Chart and ATS_1 Saved % and ANOS Saved % Compared to FP WL Chart

				Specified VSSI (n_1, n_2, h_2)					FP		ATS_1	ANOS
No.	δ_1	δ_2	δ_3	(n_1,n_2)	h_2	h_1	ATS_1	ANOS	ARL_1	ANOS	saved %	saved %
1	0.5	1.1	0	(2,6)	0.1	3.70	93.73	551.38	109.54	547.70	14.44%	-0.67%
2	0.5	1.1	1	(3,10)	0.5	1.20	21.84	169.04	38.67	193.35	43.53%	12.57%
3	0.5	1.1	2	(4,20)	0.9	1.01	12.55	104.90	29.36	146.81	57.26%	28.55%
4	0.5	1.5	0	(2,6)	0.5	2.50	5.00	38.35	7.79	38.93	35.84%	1.48%
5	0.5	1.5	1	(3,10)	0.9	1.04	3.40	28.80	5.90	29.51	42.37%	2.39%
6	0.5	1.5	2	(4,20)	0.1	1.06	2.02	32.83	6.20	31.02	67.45%	-5.83%
7	0.5	2	0	(2,6)	0.9	1.30	1.94	12.06	2.36	11.80	18.00%	-2.19%
8	0.5	2	1	(3,10)	0.1	1.36	0.53	13.73	2.31	11.55	77.05%	-18.90%
9	0.5	2	2	(4,20)	0.5	1.03	1.22	20.37	2.66	13.32	54.40%	-52.90%
10	1.2	1.1	0	(4,20)	0.1	1.06	4.62	59.55	21.67	108.35	78.69%	45.04%
11	1.2	1.1	1	(2,6)	0.5	2.50	2.09	22.78	5.10	25.51	59.12%	10.71%
12	1.2	1.1	2	(3,10)	0.9	1.04	1.39	14.94	3.60	17.99	61.50%	16.92%
13	1.2	1.5	0	(4,20)	0.5	1.03	1.18	22.93	4.14	20.72	71.58%	-10.67%
14	1.2	1.5	1	(2,6)	0.9	1.30	1.86	12.02	2.42	12.09	23.08%	0.55%
15	1.2	1.5	2	(3,10)	0.1	1.36	0.25	12.63	2.21	11.05	88.60%	-14.23%
16	1.2	2	0	(4,20)	0.9	1.01	1.06	19.04	1.88	9.41	43.47%	-102.28%
17	1.2	2	1	(2,6)	0.1	3.70	0.30	8.44	1.59	7.96	81.01%	-6.00%
18	1.2	2	2	(3,10)	0.5	1.20	0.68	11.10	1.63	8.12	57.93%	-36.68%
19	1.5	1.1	0	(3,10)	0.1	1.36	1.06	31.23	9.86	49.32	89.28%	36.67%
20	1.5	1.1	1	(4,20)	0.5	1.03	0.53	20.06	2.70	13.51	80.37%	-48.44%
21	1.5	1.1	2	(2,6)	0.9	1.30	1.48	9.84	2.04	10.19	27.32%	3.36%
22	1.5	1.5	0	(3,10)	0.5	1.20	0.91	14.75	3.01	15.07	69.70%	2.09%
23	1.5	1.5	1	(4,20)	0.9	1.01	0.94	19.72	1.79	8.97	47.43%	-119.90%
24	1.5	1.5	2	(2,6)	0.1	3.70	0.20	8.51	1.64	8.18	87.63%	-3.97%
25	1.5	2	0	(3,10)	0.9	1.04	1.06	11.04	1.67	8.34	36.33%	-32.34%
26	1.5	2	1	(4,20)	0.1	1.06	0.19	19.23	1.39	6.97	86.05%	-176.09%
27	1.5	2	2	(2,6)	0.5	2.50	0.70	7.54	1.39	6.95	49.56%	-8.46%

Note: ATS_1 saved %: $(FP\text{-}ARL_1 - VSSI\text{-}ATS_1)/FP\text{-}ARL_1$ % ,ANOS saved %: $(FP\text{-}ANOS - VSSI\text{-}ANOS)/FP\text{-}ANOS$ %

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ANOS, the response table and response diagram show the results of the sensitivity analysis (Table 7, Figure

5 and Figure 6).

Table 7. VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ANOS}$

δ_1	$\overline{ATS_1}$	$\overline{ANOS}$	δ_2	$\overline{ATS_1}$	$\overline{ANOS}$	δ_3	$\overline{ATS_1}$	$\overline{ANOS}$
0.5	18.61	111.14	1.1	19.09	115.62	0	13.82	86.11
1.2	2.16	20.19	1.5	2.52	20.21	1	5.88	35.88
1.5	2.07	15.98	2	1.23	11.48	2	3.14	25.32

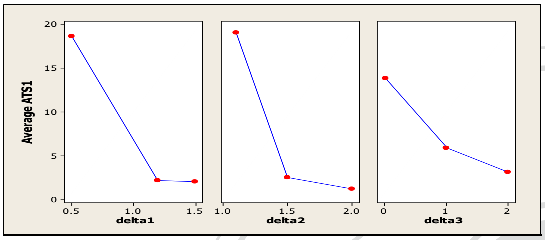

Figure 5. VSSI WL Chart –Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$

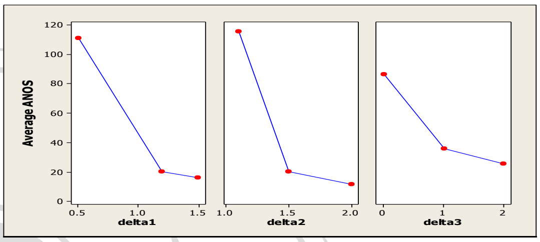

Figure 6. VSSI WL Chart –Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

From Table 6, Table 7, Figure 5 and Figure 6, we found

a. ATS_1 saved % from 14.44% to 89.28%.

b. ANOS saved % from -176.09% to 45.04%.

c. the $\overline{ATS_1}$ is decreasing when δ_1 increasing.

d. the $\overline{ATS_1}$ is decreasing when δ_2 increasing.

e. the $\overline{ATS_1}$ is decreasing when δ_3 increasing.

f. the $\overline{ANOS}$ is decreasing when δ_1 increasing.

g. the $\overline{ANOS}$ is decreasing when δ_2 increasing.

h. the $\overline{ANOS}$ is decreasing when δ_3 increasing.

(4) ATS_1 Analysis of VP WL Chart and ATS_1 Comparison between FP WL Chart and Specified VP WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or ARL_0 = 370.37), $p^* = 0.5$ and $a = 0.6$. Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2), δ_3 = (0, 1, 2), h_2 = (0.1, 0.5, 0.9), (n_1, n_2): (2, 6), (3, 10), (4, 20) and α_1 = (0.00135, 0.002025, 0.0027). The 27 combinations are arranged by $L_{27}(3^{13})$. The ATS_1s of the 27combinations of the VP chart are listed in Table 8.

Table 8. ATS_1 and ANOS for VP WL Chart and ATS_1 saved % and ANOS Saved % Compared to FP WL Chart

				Specified VP (n_1, n_2, h_2, α_1)							FP		ATS_1 saved %	ANOS saved %
No.	δ_1	δ_2	δ_3	(n_1, n_2)	h_2	h_1	α_1	α_2	ATS_1	ANOS	ARL_1	ANOS		
1	0.5	1.1	0	(2,6)	0.1	3.70	0.0027	0.0027	93.73	551.38	109.54	547.70	14.44%	-0.67%
2	0.5	1.1	1	(3,10)	0.5	1.20	0.0020	0.0044	18.02	138.14	38.67	193.35	53.39%	28.55%
3	0.5	1.1	2	(4,20)	0.9	1.01	0.0014	0.0230	8.49	64.41	29.36	146.81	71.08%	56.12%
4	0.5	1.5	0	(2,6)	0.5	2.50	0.0027	0.0027	5.00	38.35	7.79	38.93	35.84%	1.48%
5	0.5	1.5	1	(3,10)	0.9	1.04	0.0020	0.0044	3.14	26.26	5.90	29.51	46.74%	11.01%
6	0.5	1.5	2	(4,20)	0.1	1.06	0.0014	0.0230	2.13	28.22	6.20	31.02	65.64%	9.03%
7	0.5	2	0	(2,6)	0.9	1.30	0.0027	0.0027	1.94	12.06	2.36	11.80	18.00%	-2.19%
8	0.5	2	1	(3,10)	0.1	1.36	0.0020	0.0044	0.55	13.27	2.31	11.55	76.30%	-14.88%
9	0.5	2	2	(4,20)	0.5	1.03	0.0014	0.0230	1.40	19.35	2.66	13.32	47.46%	-45.26%
10	1.2	1.1	0	(4,20)	0.1	1.06	0.0020	0.0128	4.24	45.46	21.67	108.35	80.46%	58.05%
11	1.2	1.1	1	(2,6)	0.5	2.50	0.0014	0.0032	1.97	21.52	5.10	25.51	61.31%	15.65%
12	1.2	1.1	2	(3,10)	0.9	1.04	0.0027	0.0027	1.39	14.94	3.60	17.99	61.50%	16.92%
13	1.2	1.5	0	(4,20)	0.5	1.03	0.0020	0.0128	1.39	21.85	4.14	20.72	66.53%	-5.45%
14	1.2	1.5	1	(2,6)	0.9	1.30	0.0014	0.0032	1.81	11.71	2.42	12.09	25.06%	3.18%
15	1.2	1.5	2	(3,10)	0.1	1.36	0.0027	0.0027	0.25	12.63	2.21	11.05	88.60%	-14.23%
16	1.2	2	0	(4,20)	0.9	1.01	0.0020	0.0128	1.19	18.22	1.88	9.41	37.05%	-93.60%
17	1.2	2	1	(2,6)	0.1	3.70	0.0014	0.0032	0.30	8.34	1.59	7.96	80.94%	-4.75%
18	1.2	2	2	(3,10)	0.5	1.20	0.0027	0.0027	0.68	11.10	1.63	8.12	57.93%	-36.68%
19	1.5	1.1	0	(3,10)	0.1	1.36	0.0014	0.0061	0.93	24.67	9.86	49.32	90.53%	49.97%
20	1.5	1.1	1	(4,20)	0.5	1.03	0.0027	0.0027	0.53	20.06	2.70	13.51	80.37%	-48.44%
21	1.5	1.1	2	(2,6)	0.9	1.30	0.0020	0.0029	1.46	9.69	2.04	10.19	28.44%	4.86%
22	1.5	1.5	0	(3,10)	0.5	1.20	0.0014	0.0061	0.88	13.58	3.01	15.07	70.87%	9.85%
23	1.5	1.5	1	(4,20)	0.9	1.01	0.0027	0.0027	0.94	19.72	1.79	8.97	47.43%	-119.90%
24	1.5	1.5	2	(2,6)	0.1	3.70	0.0020	0.0029	0.20	8.43	1.64	8.18	87.66%	-3.06%
25	1.5	2	0	(3,10)	0.9	1.04	0.0014	0.0061	1.06	10.78	1.67	8.34	36.74%	-29.14%
26	1.5	2	1	(4,20)	0.1	1.06	0.0027	0.0027	0.19	19.23	1.39	6.97	86.05%	-176.09%
27	1.5	2	2	(2,6)	0.5	2.50	0.0020	0.0029	0.70	7.51	1.39	6.95	49.70%	-7.95%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - VP\text{-}ATS_1\right)/FP\text{-}ARL_1$ %, ANOS saved %: $\left(FP\text{-}ANOS - VP\text{-}ANOS\right)/FP\text{-}ANOS$ %

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ANOS, the response table and response diagram show the results of the sensitivity analysis (Table 9, Figure 7 and Figure 8).

Table 9. VP WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{\text{ATS}_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ANOS}$

δ_1	$\overline{ATS_1}$	$\overline{ANOS}$	δ_2	$\overline{ATS_1}$	$\overline{ANOS}$	δ_3	$\overline{ATS_1}$	$\overline{ANOS}$
0.5	14.93	99.05	1.1	14.53	98.92	0	12.26	81.82
1.2	1.47	18.42	1.5	1.75	20.08	1	3.05	30.92
1.5	0.77	14.85	2	0.89	13.32	2	1.86	19.59

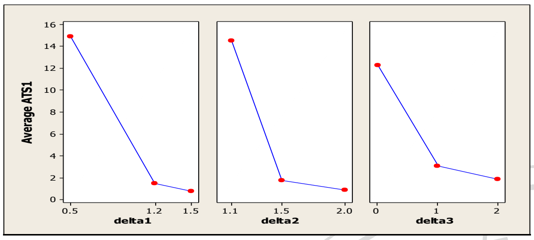

Figure 7. VP WL Chart –Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1}$

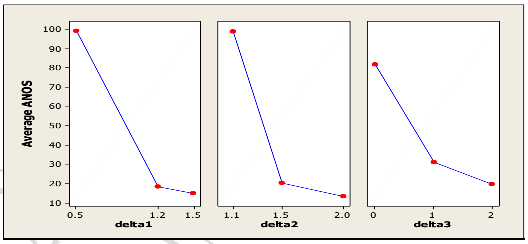

Figure 8. FP WL chart –Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

From Table 8, Table 9, Figure 7 and Figure 8, we found

a. ATS_1 saved % from 14.44% to 90.53%.

b. ANOS saved % from -176.09% to 58.05%.

c. the $\overline{\text{ATS}_1}$ is decreasing when δ_1 increasing.

d. the $\overline{\text{ATS}_1}$ is decreasing when δ_2 increasing.

e. the $\overline{\text{ATS}_1}$ is decreasing when δ_3 increasing.

f. the $\overline{ANOS}$ is decreasing when δ_1 increasing.

g. the $\overline{ANOS}$ is decreasing when δ_2 increasing.

h. the $\overline{ANOS}$ is decreasing when δ_3 increasing.

CHAPTER 4. DESIGN AND DATA ANALYSIS OF THE OPTIMAL FP, VSI, VSSI AND VP WL CHARTS

4.1 Design of the FP WL Control Chart with Optimal Weight

The design of the optimal FP WL chart is to search for an optimal value of weight a that minimizes the ARL_1 under the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0.

The optimization model:

$$\text{Min. } ARL_1$$

$$\text{s.t. } 0 \le a \le 1$$

The design algorithm has been implemented in the optimization technique PORT routine in R program.

4.2 Design of the Optimal VSI WL Control Chart

In this part, we consider two situations: (i) a is specified and (ii) a is not specified.

(i) a is specified

The design of the optimal VSI WL chart is to search for the optimal values of h_1, h_2 and p^* that minimize the out-of-control ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < h_L < h_2 < h_0 < h_1 \le h_U < \infty$$

$$0 \le P_L \le P^* \le P_U \le 1$$

(ii) a is optimal

The design of the optimal VSI WL chart is to search for the optimal values of h_1, h_2, p^* and a that minimize the out-of-control ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < h_L < h_2 < h_0 < h_1 \le h_U < \infty$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

4.3 Design of the Optimal VSSI WL Control Chart

Like section 4.2, we consider two situations: (i) a is specified and (ii) a is not specified.

(i) a is specified

The design of the optimal VSSI WL chart is to search for the optimal values of n_1, n_2, h_2 and p^* that minimize the ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < n_L \le n_1 < n_0 < n_2 \le n_U < \infty$$

$$0 < h_L < h_2 < h_0 < \infty$$

$$0 \le P_L \le P^* \le P_U \le 1$$

(ii) a is optimal

The design of the optimal VSSI WL chart is to search for the optimal values of n_1, n_2, h_2, p^* and a that minimize the ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < n_L \le n_1 < n_0 < n_2 \le n_U < \infty$$

$$0 < h_L < h_2 < h_0 < \infty$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

4.4 Design of the Optimal VP WL Control Chart

As above, we consider two situations: (i) a is specified and (ii) a is optimal.

(i) a is specified

The design of the optimal VP WL chart is to search for the optimal values of n_1, n_2, h_2, α_1 and p^* that minimize the ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < n_L \le n_1 < n_0 < n_2 \le n_U < \infty$$

$$0 < h_L < h_2 < h_0 < \infty$$

$$0 < \alpha_L \le \alpha_1 \le \alpha_0 < 1$$

$$0 \le P_L \le P^* \le P_U \le 1$$

(ii) a is optimal

The design of the optimal VP WL chart is to search for the optimal values of n_1, n_2, h_2, α_1, p^* and a that minimize the out-of-control ARL_1 at the specified δ_1, δ_2, δ_3, n_0, h_0 and α_0. The optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0 < n_L \le n_1 < n_0 < n_2 \le n_U < \infty$$

$$0 < h_L < h_2 < h_0 < \infty$$

$$0 < \alpha_L \le \alpha_1 \le \alpha_0 < 1$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

4.5 ATS_1 Analysis and ATS_1 Comparison among Optimal FP, VSI, VSSI and VP WL Charts

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$ and $\alpha_0 = 0.0027$ (or $ARL_0 = 370.37$). Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2) and δ_3 = (0, 1, 2). The 27 combinations of δ_1, δ_2 and δ_3 are arranged by $L_{27}(3^{13})$. When *a* is specified which give *a* = 0.6, the ATS_1s of the 27 combinations of the optimal VSI, VSSI and VP charts are listed in Table 12, Table 16 and Table 24. When *a* is not specified, the ARL_1s of the 27 combinations of optimal FP chart are listed in Table 10, the ATS_1s of the 27 combinations of the optimal VSI, VSSI and VP charts are listed in Table 14, Table 20 and Table 28.

（1）**ARL_1 Analysis of optimal FP WL Chart and ARL_1 Comparison between FP WL Chart and Optimal FP WL Chart with optimal *a***

Table 10. ARL_1 for Optimal FP WL Chart and ARL_1 saved % Compared to FP WL Chart

				Optimal FP (a^*)				FP	ARL_1 saved %
No.	δ_1	δ_2	δ_3	a^*	ARL_1	LCL^*	UCL^*	ARL_1	
1	0.5	1.1	0	0.80	10.30	0.04	2.01	109.54	90.60%
2	0.5	1.1	1	0.41	29.32	0.11	4.04	38.67	24.18%
3	0.5	1.1	2	0.80	23.54	0.36	3.93	29.36	19.84%
4	0.5	1.5	0	0.80	2.58	0.04	2.01	7.79	66.81%
5	0.5	1.5	1	0.60	5.90	0.11	3.58	5.90	0.03%
6	0.5	1.5	2	0.80	4.03	0.36	3.93	6.20	35.05%
7	0.5	2	0	0.80	1.45	0.04	2.01	2.36	38.43%
8	0.5	2	1	0.65	2.27	0.11	3.58	2.31	1.59%
9	0.5	2	2	0.80	1.85	0.36	3.93	2.66	30.60%
10	1.2	1.1	0	0.20	3.31	0.04	2.01	21.67	84.75%
11	1.2	1.1	1	0.28	3.25	0.09	4.63	5.10	36.26%
12	1.2	1.1	2	0.37	3.12	0.68	7.78	3.60	13.40%
13	1.2	1.5	0	0.80	2.15	0.04	2.01	4.14	48.04%
14	1.2	1.5	1	0.50	2.25	0.11	3.76	2.42	6.90%
15	1.2	1.5	2	0.59	2.21	0.56	5.75	2.21	0.12%
16	1.2	2	0	0.80	1.36	0.04	2.01	1.88	28.01%
17	1.2	2	1	0.58	1.59	0.11	3.60	1.59	0.28%
18	1.2	2	2	0.80	1.46	0.36	3.93	1.63	10.25%
19	1.5	1.1	0	0.20	1.85	0.02	1.89	9.86	81.23%
20	1.5	1.1	1	0.27	1.85	0.09	4.68	2.70	31.73%
21	1.5	1.1	2	0.37	1.80	0.69	7.84	2.04	11.65%
22	1.5	1.5	0	0.28	1.66	0.02	1.89	3.01	44.95%
23	1.5	1.5	1	0.46	1.65	0.11	3.85	1.79	8.30%
24	1.5	1.5	2	0.55	1.63	0.58	6.03	1.64	0.68%
25	1.5	2	0	0.80	1.30	0.04	2.01	1.67	22.10%
26	1.5	2	1	0.55	1.38	0.11	3.63	1.39	0.88%
27	1.5	2	2	0.80	1.33	0.36	3.93	1.39	4.27%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.FP\text{-}ARL_1\right) / FP\text{-}ARL_1\,\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ARL_1, a^* and ARL_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

11, Figure 9, Figure 10 and Figure 11).

Table 11. Optimal FP WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{\text{ARL}_1}$, δ_1, δ_2, δ_3 vs. $\overline{a*}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ARL}_1 \text{ saved \%}}$

δ_1	$\overline{\text{ARL}_1}$	$\overline{a*}$	$\overline{\text{ARL}_1 \text{ saved \%}}$	δ_2	$\overline{\text{ARL}_1}$	$\overline{a*}$	$\overline{\text{ARL}_1 \text{ saved \%}}$	δ_3	$\overline{\text{ARL}_1}$	$\overline{a*}$	$\overline{\text{ARL}_1 \text{ saved \%}}$
0.5	9.03	0.72	34.13%	1.1	8.70	0.41	43.74%	0	2.88	0.61	56.10%
1.2	2.30	0.55	25.33%	1.5	2.67	0.60	23.43%	1	5.50	0.48	12.24%
1.5	1.60	0.48	22.87%	2	1.55	0.73	15.16%	2	4.55	0.65	13.98%

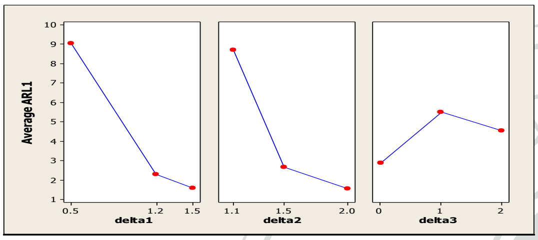

Figure 9. Optimal FP WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ARL}_1}$

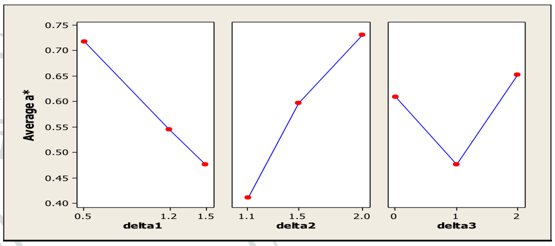

Figure 10. Optimal FP WL Chart– Response Diagram of δ_1, δ_2, δ_3 and $\overline{a*}$

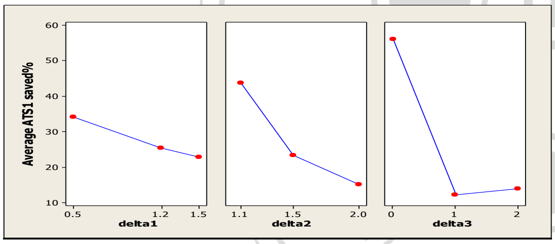

Figure 11. Optimal FP WL Chart – Response Diagram of δ_1, δ_2, δ_3 vs. $\overline{\text{ARL}_1 \text{ saved \%}}$

From Table 10, Table 11, Figure 9, Figure 10 and Figure 11, we found

a. ARL_1 saved % from 0.03% to 90.60%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is first increasing then decreasing as δ_3 increasing.

e. the $\overline{a^*}$ is decreasing as δ_1 increasing.

f. the $\overline{a^*}$ is increasing as δ_2 increasing.

g. the $\overline{a^*}$ is first decreasing then increasing as δ_3 increasing.

h. the $\overline{\text{ARL}_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

i. the $\overline{\text{ARL}_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

j. the $\overline{\text{ARL}_1 \text{ saved } \%}$ is first decreasing then increasing as δ_3 increasing.

(2) **ATS_1 analysis of Optimal VSI WL Chart and ATS_1 Comparison between FP WL Chart and Optimal VSI WL Chart with a = 0.6**

Let h_L = 0.1 and h_U = 4, then the optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0.1 < h_2 < h_0 < h_1 \le 4$$

$$0 \le P_L \le P^* \le P_U \le 1$$

According to the model, it needs a appropriate range for p^*, otherwise it can't find the optimal values. So we use the grid search method, let P_L increase from 0 with a size 0.001 and P_U decrease from 1 with a size 0.001 and use the optimization technique PORT routine in R program to calculate until the optimal values could be found. The ATS_1s of the optimal VSI chart are listed in Table 12.

Table 12. ATS_1 for Optimal VSI WL Chart and ATS_1 saved % Compared to FP WL Chart when a = 0.6

				Optimal VSI (h_1^*, h_2^*, Otp.p^*)							FP ARL_1	ATS_1 saved %
No.	δ_1	δ_2	δ_3	(h_2^*,h_1^*)	Opt.p^*	LCL^*	LWL^*	UWL^*	UCL^*	ATS_1		
1	0.5	1.1	0	(0.11,4)	0.28	0.03	0.28	0.49	2.78	77.61	109.54	29.15%
2	0.5	1.1	1	(0.11,4)	0.21	0.11	0.44	0.76	3.57	14.69	38.67	62.02%
3	0.5	1.1	2	(0.11,4)	0.27	0.55	1.44	1.94	5.63	11.27	29.36	61.62%
4	0.5	1.5	0	(0.11,4)	0.18	0.03	0.17	0.40	2.78	2.45	7.79	68.50%
5	0.5	1.5	1	(0.11,4)	0.17	0.11	0.36	0.71	3.57	1.59	5.90	72.99%
6	0.5	1.5	2	(0.11,4)	0.16	0.55	1.05	1.71	5.63	1.89	6.20	69.55%
7	0.5	2	0	(0.11,4)	0.16	0.03	0.15	0.38	2.78	0.48	2.36	79.67%
8	0.5	2	1	(0.11,4)	0.16	0.11	0.32	0.69	3.57	0.51	2.31	77.79%
9	0.5	2	2	(0.11,4)	0.16	0.55	1.05	1.71	5.63	0.79	2.66	70.18%
10	1.2	1.1	0	(0.11,4)	0.15	0.03	0.13	0.37	2.78	4.55	21.67	79.02%
11	1.2	1.1	1	(0.11,4)	0.12	0.11	0.12	0.64	3.57	0.59	5.10	88.48%
12	1.2	1.1	2	(0.11,4)	0.12	0.55	0.58	1.63	5.63	0.41	3.60	88.70%
13	1.2	1.5	0	(0.11,4)	0.16	0.03	0.15	0.38	2.78	0.74	4.14	82.02%
14	1.2	1.5	1	(0.11,4)	0.12	0.11	0.12	0.64	3.57	0.30	2.42	87.39%
15	1.2	1.5	2	(0.11,4)	0.15	0.55	0.99	1.69	5.63	0.29	2.21	86.74%
16	1.2	2	0	(0.11,4)	0.16	0.03	0.14	0.38	2.78	0.30	1.88	83.97%
17	1.2	2	1	(0.11,4)	0.21	0.11	0.44	0.76	3.57	0.25	1.59	84.57%
18	1.2	2	2	(0.11,4)	0.16	0.55	1.05	1.71	5.63	0.27	1.63	83.40%
19	1.5	1.1	0	(0.11,4)	0.12	0.03	0.04	0.34	2.78	1.28	9.86	87.03%
20	1.5	1.1	1	(0.11,4)	0.12	0.11	0.12	0.64	3.57	0.30	2.70	88.94%
21	1.5	1.1	2	(0.11,4)	0.12	0.55	0.58	1.63	5.63	0.22	2.04	88.97%
22	1.5	1.5	0	(0.11,4)	0.13	0.03	0.09	0.35	2.78	0.41	3.01	86.33%
23	1.5	1.5	1	(0.11,4)	0.12	0.11	0.12	0.64	3.57	0.21	1.79	88.53%
24	1.5	1.5	2	(0.11,4)	0.12	0.55	0.58	1.63	5.63	0.19	1.64	88.47%
25	1.5	2	0	(0.11,4)	0.16	0.03	0.14	0.38	2.78	0.24	1.67	85.84%
26	1.5	2	1	(0.11,4)	0.15	0.11	0.32	0.69	3.57	0.18	1.39	87.18%
27	1.5	2	2	(0.11,4)	0.16	0.55	1.05	1.71	5.63	0.19	1.39	86.09%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VSI\text{-}ATS_1\right)/FP\text{-}ARL_1\,\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table 13, Figure 12 and Figure 13).

Table 13. Optimal VSI WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	12.3657	80.18%	1.1	12.3236	80.18%	0	9.7851	80.18%
1.2	0.8556	89.44%	1.5	0.8983	89.44%	1	2.0684	89.44%
1.5	0.3575	90.50%	2	0.3570	90.50%	2	1.7254	90.50%

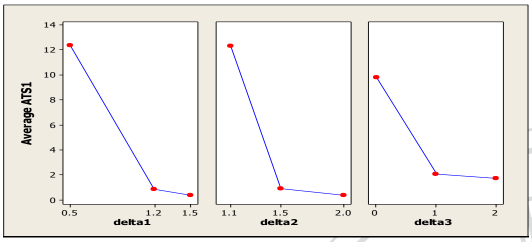

Figure 12. Optimal VSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$

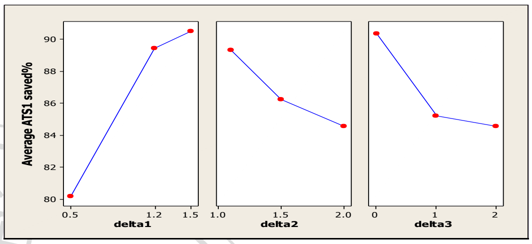

Figure 13. Optimal VSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$

From Table 12, Table 13, Figure 12 and Figure 13, we found

a. ATS_1 saved % from 29.15% to 88.97%.

b. the $\overline{ATS_1}$ is decreasing when δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing when δ_2 increasing.

d. the $\overline{ATS_1}$ is decreasing when δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is increasing when δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing when δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing when δ_3 increasing.

(3) **ATS_1 Analysis of Optimal VSI WL Chart and ATS_1 Comparison between FP WL Chart and Optimal VSI WL Chart with optimal** a

Let h_L = 0.1 and h_U = 4, then the optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 0.1 < h_2 < h_0 < h_1 \le 4$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

The method of finding the optimal values is the same as the method in the part (2) of this section. The ATS_1s of the optimal VSI chart are listed in Table 14.

Table 14. ATS_1 for Optimal VSI WL Chart and ATS_1 saved % Compared to FP WL Chart when a is Optimal

				Optimal VSI (h_1^*, h_2^*, Opt.p^*, a^*)									
No.	δ_1	δ_2	δ_3	(h_2^*,h_1^*)	Opt.p^*	a^*	LCL^*	LWL^*	UWL^*	UCL^*	ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.11,4)	0.37	0.80	0.04	0.43	0.69	2.01	9.50	109.54	91.33%
2	0.5	1.1	1	(0.11,4)	0.17	0.26	0.09	0.32	0.68	4.70	8.08	38.67	79.11%
3	0.5	1.1	2	(0.11,4)	0.18	0.43	0.66	1.41	2.16	7.18	8.34	29.36	71.59%
4	0.5	1.5	0	(0.11,4)	0.20	0.80	0.04	0.23	0.49	2.01	1.21	7.79	84.46%
5	0.5	1.5	1	(0.11,4)	0.18	0.60	0.11	0.38	0.72	3.58	1.62	5.90	72.63%
6	0.5	1.5	2	(0.11,4)	0.15	0.80	0.36	0.70	1.19	3.93	1.19	6.20	80.88%
7	0.5	2	0	(0.11,4)	0.16	0.77	0.04	0.17	0.44	2.36	0.44	2.36	81.44%
8	0.5	2	1	(0.11,4)	0.16	0.71	0.10	0.31	0.67	3.65	0.49	2.31	78.79%
9	0.5	2	2	(0.11,4)	0.16	0.80	0.36	0.71	1.19	3.93	0.50	2.66	81.42%
10	1.2	1.1	0	(0.11,4)	0.12	0.20	0.02	0.03	0.17	1.89	0.43	21.67	98.03%
11	1.2	1.1	1	(0.11,4)	0.12	0.27	0.09	0.15	0.61	4.67	0.36	5.10	92.92%
12	1.2	1.1	2	(0.11,4)	0.12	0.37	0.68	0.89	2.12	7.81	0.35	3.60	90.33%
13	1.2	1.5	0	(0.11,4)	0.16	0.23	0.02	0.08	0.20	1.88	0.36	4.14	91.34%
14	1.2	1.5	1	(0.11,4)	0.12	0.47	0.11	0.17	0.65	3.84	0.28	2.42	88.54%
15	1.2	1.5	2	(0.11,4)	0.15	0.58	0.56	1.05	1.75	5.81	0.29	2.21	86.70%
16	1.2	2	0	(0.11,4)	0.16	0.32	0.02	0.10	0.25	1.92	0.25	1.88	86.72%
17	1.2	2	1	(0.11,4)	0.16	0.56	0.11	0.34	0.70	3.62	0.23	1.59	85.43%
18	1.2	2	2	(0.11,4)	0.16	0.80	0.36	0.73	1.19	3.93	0.24	1.63	84.98%
19	1.5	1.1	0	(0.11,4)	0.12	0.20	0.02	0.03	0.17	1.89	0.21	9.86	97.89%
20	1.5	1.1	1	(0.11,4)	0.12	0.26	0.09	0.15	0.61	4.68	0.20	2.70	92.49%
21	1.5	1.1	2	(0.11,4)	0.12	0.37	0.69	0.90	2.12	7.84	0.20	2.04	90.28%
22	1.5	1.5	0	(0.11,4)	0.12	0.22	0.02	0.03	0.18	1.88	0.20	3.01	93.30%
23	1.5	1.5	1	(0.11,4)	0.12	0.45	0.11	0.17	0.65	3.91	0.19	1.79	89.63%
24	1.5	1.5	2	(0.11,4)	0.12	0.54	0.59	0.77	1.76	6.11	0.19	1.64	88.57%
25	1.5	2	0	(0.11,4)	0.16	0.28	0.02	0.09	0.23	1.88	0.19	1.67	88.67%
26	1.5	2	1	(0.11,4)	0.16	0.52	0.11	0.34	0.70	3.70	0.18	1.39	87.37%
27	1.5	2	2	(0.11,4)	0.16	0.80	0.36	0.73	1.19	3.93	0.19	1.39	86.33%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VSI\text{-}ATS_1\right)/FP\text{-}ARL_1\,\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ARL_1, a^* and ARL_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

15, Figure 14, Figure 15 and Figure 16).

Table 15. Optimal VSI WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$, δ_1, δ_2, δ_3 vs. $\overline{a*}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$

δ_1	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	3.48	0.66	65.72%	1.1	3.07	0.35	74.88%	0	1.42	0.43	75.73%
1.2	0.31	0.42	84.92%	1.5	0.61	0.52	81.17%	1	1.29	0.45	81.99%
1.5	0.19	0.41	87.49%	2	0.30	0.62	82.08%	2	1.28	0.61	80.41%

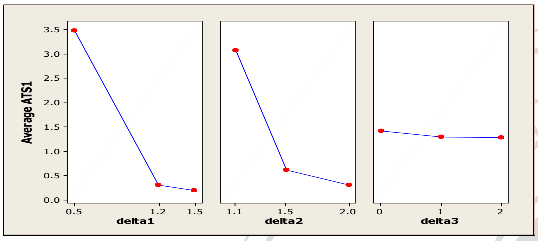

Figure 14. Optimal VSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$

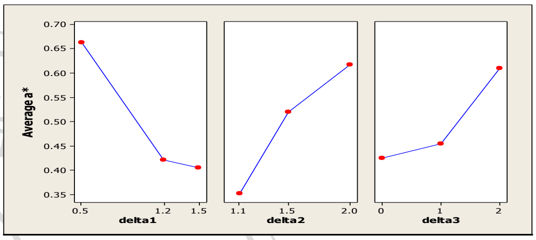

Figure 15. Optimal VSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{a*}$

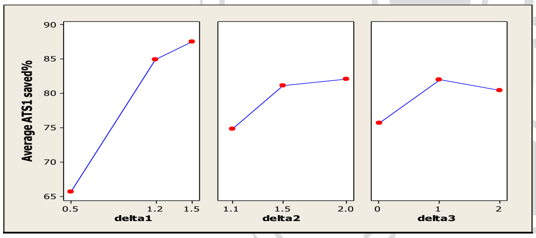

Figure 16. Optimal VSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$

From Table 14, Table 15, Figure 14, Figure 15 and Figure 16, we found

a. ATS_1 saved % from 71.59% to 98.03%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

e. the $\overline{a^*}$ is decreasing as δ_1 increasing.

f. the $\overline{a^*}$ is increasing as δ_2 increasing.

g. the $\overline{a^*}$ is increasing as δ_3 increasing.

h. the $\overline{\text{ATS}_1 \text{ saved }\%}$ is increasing as δ_1 increasing.

i. the $\overline{\text{ATS}_1 \text{ saved }\%}$ is increasing as δ_2 increasing.

j. the $\overline{\text{ATS}_1 \text{ saved }\%}$ is first decreasing then increasing as δ_3 increasing.

(4) **ATS_1 Analysis of Optimal VSSI WL Chart and ATS_1 Comparison between FP WL Chart and Optimal VSSI WL Chart with a = 0.6**

Let h_L = 0.1, n_L = 2 and n_U = 25, then the optimization model is

$$
\begin{aligned}
&\text{Min. ATS}_1 \\
&\text{s.t. } 2 \le n_1 < n_0 < n_2 \le 25 \\
&\qquad 0.1 < h_2 < h_0 = 1 \\
&\qquad 0 \le P_L \le P^* \le P_U \le 1
\end{aligned}
$$

The method of finding the optimal values is the same as the method in the part (2) of this section. The ATS_1s of the optimal VSSI chart are Listed in Table 16.

Table 16. ATS_1 and ANOS for Optimal VSSI WL Chart and ATS_1 Saved % and ANOS Saved % Compared to FP WL Chart when a = 0.6

				Optimal VSSI (n_1^*, n_2^*, h_2^*, Opt.p^*)						FP		ATS_1	ANOS
No.	δ_1	δ_2	δ_3	(n_1^*,n_2^*)	h_2^*	h_1^*	Opt. p^*	ATS_1	ANOS	ARL_1	ANOS	saved %	saved %
1	0.5	1.1	0	(2,25)	0.11	1.13	0.46	59.81	532.72	109.54	547.70	45.40%	2.73%
2	0.5	1.1	1	(2,25)	0.11	1.11	0.45	4.84	112.16	38.67	193.35	87.47%	41.99%
3	0.5	1.1	2	(2,25)	0.11	1.12	0.45	2.88	84.33	29.36	146.81	90.18%	42.55%
4	0.5	1.5	0	(3,25)	0.11	1.10	0.46	1.01	37.41	7.79	38.93	86.97%	3.88%
5	0.5	1.5	1	(3,25)	0.11	1.10	0.46	0.64	32.43	5.90	29.51	89.08%	-9.92%
6	0.5	1.5	2	(3,25)	0.11	1.10	0.45	0.77	35.20	6.20	31.02	87.63%	-13.47%
7	0.5	2	0	(3,25)	0.11	1.10	0.46	0.24	25.07	2.36	11.80	89.92%	-112.44%
8	0.5	2	1	(3,25)	0.11	1.10	0.46	0.26	25.07	2.31	11.55	88.76%	-117.07%
9	0.5	2	2	(3,25)	0.11	1.11	0.46	0.43	25.76	2.66	13.32	83.89%	-93.40%
10	1.2	1.1	0	(3,25)	0.11	1.11	0.45	1.01	48.27	21.67	108.35	95.33%	55.45%
11	1.2	1.1	1	(2,25)	0.11	1.13	0.44	0.12	25.60	5.10	25.51	97.65%	-0.38%
12	1.2	1.1	2	(2,25)	0.11	1.13	0.44	0.11	25.16	3.60	17.99	96.85%	-39.89%
13	1.2	1.5	0	(3,25)	0.11	1.10	0.45	0.23	26.65	4.14	20.72	94.37%	-28.63%
14	1.2	1.5	1	(2,25)	0.11	1.12	0.45	0.13	25.11	2.42	12.09	94.72%	-107.70%
15	1.2	1.5	2	(2,25)	0.11	1.12	0.44	0.12	25.07	2.21	11.05	94.38%	-126.86%
16	1.2	2	0	(3,25)	0.11	1.11	0.45	0.15	24.92	1.88	9.41	92.05%	-164.81%
17	1.2	2	1	(2,25)	0.11	1.12	0.45	0.13	24.92	1.59	7.96	91.79%	-212.92%
18	1.2	2	2	(2,25)	0.11	1.12	0.45	0.15	24.83	1.63	8.12	90.91%	-205.65%
19	1.5	1.1	0	(2,25)	0.11	1.13	0.46	0.24	28.91	9.86	49.32	97.61%	41.38%
20	1.5	1.1	1	(2,25)	0.11	1.13	0.44	0.11	25.01	2.70	13.51	95.92%	-85.10%
21	1.5	1.1	2	(2,25)	0.11	1.13	0.44	0.11	25.00	2.04	10.19	94.60%	-145.44%
22	1.5	1.5	0	(2,25)	0.11	1.11	0.45	0.15	25.33	3.01	15.07	95.15%	-68.11%
23	1.5	1.5	1	(2,25)	0.11	1.13	0.44	0.11	25.00	1.79	8.97	93.73%	-178.76%
24	1.5	1.5	2	(2,25)	0.11	1.13	0.44	0.11	25.00	1.64	8.18	93.13%	-205.56%
25	1.5	2	0	(2,25)	0.11	1.12	0.45	0.13	24.94	1.67	8.34	92.19%	-198.87%
26	1.5	2	1	(2,25)	0.11	1.13	0.44	0.12	24.97	1.39	6.97	91.65%	-258.40%
27	1.5	2	2	(2,25)	0.11	1.13	0.44	0.12	24.93	1.39	6.95	91.26%	-258.56%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VSSI\text{-}ATS_1\right)/FP\text{-}ARL_1\ \%$, ANOS saved %: $\left(FP\text{-}ANOS - Opt.VSSI\text{-}ANOS\right)/FP\text{-}ANOS\ \%$

Table 17. The Control Limits and Warning Limits of the Optimal VSSI Chart with Corresponding (n_1^*, h_1^*) and (n_2^*, h_2^*) which Listed in Table 18

No.	Optimal VSSI (n_1^*, n_2^*, h_2^*, Opt.p*)							
	(n_1^*, h_1^*)				(n_2^*, h_2^*)			
	LCL_1^*	LWL_1^*	UWL_1^*	UCL_1^*	LCL_2^*	LWL_2^*	UWL_2^*	UCL_2^*
1	0.00	0.02	1.79	5.82	0.22	0.32	0.84	1.27
2	0.02	0.05	2.28	5.75	0.45	0.53	1.32	1.88
3	0.22	0.40	3.99	8.12	1.28	1.44	2.67	3.47
4	0.01	0.02	1.66	4.35	0.22	0.28	0.86	1.27
5	0.03	0.09	2.32	5.35	0.45	0.56	1.34	1.88
6	0.26	0.35	3.93	7.64	1.28	1.37	2.69	3.47
7	0.00	0.02	1.68	4.55	0.22	0.29	0.86	1.27
8	0.03	0.09	2.33	5.43	0.45	0.57	1.34	1.88
9	0.25	0.51	4.04	7.76	1.28	1.50	2.71	3.47
10	0.00	0.01	1.63	4.71	0.22	0.25	0.84	1.27
11	0.01	0.02	2.28	6.51	0.45	0.49	1.29	1.88
12	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
13	0.00	0.01	1.61	4.57	0.22	0.24	0.84	1.27
14	0.01	0.06	2.34	6.24	0.45	0.56	1.31	1.88
15	0.19	0.21	3.94	8.46	1.28	1.30	2.65	3.47
16	0.00	0.01	1.64	4.88	0.22	0.27	0.84	1.27
17	0.02	0.06	2.32	6.12	0.45	0.55	1.31	1.88
18	0.21	0.40	4.00	8.17	1.28	1.45	2.67	3.47
19	0.00	0.02	1.79	5.82	0.22	0.32	0.84	1.27
20	0.01	0.03	2.33	7.11	0.45	0.52	1.29	1.88
21	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
22	0.00	0.01	1.66	5.05	0.22	0.27	0.84	1.27
23	0.01	0.03	2.33	7.11	0.45	0.52	1.29	1.88
24	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
25	0.00	0.01	1.67	5.27	0.22	0.28	0.84	1.27
26	0.01	0.02	2.29	6.66	0.45	0.50	1.29	1.88
27	0.18	0.24	3.96	8.63	1.28	1.34	2.64	3.47

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1, ANOS, ATS_1 saved % and ANOS saved %, the response table and response diagram show the results of the sensitivity analysis (Table 18, Figure 17, Figure 18, Table 19, Figure 19 and Figure 20).

Table 18. Optimal VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	7.88	83.26%	1.1	7.69	89.00%	0	7.00	87.67%
1.2	0.24	94.23%	1.5	0.36	92.13%	1	0.72	92.31%
1.5	0.13	93.92%	2	0.19	90.27%	2	0.53	91.43%

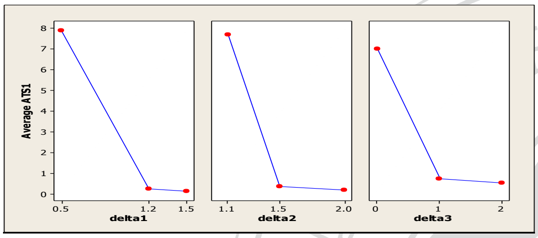

Figure 17. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$

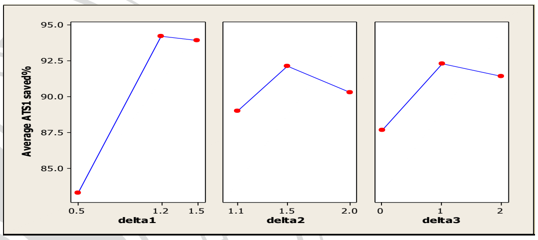

Figure 18. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$

Table 19. Optimal VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ANOS}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ANOS saved } \%}$

δ_1	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$	δ_2	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$	δ_3	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$
0.5	101.13	-28.35%	1.1	100.80	-9.63%	0	86.02	-52.16%
1.2	27.84	-92.38%	1.5	28.58	-81.68%	1	35.59	-103.14%
1.5	25.45	-150.82%	2	25.04	-180.24%	2	32.81	-116.25%

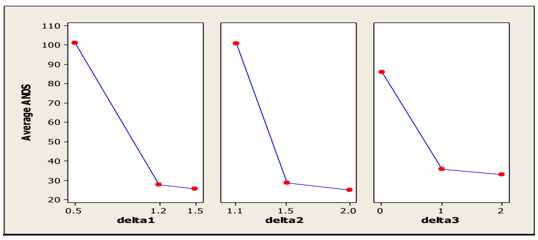

Figure 19. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

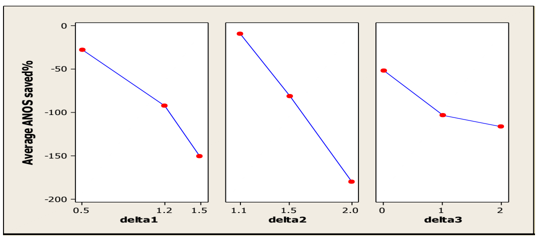

Figure 20. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ANOS saved } \%}$

From Table16, Table 18, Figure 17, Figure 18, Table 19, Figure 19 and Figure 20, we found

a. ATS_1 saved % from 45.40% to97.65%.

b. ANOS saved % from -258.56% to 55.45%.

c. the $\overline{\mathrm{ATS}_1}$ is decreasing as δ_1 increasing.

d. the $\overline{\mathrm{ATS}_1}$ is decreasing as δ_2 increasing.

e. the $\overline{\mathrm{ATS}_1}$ is decreasing as δ_3 increasing.

f. the $\overline{\mathrm{ATS}_1 \text{ saved } \%}$ is first increasing then decreasing as δ_1 increasing.

g. the $\overline{\mathrm{ATS}_1 \text{ saved } \%}$ is first increasing then decreasing as δ_2 increasing.

h. the $\overline{\mathrm{ATS}_1 \text{ saved } \%}$ is first increasing then decreasing as δ_3 increasing.

i. the $\overline{ANOS}$ is decreasing as δ_1 increasing.

j. the $\overline{ANOS}$ is decreasing as δ_2 increasing.

k. the $\overline{ANOS}$ is decreasing as δ_3 increasing.

l. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_1 increasing.

m. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_2 increasing.

n. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_3 increasing.

(5) **$\mathbf{ATS_1}$ Analysis of Optimal VSSI WL Chart and $\mathbf{ATS_1}$ Comparison between FP WL Chart and Optimal VSSI WL Chart with optimal** a

Let h_L = 0.1, n_L = 2 and n_U = 25, then the optimization model is

$$\text{Min. ATS}_1$$

$$\text{s.t. } 2 \le n_1 < n_0 < n_2 \le 25$$

$$0.1 < h_2 < h_0 = 1$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

The method of finding the optimal values is the same as the method in the part (2) of this section. The ATS_1s of the optimal VSSI chart are listed in Table 20.

Table 20. ATS_1 and ANOS for Optimal VSSI WL Chart and ATS_1 Saved % and ANOS Saved % Compared to FP WL Chart when *a* is Optimal

				Optimal VSSI (n_1^*, n_2^*, h_2^*, Opt.p^*, a^*)							FP		ATS_1 saved %	ANOS saved %
No.	δ_1	δ_2	δ_3	(n_1^*,n_2^*)	h_2^*	h_1^*	Opt. p^*	a^*	ATS	ANOS	ARL_1	ANOS		
1	0.5	1.1	0	(2,25)	0.11	1.13	0.44	0.20	5.54	96.80	109.54	547.70	94.94%	82.33%
2	0.5	1.1	1	(2,25)	0.11	1.12	0.45	0.41	2.69	88.08	38.67	193.35	93.04%	54.45%
3	0.5	1.1	2	(2,11)	0.11	1.42	0.34	0.80	2.39	22.85	29.36	146.81	91.87%	84.43%
4	0.5	1.5	0	(2,25)	0.11	1.12	0.44	0.20	0.57	37.22	7.79	38.93	92.70%	4.38%
5	0.5	1.5	1	(3,25)	0.11	1.10	0.46	0.60	0.64	32.40	5.90	29.51	89.09%	-9.79%
6	0.5	1.5	2	(3,25)	0.11	1.10	0.46	0.71	0.71	32.41	6.20	31.02	88.62%	-4.48%
7	0.5	2	0	(2,25)	0.11	1.13	0.45	0.20	0.16	25.88	2.36	11.80	93.05%	-119.28%
8	0.5	2	1	(3,25)	0.11	1.10	0.46	0.76	0.24	25.00	2.31	11.55	89.52%	-116.45%
9	0.5	2	2	(3,11)	0.11	1.33	0.37	0.75	0.36	13.92	2.66	13.32	86.68%	-4.51%
10	1.2	1.1	0	(3,25)	0.11	1.11	0.45	0.20	0.12	25.08	21.67	108.35	99.44%	76.85%
11	1.2	1.1	1	(2,25)	0.11	1.13	0.44	0.39	0.11	25.16	5.10	25.51	97.82%	1.38%
12	1.2	1.1	2	(2,25)	0.11	1.13	0.44	0.52	0.11	25.20	3.60	17.99	96.89%	-40.11%
13	1.2	1.5	0	(2,25)	0.11	1.12	0.45	0.20	0.13	25.04	4.14	20.72	96.98%	-20.87%
14	1.2	1.5	1	(2,25)	0.11	1.13	0.44	0.38	0.11	25.42	2.42	12.09	95.29%	-110.22%
15	1.2	1.5	2	(2,25)	0.11	1.13	0.44	0.52	0.12	25.61	2.21	11.05	94.75%	-131.68%
16	1.2	2	0	(2,25)	0.11	1.12	0.44	0.20	0.12	24.98	1.88	9.41	93.78%	-165.45%
17	1.2	2	1	(2,25)	0.11	1.13	0.44	0.38	0.11	25.46	1.59	7.96	92.84%	-219.74%
18	1.2	2	2	(3,12)	0.11	1.34	0.37	0.74	0.16	12.25	1.63	8.12	90.27%	-50.78%
19	1.5	1.1	0	(2,25)	0.11	1.13	0.44	0.20	0.11	25.00	9.86	49.32	98.88%	49.31%
20	1.5	1.1	1	(2,25)	0.11	1.13	0.44	0.38	0.11	25.00	2.70	13.51	95.93%	-85.02%
21	1.5	1.1	2	(2,25)	0.11	1.13	0.44	0.52	0.11	25.00	2.04	10.19	94.60%	-145.45%
22	1.5	1.5	0	(2,25)	0.11	1.13	0.44	0.20	0.11	25.00	3.01	15.07	96.28%	-65.89%
23	1.5	1.5	1	(2,25)	0.11	1.13	0.44	0.38	0.11	25.02	1.79	8.97	93.87%	-178.98%
24	1.5	1.5	2	(2,25)	0.11	1.13	0.44	0.52	0.11	25.04	1.64	8.18	93.28%	-206.12%
25	1.5	2	0	(2,25)	0.11	1.13	0.44	0.20	0.11	24.99	1.67	8.34	93.23%	-199.45%
26	1.5	2	1	(2,25)	0.11	1.13	0.44	0.38	0.11	25.07	1.39	6.97	92.03%	-259.79%
27	1.5	2	2	(2,11)	0.11	1.37	0.37	0.74	0.13	11.52	1.39	6.95	90.36%	-65.67%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VSSI\text{-}ATS_1\right)/FP\text{-}ARL_1\,\%$, ANOS saved %: $\left(FP\text{-}ANOS - Opt.VSSI\text{-}ANOS\right)/FP\text{-}ANOS\,\%$

Table 21. The Control Limits and Warning Limits of the Optimal VSSI Chart with Corresponding (n_1^*, h_1^*) and (n_2^*, h_2^*) which Listed in Table 22

No.	Optimal VSSI (n_1^*, n_2^*, h_2^*, Opt.p^*, a^*)							
	(n_1^*, h_1^*)				(n_2^*, h_2^*)			
	LCL_1^*	LWL_1^*	UWL_1^*	UCL_1^*	LCL_2^*	LWL_2^*	UWL_2^*	UCL_2^*
1	0.00	0.00	1.17	4.02	0.08	0.09	0.32	0.57
2	0.02	0.06	2.50	6.47	0.42	0.53	1.36	2.06
3	0.11	0.12	1.90	7.20	0.65	0.68	1.78	2.13
4	0.00	0.00	1.15	3.94	0.08	0.08	0.32	0.57
5	0.03	0.09	2.32	5.34	0.45	0.56	1.34	1.88
6	0.22	0.41	3.42	6.88	1.08	1.25	2.29	2.94
7	0.00	0.01	1.22	4.04	0.08	0.11	0.32	0.57
8	0.03	0.07	2.27	5.63	0.44	0.54	1.34	1.89
9	0.22	0.34	2.28	6.54	0.72	0.86	2.03	3.49
10	0.00	0.01	1.12	3.51	0.08	0.10	0.32	0.57
11	0.01	0.03	2.53	7.19	0.42	0.49	1.34	2.14
12	0.17	0.31	4.47	9.89	1.40	1.55	2.93	4.01
13	0.00	0.01	1.17	3.80	0.08	0.10	0.32	0.57
14	0.01	0.03	2.52	7.13	0.42	0.48	1.34	2.33
15	0.21	0.27	4.41	9.42	1.41	1.48	2.95	4.35
16	0.00	0.00	1.15	3.93	0.08	0.08	0.32	0.57
17	0.01	0.02	2.47	6.84	0.42	0.45	1.35	2.55
18	0.18	0.31	2.34	7.29	0.76	0.92	2.05	3.45
19	0.00	0.00	1.17	4.02	0.08	0.09	0.32	0.57
20	0.01	0.03	2.53	7.22	0.42	0.49	1.34	2.20
21	0.17	0.31	4.48	9.91	1.41	1.55	2.93	4.10
22	0.00	0.00	1.19	4.11	0.08	0.09	0.32	0.57
23	0.01	0.03	2.54	7.24	0.42	0.49	1.34	2.37
24	0.17	0.31	4.49	9.93	1.41	1.56	2.94	4.39
25	0.00	0.00	1.20	4.16	0.08	0.09	0.32	0.57
26	0.01	0.03	2.54	7.25	0.42	0.49	1.34	2.59
27	0.17	0.38	2.31	7.49	0.75	0.99	2.03	3.48

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1, ANOS, a^*, ATS_1 saved % and ANOS saved %, the response table and response diagram show the results of the sensitivity analysis (Table 22, Figure 21, Figure 22, Figure 23, Table 23, Figure 24 and Figure 25).

Table 22. Optimal VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$, δ_1, δ_2, δ_3 vs. $\overline{a*}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$

δ_1	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{a*}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	1.48	91.06%	0.514	1.1	1.26	95.93%	0.402	0	0.78	95.48%	0.200
1.2	0.12	95.34%	0.392	1.5	0.29	93.43%	0.413	1	0.47	93.27%	0.450
1.5	0.11	94.27%	0.391	2	0.17	91.31%	0.483	2	0.47	91.92%	0.647

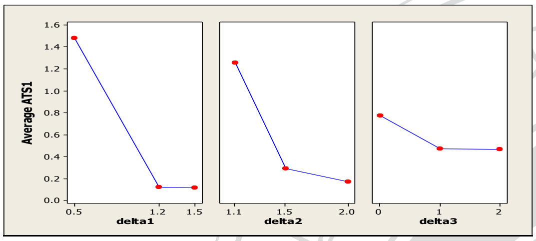

Figure 21. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$

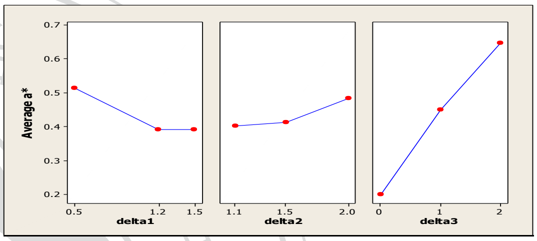

Figure 22. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{a*}$

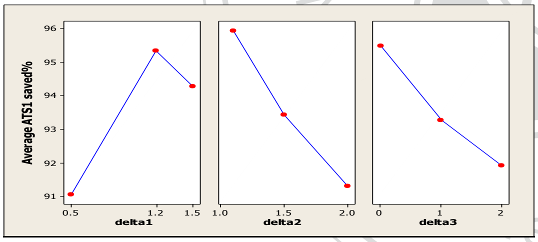

Figure 23. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$

Table 23. Optimal VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ANOS}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ANOS saved } \%}$

δ_1	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$	δ_2	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$	δ_3	$\overline{ANOS}$	$\overline{\text{ANOS saved } \%}$
0.5	41.62	-3.21%	1.1	39.80	8.69%	0	34.44	-39.79%
1.2	23.80	-73.40%	1.5	28.13	-80.41%	1	32.95	-102.68%
1.5	23.52	-128.56%	2	21.01	-133.46%	2	21.53	-62.71%

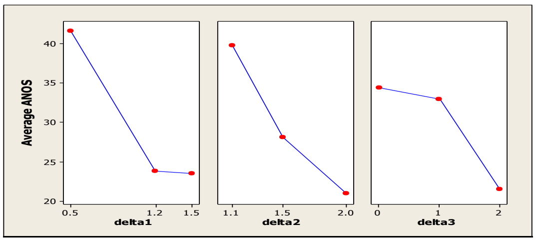

Figure 24. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

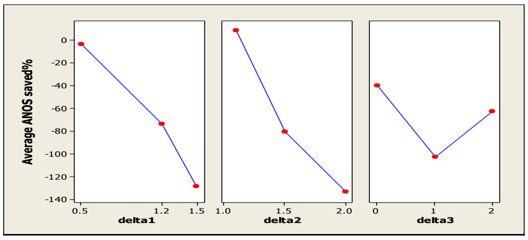

Figure 25. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ANOS saved }\%}$

From Table20, Table 22, Figure 21, Figure 22, Figure 23, Table 23, Figure 24 and Figure 25, we found

a. ATS_1 saved % from 86.68% to 99.44%.

b. ANOS saved % from -259.79% to 84.43%.

c. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

e. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

f. the $\overline{ATS_1\text{ saved }\%}$ is first increasing then decreasing as δ_1 increasing.

g. the $\overline{ATS_1\text{ saved }\%}$ is decreasing as δ_2 increasing.

h. the $\overline{ATS_1\text{ saved }\%}$ is decreasing as δ_3 increasing.

i. the $\overline{a^*}$ is decreasing as δ_1 increasing.

j. the $\overline{a^*}$ is increasing as δ_2 increasing.

k. the $\overline{a^*}$ is increasing as δ_3 increasing.

l. the $\overline{ANOS}$ is decreasing as δ_1 increasing.

m. the $\overline{ANOS}$ is decreasing as δ_2 increasing.

n. the $\overline{ANOS}$ is decreasing as δ_3 increasing.

o. the $\overline{\text{ANOS saved \%}}$ is decreasing as δ_1 increasing.

p. the $\overline{\text{ANOS saved \%}}$ is decreasing as δ_2 increasing.

q. the $\overline{\text{ANOS saved \%}}$ is first decreasing then increasing as δ_3 increasing.

(6) **ATS$_1$ Analysis of Optimal VP WL Chart and ATS$_1$ Comparison between FP WL Chart and Optimal VP WL Chart with a = 0.6**

Let h_L = 0.1, n_L = 2 , n_U = 25 and α_L = 0.001, then the optimization model is

$$\begin{aligned} &\text{Min. ATS}_1 \\ &\text{s.t. } 2 \le n_1 < n_0 < n_2 \le 25 \\ &\quad 0.1 < h_2 < h_0 = 1 \\ &\quad 0.001 \le \alpha_1 \le \alpha_0 = 0.0027 \\ &\quad 0 \le P_L \le P^* \le P_U \le 1 \end{aligned}$$

The method of finding the optimal values is the same as the method in the part (2) of this section. The ATS$_1$s of the optimal VP chart are listed in Table 24.

Table 24. ATS_1 and ANOS for Optimal VP WL Chart and ATS_1 Saved % and ANOS Saved % Compared to FP WL Chart when a = 0.6

				Optimal VP (n_1^*, n_2^*, h_2^*, α_1^*, Opt.p^*)								FP		ATS_1	ANOS
No.	δ_1	δ_2	δ_3	(n_1^*,n_2^*)	h_2^*	h_1^*	α_1^*	α_2^*	Opt. p^*	ATS	ANOS	ARL_1	ANOS	saved %	saved %
1	0.5	1.1	0	(4,25)	0.11	1.04	0.001	0.0367	0.48	34.53	243.55	109.54	547.70	68.48%	55.53%
2	0.5	1.1	1	(3,25)	0.11	1.09	0.001	0.0194	0.46	4.09	63.02	38.67	193.35	89.43%	67.41%
3	0.5	1.1	2	(3,25)	0.11	1.11	0.001	0.0167	0.46	2.79	52.25	29.36	146.81	90.50%	64.41%
4	0.5	1.5	0	(3,25)	0.11	1.10	0.0027	0.0027	0.46	1.01	37.41	7.79	38.93	86.97%	3.88%
5	0.5	1.5	1	(3,25)	0.11	1.10	0.0027	0.0027	0.46	0.64	32.43	5.90	29.51	89.08%	-9.92%
6	0.5	1.5	2	(3,25)	0.11	1.10	0.0027	0.0027	0.45	0.77	35.20	6.20	31.02	87.63%	-13.47%
7	0.5	2	0	(3,25)	0.11	1.10	0.0027	0.0027	0.46	0.24	25.07	2.36	11.80	89.92%	-112.44%
8	0.5	2	1	(3,25)	0.11	1.10	0.0027	0.0027	0.46	0.26	25.07	2.31	11.55	88.76%	-117.07%
9	0.5	2	2	(3,25)	0.11	1.11	0.0027	0.0027	0.45	0.40	25.85	2.66	13.32	85.10%	-94.06%
10	1.2	1.1	0	(3,25)	0.11	1.11	0.0027	0.0027	0.45	1.01	48.27	21.67	108.35	95.33%	55.45%
11	1.2	1.1	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.12	25.60	5.10	25.51	97.65%	-0.38%
12	1.2	1.1	2	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.11	25.16	3.60	17.99	96.85%	-39.89%
13	1.2	1.5	0	(3,25)	0.11	1.10	0.0027	0.0027	0.45	0.23	26.65	4.14	20.72	94.37%	-28.63%
14	1.2	1.5	1	(2,25)	0.11	1.12	0.0027	0.0027	0.45	0.13	25.11	2.42	12.09	94.72%	-107.70%
15	1.2	1.5	2	(2,25)	0.11	1.12	0.0027	0.0027	0.44	0.12	25.07	2.21	11.05	94.38%	-126.86%
16	1.2	2	0	(3,25)	0.11	1.11	0.0027	0.0027	0.45	0.15	24.92	1.88	9.41	92.05%	-164.81%
17	1.2	2	1	(2,25)	0.11	1.12	0.0027	0.0027	0.45	0.13	24.92	1.59	7.96	91.79%	-212.92%
18	1.2	2	2	(2,25)	0.11	1.12	0.0027	0.0027	0.45	0.15	24.83	1.63	8.12	90.91%	-205.65%
19	1.5	1.1	0	(2,25)	0.11	1.11	0.0027	0.0027	0.45	0.21	28.88	9.86	49.32	97.88%	41.44%
20	1.5	1.1	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.11	25.01	2.70	13.51	95.92%	-85.10%
21	1.5	1.1	2	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.11	25.00	2.04	10.19	94.60%	-145.44%
22	1.5	1.5	0	(2,25)	0.11	1.11	0.0027	0.0027	0.45	0.15	25.33	3.01	15.07	95.15%	-68.11%
23	1.5	1.5	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.11	25.00	1.79	8.97	93.73%	-178.76%
24	1.5	1.5	2	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.11	25.00	1.64	8.18	93.13%	-205.56%
25	1.5	2	0	(2,25)	0.11	1.12	0.0027	0.0027	0.45	0.13	24.94	1.67	8.34	92.19%	-198.87%
26	1.5	2	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.12	24.97	1.39	6.97	91.65%	-258.40%
27	1.5	2	2	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.12	24.93	1.39	6.95	91.26%	-258.56%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VP\text{-}ATS_1\right)/FP\text{-}ARL_1\,\%$, ANOS saved %: $\left(FP\text{-}ANOS - Opt.VP\text{-}ANOS\right)/FP\text{-}ANOS\,\%$

Table 25. The Control Limits and Warning Limits of the Optimal VP Chart with Corresponding (n_1^*, h_1^*, α_1^*) and (n_2^*, h_2^*, α_2^*) which Listed in Table 26

	Optimal VP (n_1^*, n_2^*, h_2^*, α_1^*, Opt.p*)							
	(n_1^*, h_1^*, α_1^*)				(n_2^*, h_2^*, α_2^*)			
	LCL_1^*	LWL_1^*	UWL_1^*	UCL_1^*	LCL_2^*	LWL_2^*	UWL_2^*	UCL_2^*
1	0.01	0.03	1.75	3.68	0.31	0.32	0.91	1.03
2	0.02	0.08	2.28	5.60	0.55	0.57	1.34	1.65
3	0.18	0.50	4.05	8.58	1.44	1.54	2.69	3.18
4	0.01	0.02	1.66	4.35	0.22	0.28	0.86	1.27
5	0.03	0.09	2.32	5.35	0.45	0.56	1.34	1.88
6	0.26	0.35	3.93	7.64	1.28	1.37	2.69	3.47
7	0.00	0.02	1.68	4.55	0.22	0.29	0.86	1.27
8	0.03	0.09	2.33	5.43	0.45	0.57	1.34	1.88
9	0.26	0.35	3.93	7.66	1.28	1.37	2.68	3.47
10	0.00	0.01	1.63	4.71	0.22	0.25	0.84	1.27
11	0.01	0.02	2.28	6.51	0.45	0.49	1.29	1.88
12	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
13	0.00	0.01	1.61	4.57	0.22	0.24	0.84	1.27
14	0.01	0.06	2.34	6.24	0.45	0.56	1.31	1.88
15	0.19	0.21	3.94	8.46	1.28	1.30	2.65	3.47
16	0.00	0.01	1.64	4.88	0.22	0.27	0.84	1.27
17	0.02	0.06	2.32	6.12	0.45	0.55	1.31	1.88
18	0.21	0.40	4.00	8.17	1.28	1.45	2.67	3.47
19	0.00	0.01	1.66	5.07	0.22	0.27	0.84	1.27
20	0.01	0.03	2.33	7.11	0.45	0.52	1.29	1.88
21	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
22	0.00	0.01	1.66	5.05	0.22	0.27	0.84	1.27
23	0.01	0.03	2.33	7.11	0.45	0.52	1.29	1.88
24	0.15	0.27	4.03	9.22	1.28	1.41	2.64	3.47
25	0.00	0.01	1.67	5.27	0.22	0.28	0.84	1.27
26	0.01	0.02	2.29	6.66	0.45	0.50	1.29	1.88
27	0.18	0.24	3.96	8.63	1.28	1.34	2.64	3.47

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1, ANOS, ATS_1 saved % and ANOS saved %, the response table and response diagram show the results of the sensitivity analysis (Table 26, Figure 26, Figure 27, Table 27, Figure 28 and Figure 29).

Table 26. Optimal VP WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{\text{ATS}_1}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ATS}_1 \text{ saved \%}}$

δ_1	$\overline{\text{ATS}_1}$	$\overline{\text{ATS}_1 \text{ saved \%}}$	δ_2	$\overline{\text{ATS}_1}$	$\overline{\text{ATS}_1 \text{ saved \%}}$	δ_3	$\overline{\text{ATS}_1}$	$\overline{\text{ATS}_1 \text{ saved \%}}$
0.5	4.97	86.21%	1.1	4.79	91.85%	0	4.18	90.26%
1.2	0.24	94.23%	1.5	0.36	92.13%	1	0.63	92.53%
1.5	0.13	93.95%	2	0.19	90.40%	2	0.52	91.60%

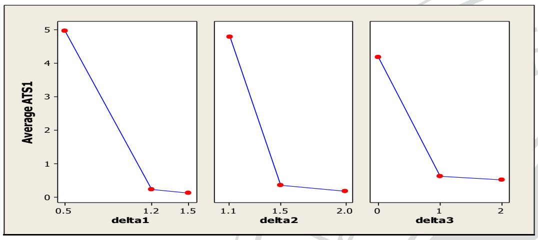

Figure 26. Optimal VP WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1}$

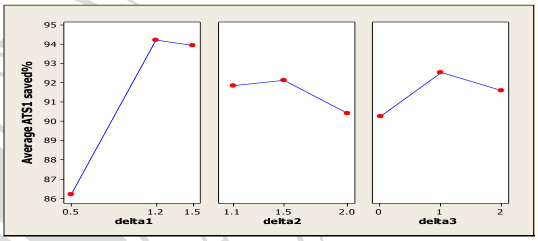

Figure 27. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1 \text{ saved \%}}$

Table 27. Optimal VP WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ANOS}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ANOS saved \%}}$

δ_1	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$	δ_2	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$	δ_3	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$
0.5	59.98	-17.30%	1.1	59.64	1.49%	0	53.89	-46.28%
1.2	27.84	-92.38%	1.5	28.58	-81.68%	1	30.13	-100.32%
1.5	25.45	-150.82%	2	25.05	-180.31%	2	29.25	-113.90%

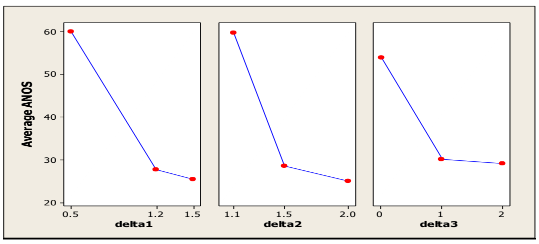

Figure 28. Optimal VP WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

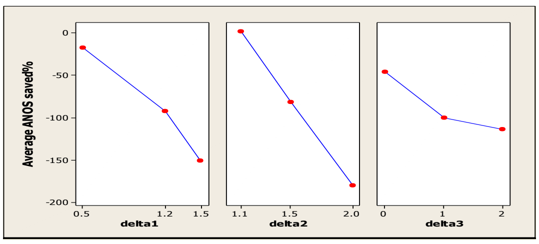

Figure 29. Optimal VSSI WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ANOS saved \%}}$

From Table 24, Table 26, Figure 26, Figure 27, Table 27, Figure 28 and Figure 29, we found

a. ATS_1 saved % from 68.48% to 97.88%.

b. ANOS saved % from -258.56% to 67.41%.

c. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

e. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is first increasing then decreasing as δ_1 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is first increasing then decreasing as δ_2 increasing.

h. the $\overline{ATS_1 \text{ saved } \%}$ is first increasing then decreasing as δ_3 increasing.

i. the $\overline{ANOS}$ is decreasing as δ_1 increasing.

j. the $\overline{ANOS}$ is decreasing as δ_2 increasing.

k. the $\overline{ANOS}$ is decreasing as δ_3 increasing.

l. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_1 increasing.

m. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_2 increasing.

n. the $\overline{\text{ANOS saved } \%}$ is decreasing as δ_3 increasing.

(7) **ATS_1 Analysis of Optimal VP WL Chart and ATS_1 Comparison between FP WL Chart and Optimal VP WL Chart with optimal** a

Let $h_L = 0.1$, $n_L = 2$, $n_U = 25$ and $\alpha_L = 0.001$, then the optimization model is

$$\text{Min. } ATS_1$$

$$\text{s.t. } 2 \le n_1 < n_0 < n_2 \le 25$$

$$0.1 < h_2 < h_0 = 1$$

$$0.001 \le \alpha_1 \le \alpha_0 = 0.0027$$

$$0 \le P_L \le P^* \le P_U \le 1$$

$$0 \le a \le 1$$

The method of finding the optimal values is the same as the method in the part (2) of this section. The ATS_1s of the optimal VP chart are listed in Table 28.

Table 28. ATS_1 for Optimal VP WL Chart and ATS_1 Saved % Compared to FP WL Chart when *a* is Optimal

				Optimal VP (n_1^*, n_2^*, h_2^*, α_1^*, Opt.p^*, a^*)									ATS_1 saved %	ANOS saved %
No.	δ_1	δ_2	δ_3	(n_1^*,n_2^*)	h_2^*	h_1^*	α_1^*	α_2^*	Opt. p^*	a^*	ATS	ANOS		
1	0.5	1.1	0	(2,25)	0.11	1.13	0.0010	0.0140	0.46	0.20	5.29	64.65	95.17%	88.20%
2	0.5	1.1	1	(3,25)	0.11	1.11	0.0010	0.0171	0.45	0.36	2.15	54.54	94.45%	71.79%
3	0.5	1.1	2	(3,25)	0.11	1.11	0.0010	0.0172	0.45	0.51	2.23	52.87	92.40%	63.99%
4	0.5	1.5	0	(2,25)	0.11	1.12	0.0027	0.0027	0.44	0.20	0.57	37.22	92.70%	4.38%
5	0.5	1.5	1	(3,25)	0.11	1.10	0.0027	0.0027	0.46	0.60	0.64	32.40	89.09%	-9.79%
6	0.5	1.5	2	(2,20)	0.11	1.18	0.0027	0.0027	0.43	0.72	0.67	29.93	89.17%	3.54%
7	0.5	2	0	(2,25)	0.11	1.13	0.0027	0.0027	0.45	0.20	0.16	25.88	93.05%	-119.28%
8	0.5	2	1	(3,25)	0.11	1.10	0.0027	0.0027	0.46	0.76	0.24	25.00	89.52%	-116.45%
9	0.5	2	2	(3,11)	0.11	1.35	0.0027	0.0027	0.36	0.75	0.35	13.99	87.05%	-5.02%
10	1.2	1.1	0	(3,25)	0.11	1.11	0.0027	0.0027	0.45	0.20	0.12	25.08	99.44%	76.85%
11	1.2	1.1	1	(2,25)	0.11	1.13	0.0010	0.0142	0.44	0.35	0.11	25.11	97.82%	1.57%
12	1.2	1.1	2	(2,25)	0.11	1.13	0.0010	0.0142	0.44	0.50	0.11	25.17	96.89%	-39.96%
13	1.2	1.5	0	(2,25)	0.11	1.12	0.0027	0.0027	0.45	0.20	0.13	25.04	96.98%	-20.87%
14	1.2	1.5	1	(2,25)	0.11	1.13	0.0010	0.0143	0.44	0.35	0.11	25.41	95.29%	-110.19%
15	1.2	1.5	2	(2,25)	0.11	1.12	0.0010	0.0151	0.44	0.50	0.12	25.60	94.80%	-131.59%
16	1.2	2	0	(2,25)	0.11	1.12	0.0027	0.0027	0.44	0.20	0.12	24.98	93.78%	-165.45%
17	1.2	2	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.38	0.11	25.46	92.84%	-219.74%
18	1.2	2	2	(2,12)	0.11	1.34	0.0027	0.0027	0.37	0.74	0.16	12.41	90.34%	-52.76%
19	1.5	1.1	0	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.20	0.11	25.00	98.88%	49.31%
20	1.5	1.1	1	(2,25)	0.11	1.13	0.0010	0.0141	0.44	0.35	0.11	25.00	95.93%	-85.01%
21	1.5	1.1	2	(2,25)	0.11	1.13	0.0010	0.0140	0.44	0.50	0.11	25.00	94.60%	-145.44%
22	1.5	1.5	0	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.20	0.11	25.00	96.28%	-65.89%
23	1.5	1.5	1	(2,25)	0.11	1.13	0.0010	0.0140	0.44	0.35	0.11	25.02	93.87%	-178.97%
24	1.5	1.5	2	(2,25)	0.11	1.13	0.0010	0.0140	0.45	0.50	0.11	25.05	93.28%	-206.17%
25	1.5	2	0	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.20	0.11	24.99	93.23%	-199.45%
26	1.5	2	1	(2,25)	0.11	1.13	0.0027	0.0027	0.44	0.38	0.11	25.07	92.03%	-259.79%
27	1.5	2	2	(3,11)	0.11	1.36	0.0027	0.0027	0.36	0.74	0.14	10.96	90.22%	-57.71%

Note: ATS_1 saved %: $\left(FP\text{-}ARL_1 - Opt.VSSI\text{-}ATS_1\right)/FP\text{-}ARL_1$ %, ANOS saved %: $\left(FP\text{-}ANOS - Opt.VSSI\text{-}ANOS\right)/FP\text{-}ANOS$ %

Table 29. The Control Limits and Warning Limits of the Optimal VP Chart with Corresponding (n_1^*, h_1^*, α_1^*) and (n_2^*, h_2^*, α_2^*) which Listed in Table 22

No.	Optimal VP (n_1^*, n_2^*, h_2^*, α_1^*, Opt.p^*, a^*)							
	(n_1^*, h_1^*, α_1^*)				(n_2^*, h_2^*, α_2^*)			
	LCL_1^*	LWL_1^*	UWL_1^*	UCL_1^*	LCL_2^*	LWL_2^*	UWL_2^*	UCL_2^*
1	0.00	0.02	1.36	5.14	0.10	0.12	0.32	0.47
2	0.02	0.04	2.47	7.02	0.50	0.51	1.38	1.88
3	0.22	0.37	4.42	9.41	1.60	1.62	3.00	3.65
4	0.00	0.00	1.15	3.94	0.08	0.08	0.32	0.57
5	0.03	0.09	2.32	5.34	0.45	0.56	1.34	1.88
6	0.13	0.31	3.10	8.62	0.98	1.18	2.20	3.07
7	0.00	0.01	1.22	4.04	0.08	0.11	0.32	0.57
8	0.03	0.07	2.27	5.63	0.44	0.54	1.34	1.89
9	0.20	0.26	2.24	6.84	0.72	0.79	2.01	3.48
10	0.00	0.01	1.12	3.51	0.08	0.10	0.32	0.57
11	0.01	0.03	2.56	8.37	0.49	0.51	1.35	2.19
12	0.13	0.30	4.56	11.07	1.59	1.64	2.99	4.11
13	0.00	0.01	1.17	3.80	0.08	0.10	0.32	0.57
14	0.01	0.03	2.55	8.32	0.49	0.51	1.35	2.36
15	0.16	0.18	4.48	10.42	1.60	1.60	3.00	4.43
16	0.00	0.00	1.15	3.93	0.08	0.08	0.32	0.57
17	0.01	0.02	2.47	6.84	0.42	0.45	1.35	2.55
18	0.17	0.31	2.35	7.53	0.77	0.94	2.04	3.44
19	0.00	0.00	1.21	4.23	0.08	0.09	0.31	0.57
20	0.01	0.03	2.57	8.49	0.48	0.51	1.34	2.27
21	0.13	0.30	4.58	11.22	1.59	1.65	2.99	4.20
22	0.00	0.00	1.19	4.11	0.08	0.09	0.32	0.57
23	0.01	0.03	2.58	8.52	0.48	0.51	1.34	2.40
24	0.12	0.45	4.70	11.23	1.59	1.73	3.01	4.50
25	0.00	0.00	1.20	4.16	0.08	0.09	0.32	0.57
26	0.01	0.03	2.54	7.25	0.42	0.49	1.34	2.59
27	0.20	0.30	2.25	6.78	0.72	0.83	2.03	3.54

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1, ANOS, a^*, ATS_1 saved % and ANOS saved %, the response table and response diagram show the results of the sensitivity analysis (Table 30, Figure 30, Figure 31, Figure 32, Table 31, Figure 33 and Figure 34).

Table 30. Optimal VP WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{\text{ATS}_1}$, δ_1, δ_2, δ_3 vs. $\overline{a*}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ATS}_1 \text{ saved \%}}$

δ_1	$\overline{\text{ATS}_1}$	$\overline{a*}$	$\overline{\text{ATS}_1 \text{ saved \%}}$	δ_2	$\overline{\text{ATS}_1}$	$\overline{a*}$	$\overline{\text{ATS}_1 \text{ saved \%}}$	δ_3	$\overline{\text{ATS}_1}$	$\overline{a*}$	$\overline{\text{ATS}_1 \text{ saved \%}}$
0.5	1.37	0.48	91.40%	1.1	1.15	0.35	96.18%	0	0.75	0.20	95.50%
1.2	0.12	0.38	95.35%	1.5	0.29	0.40	93.50%	1	0.41	0.43	93.43%
1.5	0.11	0.38	94.26%	2	0.17	0.48	91.34%	2	0.44	0.61	92.08%

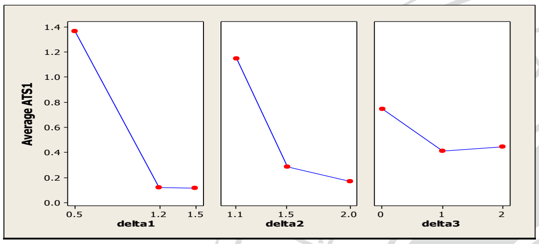

Figure 30. Optimal VP WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1}$

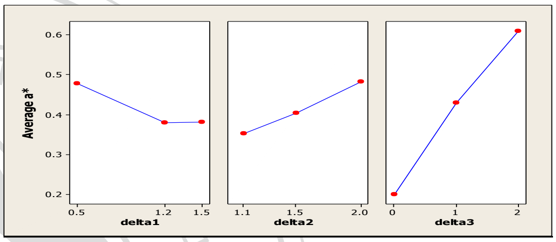

Figure 31. Optimal VP WL Chart– Response Diagram of δ_1, δ_2, δ_3 and $\overline{a*}$

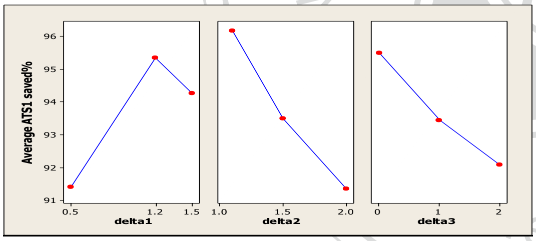

Figure 32. Optimal VP WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ATS}_1 \text{ saved \%}}$

Table 31. Optimal VSSI WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ANOS}$ and δ_1, δ_2, δ_3 vs. $\overline{\text{ANOS saved \%}}$

δ_1	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$	δ_2	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$	δ_3	$\overline{ANOS}$	$\overline{\text{ANOS saved \%}}$
0.5	37.38	-2.07%	1.1	35.82	9.03%	0	30.87	-39.13%
1.2	23.81	-73.57%	1.5	27.85	-79.51%	1	29.22	-100.73%
1.5	23.45	-127.68%	2	20.97	-132.85%	2	24.55	-63.46%

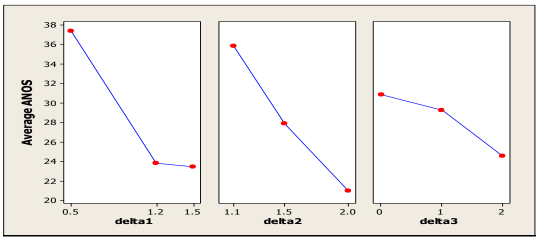

Figure 33. Optimal VPWL Chart –
Response Diagram of δ_1, δ_2, δ_3 and $\overline{ANOS}$

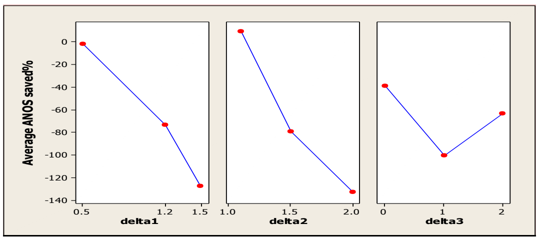

Figure 34. Optimal VP WL Chart –
Response Diagram of δ_1, δ_2, δ_3 and $\overline{\text{ANOS saved \%}}$

From Table 28, Table 30, Figure 30, Figure 31, Figure 32, Table 31, Figure 33 and Figure 34, we found

a. ATS_1 saved % from 90.22% to 99.44%.

b. ANOS saved % from -259.79% to 88.20%.

c. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

e. the $\overline{ATS_1}$ is first decreasing then increasing as δ_3 increasing.

f. the $\overline{ATS_1 \text{ saved \%}}$ is first increasing then decreasing as δ_1 increasing.

g. the $\overline{ATS_1 \text{ saved \%}}$ is decreasing as δ_2 increasing.

h. the $\overline{ATS_1 \text{ saved \%}}$ is decreasing as δ_3 increasing.

i. the $\overline{a^*}$ is decreasing as δ_1 increasing.

j. the $\overline{a^*}$ is increasing as δ_2 increasing.

k. the $\overline{a^*}$ is increasing as δ_3 increasing.

l. the $\overline{ANOS}$ is decreasing as δ_1 increasing.

m. the $\overline{ANOS}$ is decreasing as δ_2 increasing.

n. the $\overline{ANOS}$ is decreasing as δ_3 increasing.

o. the $\overline{\text{ANOS saved \%}}$ is decreasing as δ_1 increasing.

p. the $\overline{\text{ANOS saved \%}}$ is decreasing as δ_2 increasing.

q. the $\overline{\text{ANOS saved \%}}$ is first decreasing then increasing as δ_3 increasing.

4.6 ARL$_1$ Comparison among the WT-WL, Max and One-Sided FP WL Control Charts

(1) One-sided Optimal FP WL Chart, WT-WL Chart and $\bar{X}-S$ Chart

Wu and Tian (2006) proposed the WL chart (say WT-WL chart) to detect mean and variance simultaneously. They let $T=\mu_0=0\ (\delta_3=0)$, $\sigma_0^2=1$, n = 5 and $\alpha=0.0027$. The plotted statistic is $\text{WT-WL}=aS^2+(1-a)(\bar{X}-\mu_0)^2$. The statistic of the WT-WL chart is greater than Γ when $\bar{x}<\mu_0-z$ or $\bar{x}>\mu_0+z$,

or $\mu_0-z<\bar{X}<\mu_0+z$ and $s^2>\dfrac{\Gamma-(1-a)(\bar{x}-\mu_0)^2}{a}$ where $z=\sqrt{\dfrac{\Gamma}{1-a}}$.

Thus, $P(\text{WT-WL}>\Gamma)=P_1+P_2+P_3$

where $\text{P}_1=\text{P}(\bar{x}<\mu_0-\text{z})=\Phi(\dfrac{-\sqrt{n}(z+\delta_1\sigma_0)}{\delta_2\sigma_0})$,

$$\text{P}_2=\text{P}(\bar{x}>\mu_0+\text{z})=1\text{-}\Phi(\frac{\sqrt{n}(z-\delta_1\sigma_0)}{\delta_2\sigma_0}),$$

and $\text{P}_3=\displaystyle\int_{\mu_0-z}^{\mu_0+z}p(\bar{x})f(\bar{x})d\bar{x}$,

where $\bar{x}\sim\text{N}(\mu_0+\delta_1\sigma_0,\dfrac{(\delta_2\sigma_0)^2}{n})$ and

$p(\bar{x})=P(\text{WT-WL}>\Gamma)$

$$= P(s^2 > \frac{\Gamma - (1-a)(\bar{x} - \mu_0)^2}{a})$$

$$= P(\frac{(n-1)s^2}{(\delta_2\sigma_0)^2} > \frac{(n-1)\left[\Gamma - (1-a)(\bar{x} - \mu_0)\right]^2}{a(\delta_2\sigma_0)^2})$$

$$= 1 - \chi_{n-1}(\frac{(n-1)\left[\Gamma - (1-a)(\bar{x} - \mu_0)\right]^2}{a(\delta_2\sigma_0)^2})$$

where $\chi_{n-1}(\cdot)$ is a chi-square r.v. with d.f. (n-1).

The ARL_1 is calculated by

$$ARL_1 = \frac{1}{P(WT - WL > UCL_{WL})}$$

under $ARL_0 = 370$.

For various (δ_1, δ_2), the ARL_1 are listed in Table 32.

Table 32. ARL_1 Comparison among the $\bar{X}-S$, WT-WL and One-sided Optimal FP WL Control Charts with Optimal a

$\delta_2 \setminus \delta_1$		0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
1.0	$\bar{X}-S$	369.99	230.34	86.56	31.40	12.79	6.06	3.35	2.14	1.56	1.26	1.12
	WT-WL	370.00	210.97	76.93	29.15	12.49	6.13	3.46	2.22	1.61	1.30	1.14
	Opt. FP WL	370.35	177.77	56.86	21.76	9.78	5.01	2.92	1.92	1.43	1.19	1.07
	a^*		0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
1.2	$\bar{X}-S$	44.46	37.42	23.99	13.58	7.65	4.56	2.95	2.08	1.60	1.33	1.17
	WT-WL	41.38	32.66	19.27	10.75	6.25	3.91	2.65	1.94	1.53	1.30	1.16
	Opt. FP WL	37.08	31.65	19.49	10.77	6.17	3.79	2.53	1.83	1.44	1.22	1.10
	a^*	0.44	0.39	0.32	0.27	0.24	0.22	0.20	0.20	0.20	0.20	0.20
1.4	$\bar{X}-S$	12.43	11.51	9.31	6.87	4.87	3.48	2.56	1.97	1.60	1.36	1.21
	WT-WL	11.47	10.29	7.83	5.55	3.93	2.87	2.18	1.75	1.46	1.28	1.16
	Opt. FP WL	10.19	9.50	7.71	5.69	4.06	2.93	2.19	1.72	1.42	1.23	1.12
	a^*	0.44	0.42	0.38	0.34	0.32	0.29	0.27	0.26	0.24	0.23	0.21
1.6	$\bar{X}-S$	5.61	5.39	4.81	4.07	3.32	2.68	2.18	1.81	1.55	1.36	1.23
	WT-WL	5.21	4.93	4.26	3.49	2.80	2.27	1.88	1.60	1.40	1.26	1.16
	Opt. FP WL	4.71	4.53	4.07	3.46	2.84	2.31	1.89	1.59	1.37	1.23	1.13
	a^*	0.44	0.43	0.41	0.38	0.36	0.33	0.31	0.30	0.28	0.27	0.25
1.8	$\bar{X}-S$	3.35	3.27	3.07	2.78	2.46	2.15	1.87	1.65	1.47	1.33	1.23
	WT-WL	3.15	3.06	2.81	2.50	2.18	1.90	1.66	1.48	1.34	1.23	1.16
	Opt. FP WL	2.90	2.84	2.67	2.43	2.16	1.89	1.66	1.47	1.32	1.21	1.13
	a^*	0.44	0.44	0.42	0.40	0.38	0.36	0.34	0.32	0.31	0.30	0.28
2.0	$\bar{X}-S$	2.38	2.34	2.25	2.12	1.96	1.80	1.64	1.50	1.38	1.29	1.21
	WT-WL	2.26	2.22	2.11	1.97	1.81	1.65	1.50	1.38	1.28	1.21	1.15
	Opt. FP WL	2.11	2.09	2.01	1.90	1.76	1.62	1.48	1.36	1.26	1.18	1.12
	a^*	0.44	0.44	0.43	0.41	0.40	0.38	0.36	0.35	0.33	0.32	0.31

From Table 32, we found that the ARL_1 of the one-sided FP WL chart with optimal a is always smaller than Shewhart $\bar{X}-S$ chart and most ARL_1 of the one-sided FP WL chart with optimal a are smaller than or same as the ARL_1 of WT-WL chart.

(2) One-sided Optimal FP WL Chart vs. Max Chart

Chen and Cheng (1998) proposed the Max chart to monitor the mean and variance simultaneously. They let $T = \mu_0 = 0\ (\delta_3 = 0)$, $\sigma_0^2 = 1$, n = 5 and $\alpha = 0.0054$.

The construction of the Max chart is as follows:

Define

$$U = \frac{(\bar{X} - \mu)}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$V = \Phi^{-1}\left\{ H\left(\frac{(n-1)S^2}{\sigma^2}; n-1\right) \right\} \sim N(0,1)$$

where $H(w;v) = P(W \le w \mid v)$ for $W \sim \chi_v^2$.

And the statistic of the Max chart is $M(n) = max\{|U|,|V|\}$. The distribution of $M(n)$ is

$$F(y;n) = \{\Phi(y) - \Phi(-y)\}^2 = P(\chi_1^2 \le y^2)^2$$

The control limits of the Max chart are

$$UCL_{Max} = \left\{ \chi^2_{\sqrt{1-\alpha},1} \right\}^{\frac{1}{2}}$$

$$LCL_{Max} = 0$$

And the ARL_1 is calculated by

$$ARL_1 = \frac{1}{P(Max > UCL_{Max})}$$

under $ARL_0 \doteq 185$.

For various (δ_1, δ_2), the ARL_1 are listed in Table 35.

Table 33. ARL_1 Comparison among the Max and One-Sided Optimal FP WL Charts with Optimal a

$\delta_2 \backslash \delta_1$		0	0.5	1	2
1	Max.	185.2	30.7	4.5	1.6
	Opt. FP WL	185.19	22.25	3.82	1.04
	a^*		0.2	0.2	0.2
1.5	Max.	7.3	5.2	2.7	1.6
	Opt. FP WL	5.13	3.93	2.23	1.09
	a^*	0.44	0.38	0.31	0.22
2	Max.	2.3	2.1	1.7	1.4
	Opt. FP WL	1.89	1.77	1.5	1.09
	a^*	0.44	0.42	0.38	0.29

From Table 33, we found that the ARL_1 of the one-sided FP WL chart with optimal a is always smaller than the Max chart.

CHAPTER 5. DESIGN AND ATS_1 ANALYSIS OF THE FP AND VSI EWMA WL CHARTS

5.1 Design of The FP EWMA-WL Chart

Exponentially weighted moving average (EWMA) control chart is effective in detecting small changes in process parameters. Two EWMA charts are generally used for monitoring the process mean and the process variance simultaneously. However, this is inconvenient and time-consuming. So we propose a single EWMA chart based on statistic WL for monitoring both the process mean and process variance.

The FP EWMA WL control chart uses fixed sampling interval h_0, sample size n_0 and false alarm rate α_0. The EWMA statistic is

$$EWMA_i = \lambda WL + (1-\lambda) EWMA_{i-1},$$

where, $0 < \lambda < 1$.

The mean and variance of $WL = aS^2 + (1-a)(\bar{X} - T)^2$ are derived as follows.

$$E\left(WL\right) = E(aS^2 + (1-a)(\bar{X} - T)^2)$$
$$= a\sigma_0^2 + (1-a)\frac{(1+\tau)\sigma_0^2}{n_0}$$

$$Var\left(WL\right) = Var(aS^2 + (1-a)(\bar{X} - T)^2)$$
$$= a^2 \frac{2\sigma_0^4}{n_0 - 1} + (1-a)^2 \frac{2(1+2\tau)\sigma_0^4}{n_0^2}$$

Hence the control limits of the FP EWMA WL chart are

$$UCL = E(WL) + L_1 \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right] Var(WL)}$$

and

$$LCL = E(WL) - L_2 \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right] Var(WL)}$$

where, the L_1 and L_2 are the control factors.

When i is large, the control limits converge to

$$UCL = E(WL) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)} \tag{26}$$

and

$$LCL = E(WL) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)} \tag{27}$$

We use the Markov chain approach (Saccucci and Lucas (1990)) to derive the ARL_1. Before calculating the ARL_1 we have to find the control factors L_1 and L_2 to set the EWMA chart with specified ARL_0.The procedure to determine L_1 and L_2 is:

First, we divide the interval between the UCL and LCL into t subintervals of width 2δ where $\delta = \frac{UCL - LCL}{2t}$ and define the t transient states in Figure 35.

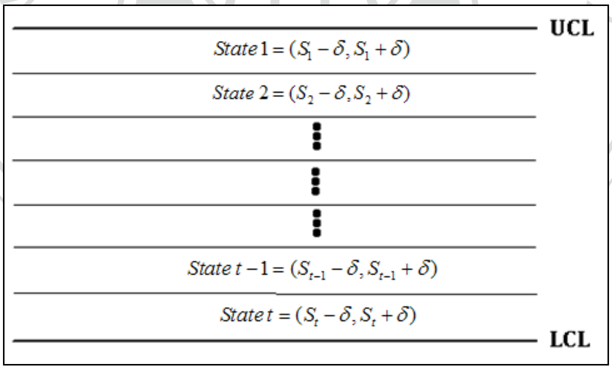

Figure 35. The Structure of FP EWMA WL Chart

Therefore, there have t transient states which

State (j): $S_j - \delta < EWMA_i \le S_j + \delta$ where S_j is the midpoint of the j^{th} interval.

j = 1, 2, • • •, t.

After having defined the states, the transition probability from state j transient to k, p_{jk} could be evaluated:

$$
\begin{aligned}
p_{jk} &= P(\text{going to } S_k \mid \text{in } S_j) \\
&= P\left((S_k - \delta) \le EWMA_i \le (S_k + \delta) \middle| EWMA_{i-1} = S_j \right) \\
&= P\left((S_k - \delta) \le \lambda WL_i + (1-\lambda) EWMA_{i-1} \le (S_k + \delta) \middle| EWMA_{i-1} = S_j \right) \\
&= P\left(\frac{(S_k - \delta) - (1-\lambda) EWMA_{i-1}}{\lambda} \le WL_i \le \frac{(S_k + \delta) - (1-\lambda) EWMA_{i-1}}{\lambda} \middle| EWMA_{i-1} = S_j , WL_i \sim Q_{n_0,\tau_0} \right) \\
&= P\left(\frac{(S_k - \delta) - (1-\lambda) S_j}{\lambda} \le WL_i \le \frac{(S_k + \delta) - (1-\lambda) S_j}{\lambda} \mid WL_i \sim Q_{n_0,\tau_0} \right)
\end{aligned}
$$

The transition probability matrix which contains the transient probability from state i transient to state j is

$$
Q = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & \cdots & p_{1t} \\
p_{21} & p_{22} & & & p_{2t} \\
p_{31} & & \ddots & & \vdots \\
 & & & \ddots & \vdots \\
p_{t1} & p_{t2} & \cdots & \cdots & p_{tt}
\end{bmatrix}
$$

The ARL_0 is,

$$
ARL_0 = \mathbf{r}'(\mathbf{I} - \mathbf{Q})^{-1},
$$

where $\mathbf{I}$ is the identity matrix of order t and $\mathbf{r}'$ is a (1xt) vector with the steady-state starting probability for transient state which $\mathbf{r}' = (r_1, r_2, \ldots\ldots, r_t)$ s.t. $\mathbf{r}'\mathbf{Q} = \mathbf{r}'$ s.t. $\sum_{j=1}^{t} r_j = 1$.

The procedure in getting the control factors L_1 and L_2 is as follows:

Step 1: Determine L_1 value.

Set false alarm rate α_0, and let $ARL_0 = \frac{2}{\alpha_0}$. The control limits of the chart are $UCL = E(WL) + L_1\sqrt{\frac{\lambda}{(2-\lambda)} Var(WL)}$ and $LCL = 0$. Using Zero(s) in Fortran IMSL subroutine which applied Muller's method to solve the equation, $\mathbf{r}'(\mathbf{I} - \mathbf{Q})^{-1} - \frac{2}{\alpha_0} = 0$.

The L_1 is thus calculated, and UCL is obtained.

Step 2: Determine L_2 value.

Given UCL in step 1, and $LCL = E(WL) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)}$. Using the same approach in step 1 to solve the equation, $\mathbf{r}'(\mathbf{I}-\mathbf{Q})^{-1} - \frac{1}{\alpha_0} = 0$, to get L_2.

5.2 Design of The VSI EWMA-WL Chart

The variable sampling intervals (VSI) EWMA WL control chart uses variable sampling intervals h_q, q = 1, 2, fixed sample size n_0 and false alarm rate α_0. Then the VSI EWMA WL control limits and warning limits are

$$UCL = E(WL) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]Var(WL)}$$

$$UWL = E(WL) + W_1\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]Var(WL)}$$

$$LWL = E(WL) - W_2\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]Var(WL)}$$

$$LCL = E(WL) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]Var(WL)}$$

where $0 < \lambda \le 1$ and the L_1, L_2, W_1 and W_2 are the control factors and warning factors. When i is large, the control limits and warning limits converge to

$$UCL = E(WL) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)}$$

$$W_U = E(WL) + W_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)}$$

$$W_L = E(WL) - W_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)}$$

$$LCL = E(WL) - L_2 \sqrt{\frac{\lambda}{(2-\lambda)} Var(WL)}$$

The variable sampling intervals h_q, q = 1, 2, are adopted where $0 < h_2 < h_0 < h_1 < \infty$ and h_0 is the sampling interval for FP EWMA WL chart.

Use the same approach above to derive the ATS_0.

First, we divide the interval between the UCL and LCL into t subintervals and each interval with width 2δ, where $\delta = \frac{UCL - LCL}{2t}$, and midpoint S_j, j = 1, 2, …, t. For the interval including UWL, we divide it into 2 subintervals and let the subintervals be state t_1 and t_2, where state t_1 = $(S_{t_1} - \gamma_1, S_{t_1} + \gamma_1)$ and t_2 = $(S_{t_2} - \gamma_2, S_{t_2} + \gamma_2)$, where $t_2 = t_1 + 1$. Same to the interval including LWL, where $\gamma_1 + \gamma_2 = \gamma_3 + \gamma_4 = \delta$. Hence the total number of intervals is t+2, or the process with t+2 states, See Figure 36.

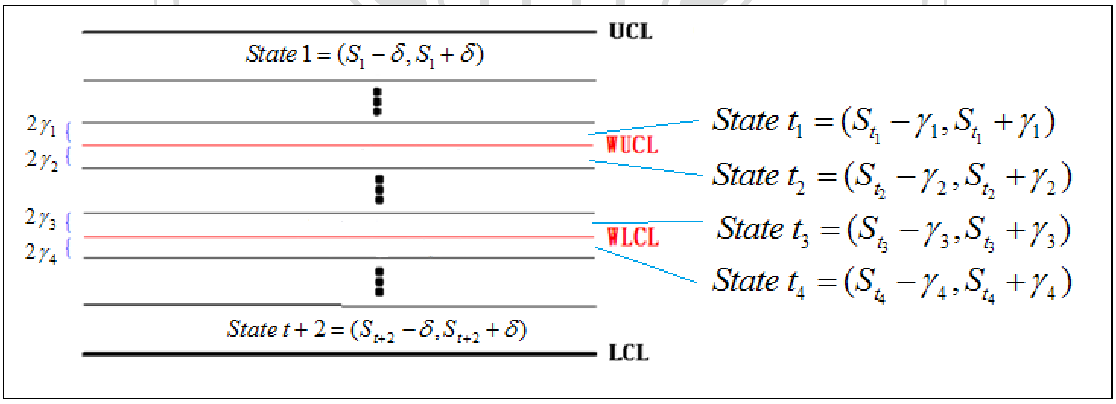

Figure 36. The Structure of VSI EWMA WL Chart

If the statistic EWMA WL falls into the central region, the next sampling interval should have long interval h_1. If EWMA WL falls into the warning region, the next sampling interval should have short interval h_2. If EWMA WL falls into the action region, an out-of-control signal would have occurred. The relationship between the position of the current sample and the next sampling interval could be expressed in

Table 34.

Table 34. Definition of Process States for the VSI EWMA WL Chart

State	Region	Alarm	Next Sampling Interval (h_q)
state t_2 to state t_3	Central	No	h_1
state 1 to state t_1 and state t_4 to state $t+2$	Warning	No	h_2

After having defined the states, the transition probability from state j transient to k,

p_{jk} is:

$$
\begin{aligned}
p_{jk} &= P(\text{going to } S_k \mid \text{in } S_j) \\
&= P\left((S_k-\delta)\le EWMA_i \le (S_k+\delta)\middle| EWMA_{i-1}=S_j\right) \\
&= P\left((S_k-\delta)\le \lambda WL_i+(1-\lambda)EWMA_{i-1} \le (S_k+\delta)\middle| EWMA_{i-1}=S_j\right) \\
&= P\left(\frac{(S_k-\delta)-(1-\lambda)EWMA_{i-1}}{\lambda}\le WL_i \le \frac{(S_k+\delta)-(1-\lambda)EWMA_{i-1}}{\lambda}\middle| EWMA_{i-1}=S_j, WL_i \sim Q_{n_0,\tau_0}\right) \\
&= P\left(\frac{(S_k-\delta)-(1-\lambda)S_j}{\lambda}\le WL_i \le \frac{(S_k+\delta)-(1-\lambda)S_j}{\lambda} \mid WL_i \sim Q_{n_0,\tau_0}\right)
\end{aligned}
$$

for $j=1,2,......,t+2$ and $k\ne t_1,t_2,t_3,t_4$.

$$
\begin{aligned}
p_{jt_q} &= P(\text{going to } S_{t_q} \mid \text{in } S_j) \\
&= P\left((S_{t_q}-\gamma_q)\le EWMA_i \le (S_{t_q}+\gamma_q)\middle| EWMA_{i-1}=S_j\right) \\
&= P\left((S_{t_q}-\gamma_q)\le \lambda WL_i+(1-\lambda)EWMA_{i-1} \le (S_{t_q}+\gamma_q)\middle| EWMA_{i-1}=S_j\right) \\
&= P\left(\frac{(S_{t_q}-\gamma_q)-(1-\lambda)EWMA_{i-1}}{\lambda}\le WL_i \le \frac{(S_{t_q}+\gamma_q)-(1-\lambda)EWMA_{i-1}}{\lambda}\middle| EWMA_{i-1}=S_j, WL_i \sim Q_{n_0,\tau_0}\right) \\
&= P\left(\frac{(S_{t_q}-\gamma_q)-(1-\lambda)S_j}{\lambda}\le WL_i \le \frac{(S_{t_q}+\gamma_q)-(1-\lambda)S_j}{\lambda} \mid WL_i \sim Q_{n_0,\tau_0}\right)
\end{aligned}
$$

for $j=1,2,......,t+2$ and $q=1,2,3,4$.

The transition probability matrix which contains the transient probability from state i to state j is

$$
\mathbf{Q} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1,t+2} \\ p_{21} & p_{22} & & & p_{2,t+2} \\ p_{31} & & \ddots & & \vdots \\ & & & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & \cdots & p_{t+2,t+2} \end{bmatrix}
$$

The ATS is obtained by

$$
ATS = \mathbf{r}'(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{h}
$$

where $\mathbf{I}$ is the identity matrix of order t+2, $\mathbf{h}' = (\underbrace{h_2,\ldots,h_2}_{t_1}, \underbrace{h_1,\ldots,h_1}_{t_3-t_1}, \underbrace{h_2,\ldots,h_2}_{t+2-t_3})$ is a $1\times(t+2)$ vector of sampling time for transient state i, i = 1, • • •, t+2 and $\mathbf{r}'$ is a $1\times(t+2)$ vector with the steady-state starting probability for transient state where $\mathbf{r}' = (r_1, r_2, \ldots\ldots, r_{t+2})$ s.t. $\mathbf{r}'\mathbf{Q} = \mathbf{r}'$ s.t. $\sum_{j=1}^{t+2} r_j = 1$. The transition probabilities, steady-state starting probabilities and ATS could be calculated by Fortran program.

The procedure in getting the control factors L_1, L_2, W_1 and W_2 is as follows:

Step 1: Determine L_1 and L_2 values.

The L_1 and L_2 are known by FP EWMA WL chart.

Step 2: Determine W_1 and W_2 values.

Give W_2 = 0.5, 1.0, 1.5… < L_2. The false alarm rate α_0, so $ARL_0 = \frac{1}{\alpha_0}$.

Using Zero(s) in Fortran IMSL subroutine which use Muller's method to solve the function $\mathbf{r}'(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{h} - \frac{1}{\alpha_0} = 0$. Then we can find W_1 under different W_2.

We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 to construct the VSI EWMA WL chart under the given combination of δ_3 and (h_2, h_1).

5.3 Data Analysis and ATS_1 Comparison among the FP EWMA, VSI EWMA and FP WL Charts

(1) ARL_1 Analysis of FP EWMA WL Chart and ARL_1 Comparison between FP EWMA Chart and FP WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or $ARL_0 = 370.37$) and $a = 0.6$. Consider the levels of parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2), δ_3 = (0, 1, 2) and h_2 = (0.1, 0.5, 0.9). The 27 combinations are arranged by $L_{27}(3^{13})$.

Similar to Section 5.1, the values of L_1 and L_2 under the combinations of λ= 0.05, 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and δ_3 = 0, 1, 2 are listed in Table 35.

Table 35. The Values of L_1 and L_2 under Various Values of λ and δ_3

	λ	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\delta_3 = 0$	L_1	2.74	3.15	3.41	3.61	3.78	3.92	4.16	4.35	4.50	4.62	4.71	4.76
	L_2	2.27	2.29	2.26	2.21	2.16	2.10	1.99	1.89	1.79	1.70	1.61	1.53
$\delta_3 = 1$	L_1	2.68	3.05	3.28	3.45	3.59	3.71	3.90	4.06	4.18	4.28	4.35	4.39
	L_2	2.32	2.38	2.36	2.33	2.29	2.25	2.16	2.07	1.98	1.90	1.82	1.75
$\delta_3 = 2$	L_1	2.63	2.95	3.14	3.27	3.38	3.47	3.61	3.73	3.82	3.89	3.95	3.98
	L_2	2.38	2.48	2.49	2.48	2.46	2.44	2.38	2.31	2.25	2.19	2.13	2.08

The ARL_1s of the 27 combinations of δ_1, δ_2 and δ_3 under λ = 0.05 are listed in Table 36.

Table 36. ARL_1 for FP EWMA WL and ATS_1 Saved % Compared to FP WL Chart under $\lambda = 0.05$

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	2.74	2.27	0.87	0.52	25.50	109.54	76.72%
2	0.5	1.1	1	2.68	2.32	1.32	0.87	10.23	38.67	73.55%
3	0.5	1.1	2	2.63	2.38	2.63	1.96	8.92	29.36	69.61%
4	0.5	1.5	0	2.74	2.27	0.87	0.52	5.47	7.79	29.74%
5	0.5	1.5	1	2.68	2.32	1.32	0.87	4.81	5.90	18.43%
6	0.5	1.5	2	2.63	2.38	2.63	1.96	5.30	6.20	14.60%
7	0.5	2	0	2.74	2.27	0.87	0.52	2.71	2.36	-15.00%
8	0.5	2	1	2.68	2.32	1.32	0.87	2.81	2.31	-21.77%
9	0.5	2	2	2.63	2.38	2.63	1.96	3.36	2.66	-26.09%
10	1.2	1.1	0	2.74	2.27	0.87	0.52	7.02	21.67	67.59%
11	1.2	1.1	1	2.68	2.32	1.32	0.87	3.75	5.10	26.44%
12	1.2	1.1	2	2.63	2.38	2.63	1.96	3.44	3.60	4.34%
13	1.2	1.5	0	2.74	2.27	0.87	0.52	3.69	4.14	10.93%
14	1.2	1.5	1	2.68	2.32	1.32	0.87	2.83	2.42	-16.83%
15	1.2	1.5	2	2.63	2.38	2.63	1.96	2.86	2.21	-29.40%
16	1.2	2	0	2.74	2.27	0.87	0.52	2.30	1.88	-21.94%
17	1.2	2	1	2.68	2.32	1.32	0.87	2.11	1.59	-32.64%
18	1.2	2	2	2.63	2.38	2.63	1.96	2.30	1.63	-41.35%
19	1.5	1.1	0	2.74	2.27	0.87	0.52	4.79	9.86	51.49%
20	1.5	1.1	1	2.68	2.32	1.32	0.87	2.88	2.70	-6.44%
21	1.5	1.1	2	2.63	2.38	2.63	1.96	2.71	2.04	-32.89%
22	1.5	1.5	0	2.74	2.27	0.87	0.52	3.06	3.01	-1.56%
23	1.5	1.5	1	2.68	2.32	1.32	0.87	2.36	1.79	-31.27%
24	1.5	1.5	2	2.63	2.38	2.63	1.96	2.38	1.64	-45.17%
25	1.5	2	0	2.74	2.27	0.87	0.52	2.09	1.67	-25.16%
26	1.5	2	1	2.68	2.32	1.32	0.87	1.88	1.39	-35.25%
27	1.5	2	2	2.63	2.38	2.63	1.96	2.01	1.39	-44.75%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ARL_1 and ARL_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table 37, Figure 37 and Figure 38).

Table 37. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.05

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	7.68	24.42%	1.1	7.69	36.71%	0	6.29	19.20%
1.2	3.37	-3.65%	1.5	3.64	-5.61%	1	3.74	-2.86%
1.5	2.68	-19.00%	2	2.40	-29.33%	2	3.70	-14.57%

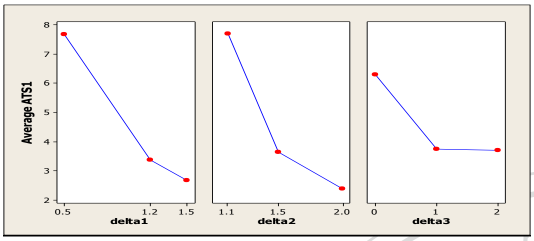

Figure 37. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.05

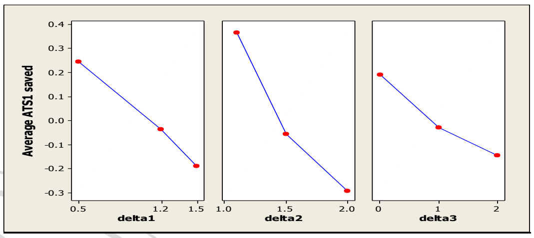

Figure 38. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.05

From Table 36, Table 37, Figure 37 and Figure 38, we found

a. ARL_1 saved % from -45.17% to 76.72%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.1 are listed in Table 38.

Table 38. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.1

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	3.15	2.29	1.00	0.45	28.42	109.54	74.06%
2	0.5	1.1	1	3.05	2.38	1.48	0.77	9.73	38.67	74.84%
3	0.5	1.1	2	2.95	2.48	2.85	1.80	8.20	29.36	72.06%
4	0.5	1.5	0	3.15	2.29	1.00	0.45	4.93	7.79	36.63%
5	0.5	1.5	1	3.05	2.38	1.48	0.77	4.26	5.90	27.80%
6	0.5	1.5	2	2.95	2.48	2.85	1.80	4.64	6.20	25.19%
7	0.5	2	0	3.15	2.29	1.00	0.45	2.42	2.36	-2.42%
8	0.5	2	1	3.05	2.38	1.48	0.77	2.48	2.31	-7.23%
9	0.5	2	2	2.95	2.48	2.85	1.80	2.92	2.66	-9.53%
10	1.2	1.1	0	3.15	2.29	1.00	0.45	6.47	21.67	70.13%
11	1.2	1.1	1	3.05	2.38	1.48	0.77	3.27	5.10	35.85%
12	1.2	1.1	2	2.95	2.48	2.85	1.80	2.95	3.60	17.96%
13	1.2	1.5	0	3.15	2.29	1.00	0.45	3.27	4.14	20.99%
14	1.2	1.5	1	3.05	2.38	1.48	0.77	2.46	2.42	-1.90%
15	1.2	1.5	2	2.95	2.48	2.85	1.80	2.46	2.21	-11.35%
16	1.2	2	0	3.15	2.29	1.00	0.45	2.05	1.88	-8.66%
17	1.2	2	1	3.05	2.38	1.48	0.77	1.87	1.59	-17.07%
18	1.2	2	2	2.95	2.48	2.85	1.80	2.00	1.63	-22.83%
19	1.5	1.1	0	3.15	2.29	1.00	0.45	4.28	9.86	56.66%
20	1.5	1.1	1	3.05	2.38	1.48	0.77	2.49	2.70	7.81%
21	1.5	1.1	2	2.95	2.48	2.85	1.80	2.31	2.04	-13.55%
22	1.5	1.5	0	3.15	2.29	1.00	0.45	2.70	3.01	10.32%
23	1.5	1.5	1	3.05	2.38	1.48	0.77	2.05	1.79	-14.44%
24	1.5	1.5	2	2.95	2.48	2.85	1.80	2.04	1.64	-24.82%
25	1.5	2	0	3.15	2.29	1.00	0.45	1.87	1.67	-11.74%
26	1.5	2	1	3.05	2.38	1.48	0.77	1.67	1.39	-19.67%
27	1.5	2	2	2.95	2.48	2.85	1.80	1.75	1.39	-26.04%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\ ARL_1\right) / FP\ ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 39, Figure 39 and Figure 40).

Table 39. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved }\%}$ under λ=0.1

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved }\%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved }\%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved }\%}$
0.5	7.56	32.38%	1.1	7.57	43.98%	0	6.27	27.33%
1.2	2.98	9.24%	1.5	3.20	7.60%	1	3.36	9.55%
1.5	2.35	-3.94%	2	2.11	-13.91%	2	3.25	0.79%

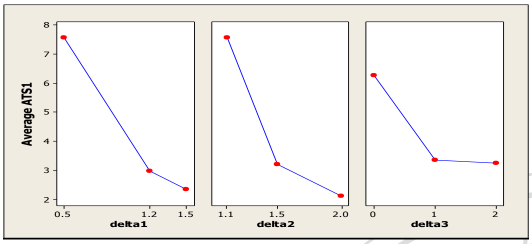

Figure 39. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.1

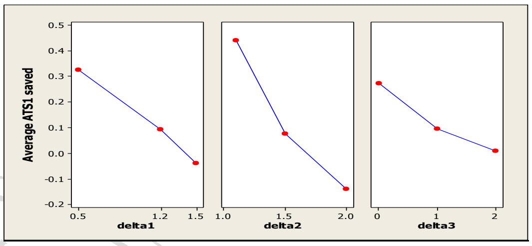

Figure 40. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved }\%}$ under λ = 0.1

From Table 38, Table 39, Figure 39 and Figure 40, we found

a. ARL_1 saved % from -26.04% to 74.84%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved }\%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved }\%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved }\%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.15 are listed in Table 40.

Table 40. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.15

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	3.41	2.26	1.11	0.40	32.85	109.54	70.02%
2	0.5	1.1	1	3.28	2.36	1.61	0.70	9.97	38.67	74.21%
3	0.5	1.1	2	3.14	2.49	3.03	1.68	8.15	29.36	72.24%
4	0.5	1.5	0	3.41	2.26	1.11	0.40	4.75	7.79	38.99%
5	0.5	1.5	1	3.28	2.36	1.61	0.70	4.04	5.90	31.60%
6	0.5	1.5	2	3.14	2.49	3.03	1.68	4.37	6.20	29.63%
7	0.5	2	0	3.41	2.26	1.11	0.40	2.28	2.36	3.31%
8	0.5	2	1	3.28	2.36	1.61	0.70	2.32	2.31	-0.48%
9	0.5	2	2	3.14	2.49	3.03	1.68	2.71	2.66	-1.76%
10	1.2	1.1	0	3.41	2.26	1.11	0.40	6.42	21.67	70.38%
11	1.2	1.1	1	3.28	2.36	1.61	0.70	3.06	5.10	39.96%
12	1.2	1.1	2	3.14	2.49	3.03	1.68	2.72	3.60	24.35%
13	1.2	1.5	0	3.41	2.26	1.11	0.40	3.10	4.14	25.24%
14	1.2	1.5	1	3.28	2.36	1.61	0.70	2.30	2.42	5.09%
15	1.2	1.5	2	3.14	2.49	3.03	1.68	2.27	2.21	-2.62%
16	1.2	2	0	3.41	2.26	1.11	0.40	1.93	1.88	-2.50%
17	1.2	2	1	3.28	2.36	1.61	0.70	1.75	1.59	-9.60%
18	1.2	2	2	3.14	2.49	3.03	1.68	1.85	1.63	-13.85%
19	1.5	1.1	0	3.41	2.26	1.11	0.40	4.10	9.86	58.46%
20	1.5	1.1	1	3.28	2.36	1.61	0.70	2.31	2.70	14.54%
21	1.5	1.1	2	3.14	2.49	3.03	1.68	2.12	2.04	-4.07%
22	1.5	1.5	0	3.41	2.26	1.11	0.40	2.54	3.01	15.63%
23	1.5	1.5	1	3.28	2.36	1.61	0.70	1.91	1.79	-6.30%
24	1.5	1.5	2	3.14	2.49	3.03	1.68	1.88	1.64	-14.85%
25	1.5	2	0	3.41	2.26	1.11	0.40	1.76	1.67	-5.45%
26	1.5	2	1	3.28	2.36	1.61	0.70	1.56	1.39	-12.28%
27	1.5	2	2	3.14	2.49	3.03	1.68	1.63	1.39	-16.91%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 41, Figure 41 and Figure 42).

Table 41. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.15

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	7.94	35.31%	1.1	7.97	46.68%	0	6.64	30.45%
1.2	2.82	15.16%	1.5	3.02	13.60%	1	3.25	15.19%
1.5	2.20	3.20%	2	1.98	-6.61%	2	3.08	8.02%

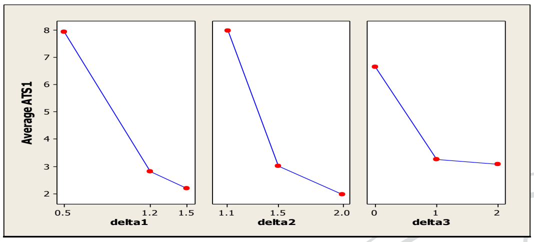

Figure 41. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.15

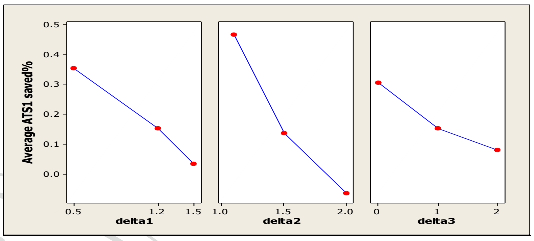

Figure 42. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.15

From Table 40, Table 41, Figure 41 and Figure 42, we found

a. ARL_1 saved % from -16.91% to 74.21%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.2 are listed in Table 42.

Table 42. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.2

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	3.61	2.21	1.21	0.36	37.94	109.54	65.37%
2	0.5	1.1	1	3.45	2.33	1.73	0.64	10.58	38.67	72.65%
3	0.5	1.1	2	3.27	2.48	3.20	1.59	8.39	29.36	71.41%
4	0.5	1.5	0	3.61	2.21	1.21	0.36	4.70	7.79	39.58%
5	0.5	1.5	1	3.45	2.33	1.73	0.64	3.94	5.90	33.26%
6	0.5	1.5	2	3.27	2.48	3.20	1.59	4.24	6.20	31.74%
7	0.5	2	0	3.61	2.21	1.21	0.36	2.21	2.36	6.48%
8	0.5	2	1	3.45	2.33	1.73	0.64	2.23	2.31	3.42%
9	0.5	2	2	3.27	2.48	3.20	1.59	2.59	2.66	2.78%
10	1.2	1.1	0	3.61	2.21	1.21	0.36	6.59	21.67	69.60%
11	1.2	1.1	1	3.45	2.33	1.73	0.64	2.96	5.10	42.06%
12	1.2	1.1	2	3.27	2.48	3.20	1.59	2.59	3.60	27.97%
13	1.2	1.5	0	3.61	2.21	1.21	0.36	3.01	4.14	27.29%
14	1.2	1.5	1	3.45	2.33	1.73	0.64	2.20	2.42	9.10%
15	1.2	1.5	2	3.27	2.48	3.20	1.59	2.15	2.21	2.58%
16	1.2	2	0	3.61	2.21	1.21	0.36	1.86	1.88	1.01%
17	1.2	2	1	3.45	2.33	1.73	0.64	1.68	1.59	-5.21%
18	1.2	2	2	3.27	2.48	3.20	1.59	1.76	1.63	-8.37%
19	1.5	1.1	0	3.61	2.21	1.21	0.36	4.05	9.86	58.90%
20	1.5	1.1	1	3.45	2.33	1.73	0.64	2.20	2.70	18.46%
21	1.5	1.1	2	3.27	2.48	3.20	1.59	2.00	2.04	1.72%
22	1.5	1.5	0	3.61	2.21	1.21	0.36	2.46	3.01	18.45%
23	1.5	1.5	1	3.45	2.33	1.73	0.64	1.82	1.79	-1.45%
24	1.5	1.5	2	3.27	2.48	3.20	1.59	1.78	1.64	-8.68%
25	1.5	2	0	3.61	2.21	1.21	0.36	1.70	1.67	-1.74%
26	1.5	2	1	3.45	2.33	1.73	0.64	1.50	1.39	-7.82%
27	1.5	2	2	3.27	2.48	3.20	1.59	1.55	1.39	-11.37%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right)\big/\,FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 43, Figure 43 and Figure 44).

Table 43. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.2

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	8.54	36.30%	1.1	8.59	47.57%	0	7.17	31.66%
1.2	2.76	18.45%	1.5	2.92	16.87%	1	3.23	18.27%
1.5	2.12	7.39%	2	1.90	-2.31%	2	3.01	12.20%

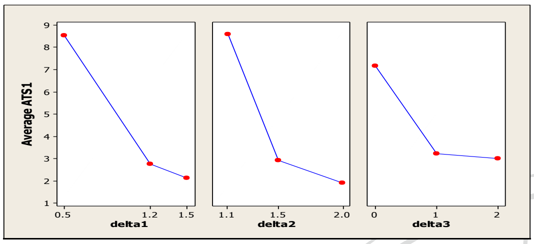

Figure 43. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.2

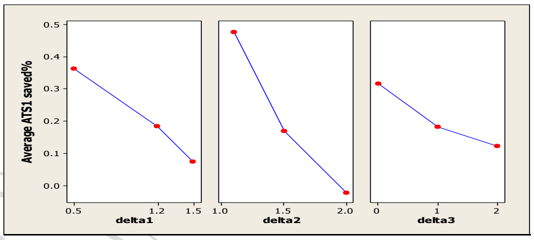

Figure 44. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.2

From Table 42, Table 43, Figure 43 and Figure 44, we found

a. ARL_1 saved % from -11.37% to 72.65%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.25 are listed in Table 44.

Table 44. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.25

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	3.78	2.16	1.31	0.32	43.38	109.54	60.40%
2	0.5	1.1	1	3.59	2.29	1.85	0.59	11.43	38.67	70.44%
3	0.5	1.1	2	3.38	2.46	3.35	1.50	8.83	29.36	69.93%
4	0.5	1.5	0	3.78	2.16	1.31	0.32	4.73	7.79	39.20%
5	0.5	1.5	1	3.59	2.29	1.85	0.59	3.91	5.90	33.79%
6	0.5	1.5	2	3.38	2.46	3.35	1.50	4.18	6.20	32.62%
7	0.5	2	0	3.78	2.16	1.31	0.32	2.16	2.36	8.39%
8	0.5	2	1	3.59	2.29	1.85	0.59	2.18	2.31	5.84%
9	0.5	2	2	3.38	2.46	3.35	1.50	2.51	2.66	5.63%
10	1.2	1.1	0	3.78	2.16	1.31	0.32	6.90	21.67	68.14%
11	1.2	1.1	1	3.59	2.29	1.85	0.59	2.91	5.10	43.04%
12	1.2	1.1	2	3.38	2.46	3.35	1.50	2.51	3.60	30.19%
13	1.2	1.5	0	3.78	2.16	1.31	0.32	2.98	4.14	28.21%
14	1.2	1.5	1	3.59	2.29	1.85	0.59	2.14	2.42	11.62%
15	1.2	1.5	2	3.38	2.46	3.35	1.50	2.08	2.21	5.97%
16	1.2	2	0	3.78	2.16	1.31	0.32	1.82	1.88	3.24%
17	1.2	2	1	3.59	2.29	1.85	0.59	1.63	1.59	-2.32%
18	1.2	2	2	3.38	2.46	3.35	1.50	1.70	1.63	-4.74%
19	1.5	1.1	0	3.78	2.16	1.31	0.32	4.09	9.86	58.53%
20	1.5	1.1	1	3.59	2.29	1.85	0.59	2.14	2.70	20.90%
21	1.5	1.1	2	3.38	2.46	3.35	1.50	1.92	2.04	5.65%
22	1.5	1.5	0	3.78	2.16	1.31	0.32	2.41	3.01	20.01%
23	1.5	1.5	1	3.59	2.29	1.85	0.59	1.76	1.79	1.73%
24	1.5	1.5	2	3.38	2.46	3.35	1.50	1.71	1.64	-4.52%
25	1.5	2	0	3.78	2.16	1.31	0.32	1.66	1.67	0.60%
26	1.5	2	1	3.59	2.29	1.85	0.59	1.46	1.39	-4.88%
27	1.5	2	2	3.38	2.46	3.35	1.50	1.50	1.39	-7.70%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 45, Figure 45 and Figure 46).

Table 45. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.25

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	9.26	36.25%	1.1	9.35	47.47%	0	7.79	31.86%
1.2	2.74	20.37%	1.5	2.88	18.74%	1	3.28	20.02%
1.5	2.07	10.04%	2	1.85	0.45%	2	2.99	14.78%

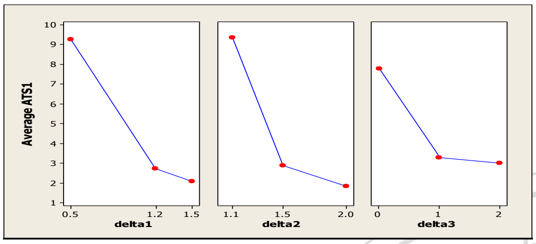

Figure 45. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.25

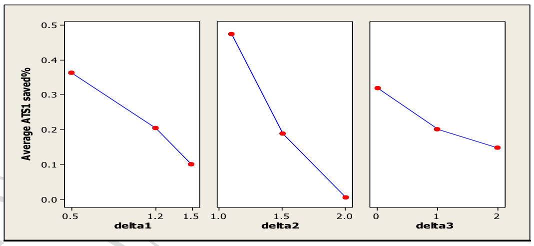

Figure 46. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.25

From Table 44, Table 45, Figure 45 and Figure 46, we found

a. ARL_1 saved % from -7.70% to 70.44%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.3 are listed in Table 46.

Table 46. ARL_1 for FP EWMA WL and ARL_1 saved % Compared to FP WL Chart under λ=0.3

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	3.92	2.10	1.40	0.29	48.99	109.54	55.27%
2	0.5	1.1	1	3.71	2.25	1.96	0.55	12.49	38.67	67.70%
3	0.5	1.1	2	3.47	2.44	3.50	1.42	9.42	29.36	67.91%
4	0.5	1.5	0	3.92	2.10	1.40	0.29	4.81	7.79	38.20%
5	0.5	1.5	1	3.71	2.25	1.96	0.55	3.92	5.90	33.58%
6	0.5	1.5	2	3.47	2.44	3.50	1.42	4.18	6.20	32.70%
7	0.5	2	0	3.92	2.10	1.40	0.29	2.14	2.36	9.49%
8	0.5	2	1	3.71	2.25	1.96	0.55	2.14	2.31	7.36%
9	0.5	2	2	3.47	2.44	3.50	1.42	2.46	2.66	7.51%
10	1.2	1.1	0	3.92	2.10	1.40	0.29	7.34	21.67	66.11%
11	1.2	1.1	1	3.71	2.25	1.96	0.55	2.89	5.10	43.30%
12	1.2	1.1	2	3.47	2.44	3.50	1.42	2.46	3.60	31.50%
13	1.2	1.5	0	3.92	2.10	1.40	0.29	2.97	4.14	28.35%
14	1.2	1.5	1	3.71	2.25	1.96	0.55	2.10	2.42	13.19%
15	1.2	1.5	2	3.47	2.44	3.50	1.42	2.03	2.21	8.28%
16	1.2	2	0	3.92	2.10	1.40	0.29	1.79	1.88	4.68%
17	1.2	2	1	3.71	2.25	1.96	0.55	1.60	1.59	-0.31%
18	1.2	2	2	3.47	2.44	3.50	1.42	1.66	1.63	-2.22%
19	1.5	1.1	0	3.92	2.10	1.40	0.29	4.19	9.86	57.57%
20	1.5	1.1	1	3.71	2.25	1.96	0.55	2.10	2.70	22.42%
21	1.5	1.1	2	3.47	2.44	3.50	1.42	1.87	2.04	8.39%
22	1.5	1.5	0	3.92	2.10	1.40	0.29	2.39	3.01	20.77%
23	1.5	1.5	1	3.71	2.25	1.96	0.55	1.72	1.79	3.90%
24	1.5	1.5	2	3.47	2.44	3.50	1.42	1.66	1.64	-1.53%
25	1.5	2	0	3.92	2.10	1.40	0.29	1.63	1.67	2.16%
26	1.5	2	1	3.71	2.25	1.96	0.55	1.43	1.39	-2.80%
27	1.5	2	2	3.47	2.44	3.50	1.42	1.46	1.39	-5.11%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\ ARL_1\right) / FP\ ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 47, Figure 47 and Figure 48).

Table 47. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.3

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	10.06	35.52%	1.1	10.20	46.69%	0	8.47	31.40%
1.2	2.76	21.43%	1.5	2.86	19.72%	1	3.38	20.93%
1.5	2.05	11.75%	2	1.81	2.31%	2	3.02	16.38%

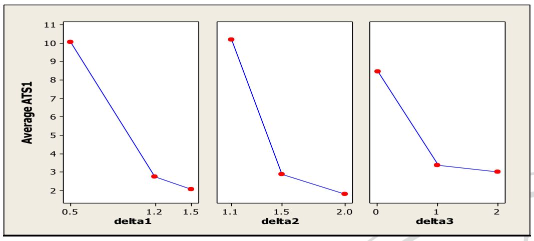

Figure 47. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.3

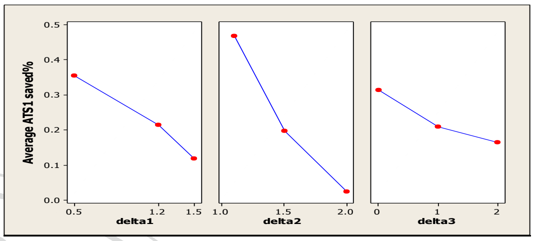

Figure 48. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.3

From Table 46, Table 47, Figure 47 and Figure 48, we found

a. ARL_1 saved % from -5.11% to 67.91%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.4 are listed in Table 48.

Table 48. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.4

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.16	1.99	1.59	0.24	60.32	109.54	44.93%
2	0.5	1.1	1	3.90	2.16	2.19	0.47	15.14	38.67	60.85%
3	0.5	1.1	2	3.61	2.38	3.80	1.28	11.02	29.36	62.47%
4	0.5	1.5	0	4.16	1.99	1.59	0.24	5.07	7.79	34.93%
5	0.5	1.5	1	3.90	2.16	2.19	0.47	4.03	5.90	31.72%
6	0.5	1.5	2	3.61	2.38	3.80	1.28	4.26	6.20	31.33%
7	0.5	2	0	4.16	1.99	1.59	0.24	2.12	2.36	10.34%
8	0.5	2	1	3.90	2.16	2.19	0.47	2.11	2.31	8.83%
9	0.5	2	2	3.61	2.38	3.80	1.28	2.41	2.66	9.38%
10	1.2	1.1	0	4.16	1.99	1.59	0.24	8.55	21.67	60.57%
11	1.2	1.1	1	3.90	2.16	2.19	0.47	2.95	5.10	42.26%
12	1.2	1.1	2	3.61	2.38	3.80	1.28	2.44	3.60	32.28%
13	1.2	1.5	0	4.16	1.99	1.59	0.24	3.02	4.14	27.22%
14	1.2	1.5	1	3.90	2.16	2.19	0.47	2.07	2.42	14.60%
15	1.2	1.5	2	3.61	2.38	3.80	1.28	1.97	2.21	10.85%
16	1.2	2	0	4.16	1.99	1.59	0.24	1.77	1.88	6.16%
17	1.2	2	1	3.90	2.16	2.19	0.47	1.56	1.59	2.07%
18	1.2	2	2	3.61	2.38	3.80	1.28	1.61	1.63	0.98%
19	1.5	1.1	0	4.16	1.99	1.59	0.24	4.52	9.86	54.20%
20	1.5	1.1	1	3.90	2.16	2.19	0.47	2.06	2.70	23.68%
21	1.5	1.1	2	3.61	2.38	3.80	1.28	1.80	2.04	11.78%
22	1.5	1.5	0	4.16	1.99	1.59	0.24	2.39	3.01	20.80%
23	1.5	1.5	1	3.90	2.16	2.19	0.47	1.68	1.79	6.41%
24	1.5	1.5	2	3.61	2.38	3.80	1.28	1.60	1.64	2.20%
25	1.5	2	0	4.16	1.99	1.59	0.24	1.60	1.67	3.89%
26	1.5	2	1	3.90	2.16	2.19	0.47	1.40	1.39	-0.29%
27	1.5	2	2	3.61	2.38	3.80	1.28	1.41	1.39	-1.73%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 49, Figure 49 and Figure 50).

Table 49. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.4

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	11.83	32.75%	1.1	12.09	43.67%	0	9.93	29.23%
1.2	2.88	21.89%	1.5	2.90	20.01%	1	3.67	21.13%
1.5	2.05	13.44%	2	1.78	4.40%	2	3.17	17.73%

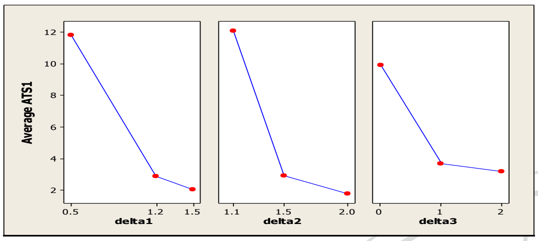

Figure 49. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.4

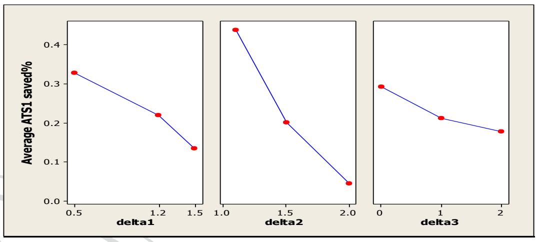

Figure 50. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.4

From Table 48, Table 49, Figure 49 and Figure 50, we found

a. ARL_1 saved % from -1.73% to 62.47%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.5 are listed in Table 50.

Table 50. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.5

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.35	1.89	1.78	0.20	71.30	109.54	34.91%
2	0.5	1.1	1	4.06	2.07	2.41	0.40	18.39	38.67	52.45%
3	0.5	1.1	2	3.73	2.31	4.09	1.16	13.12	29.36	55.31%
4	0.5	1.5	0	4.35	1.89	1.78	0.20	5.42	7.79	30.44%
5	0.5	1.5	1	4.06	2.07	2.41	0.40	4.22	5.90	28.45%
6	0.5	1.5	2	3.73	2.31	4.09	1.16	4.44	6.20	28.47%
7	0.5	2	0	4.35	1.89	1.78	0.20	2.12	2.36	10.00%
8	0.5	2	1	4.06	2.07	2.41	0.40	2.10	2.31	9.00%
9	0.5	2	2	3.73	2.31	4.09	1.16	2.40	2.66	9.76%
10	1.2	1.1	0	4.35	1.89	1.78	0.20	10.14	21.67	53.22%
11	1.2	1.1	1	4.06	2.07	2.41	0.40	3.09	5.10	39.49%
12	1.2	1.1	2	3.73	2.31	4.09	1.16	2.47	3.60	31.28%
13	1.2	1.5	0	4.35	1.89	1.78	0.20	3.12	4.14	24.71%
14	1.2	1.5	1	4.06	2.07	2.41	0.40	2.07	2.42	14.47%
15	1.2	1.5	2	3.73	2.31	4.09	1.16	1.95	2.21	11.67%
16	1.2	2	0	4.35	1.89	1.78	0.20	1.76	1.88	6.43%
17	1.2	2	1	4.06	2.07	2.41	0.40	1.54	1.59	3.20%
18	1.2	2	2	3.73	2.31	4.09	1.16	1.58	1.63	2.58%
19	1.5	1.1	0	4.35	1.89	1.78	0.20	5.03	9.86	49.04%
20	1.5	1.1	1	4.06	2.07	2.41	0.40	2.07	2.70	23.27%
21	1.5	1.1	2	3.73	2.31	4.09	1.16	1.77	2.04	13.25%
22	1.5	1.5	0	4.35	1.89	1.78	0.20	2.43	3.01	19.44%
23	1.5	1.5	1	4.06	2.07	2.41	0.40	1.66	1.79	7.41%
24	1.5	1.5	2	3.73	2.31	4.09	1.16	1.57	1.64	4.16%
25	1.5	2	0	4.35	1.89	1.78	0.20	1.59	1.67	4.49%
26	1.5	2	1	4.06	2.07	2.41	0.40	1.38	1.39	1.08%
27	1.5	2	2	3.73	2.31	4.09	1.16	1.39	1.39	0.14%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 51, Figure 51 and Figure 52).

Table 51. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved \%}}$ under λ=0.5

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved \%}}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved \%}}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved \%}}$
0.5	13.72	28.75%	1.1	14.15	39.14%	0	11.43	25.85%
1.2	3.08	20.78%	1.5	2.99	18.80%	1	4.06	19.87%
1.5	2.10	13.59%	2	1.76	5.19%	2	3.41	17.40%

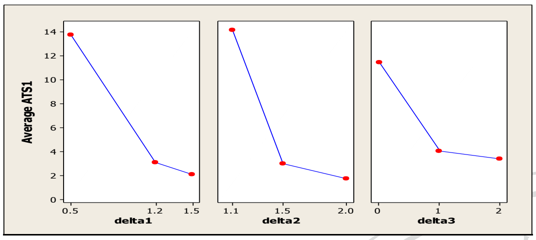

Figure 51. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.5

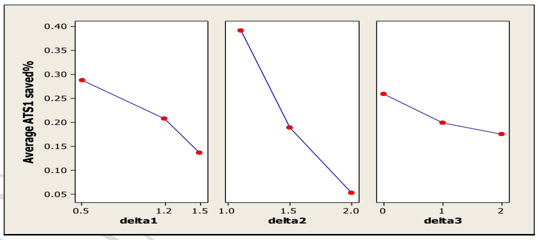

Figure 52. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved \%}}$ under λ = 0.5

From Table 50, Table 51, Figure 51 and Figure 52, we found

a. ARL_1 saved % from 0.14% to 55.31%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved \%}}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved \%}}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved \%}}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.6 are listed in Table 52.

Table 52. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.6

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.50	1.79	1.98	0.17	81.53	109.54	25.57%
2	0.5	1.1	1	4.18	1.98	2.63	0.35	22.11	38.67	42.83%
3	0.5	1.1	2	3.82	2.25	4.38	1.05	15.70	29.36	46.54%
4	0.5	1.5	0	4.50	1.79	1.98	0.17	5.83	7.79	25.10%
5	0.5	1.5	1	4.18	1.98	2.63	0.35	4.48	5.90	24.13%
6	0.5	1.5	2	3.82	2.25	4.38	1.05	4.69	6.20	24.45%
7	0.5	2	0	4.50	1.79	1.98	0.17	2.15	2.36	8.90%
8	0.5	2	1	4.18	1.98	2.63	0.35	2.12	2.31	8.27%
9	0.5	2	2	3.82	2.25	4.38	1.05	2.42	2.66	9.08%
10	1.2	1.1	0	4.50	1.79	1.98	0.17	12.07	21.67	44.28%
11	1.2	1.1	1	4.18	1.98	2.63	0.35	3.31	5.10	35.08%
12	1.2	1.1	2	3.82	2.25	4.38	1.05	2.57	3.60	28.69%
13	1.2	1.5	0	4.50	1.79	1.98	0.17	3.27	4.14	21.16%
14	1.2	1.5	1	4.18	1.98	2.63	0.35	2.10	2.42	13.23%
15	1.2	1.5	2	3.82	2.25	4.38	1.05	1.96	2.21	11.26%
16	1.2	2	0	4.50	1.79	1.98	0.17	1.77	1.88	6.00%
17	1.2	2	1	4.18	1.98	2.63	0.35	1.54	1.59	3.52%
18	1.2	2	2	3.82	2.25	4.38	1.05	1.57	1.63	3.26%
19	1.5	1.1	0	4.50	1.79	1.98	0.17	5.71	9.86	42.14%
20	1.5	1.1	1	4.18	1.98	2.63	0.35	2.12	2.70	21.46%
21	1.5	1.1	2	3.82	2.25	4.38	1.05	1.77	2.04	13.30%
22	1.5	1.5	0	4.50	1.79	1.98	0.17	2.50	3.01	17.02%
23	1.5	1.5	1	4.18	1.98	2.63	0.35	1.66	1.79	7.36%
24	1.5	1.5	2	3.82	2.25	4.38	1.05	1.56	1.64	4.95%
25	1.5	2	0	4.50	1.79	1.98	0.17	1.60	1.67	4.43%
26	1.5	2	1	4.18	1.98	2.63	0.35	1.37	1.39	1.65%
27	1.5	2	2	3.82	2.25	4.38	1.05	1.37	1.39	1.15%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / \, FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 53, Figure 53 and Figure 54).

Table 53. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.6

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	15.67	23.87%	1.1	16.32	33.32%	0	12.94	21.62%
1.2	3.35	18.50%	1.5	3.12	16.52%	1	4.53	17.50%
1.5	2.18	12.61%	2	1.77	5.14%	2	3.73	15.85%

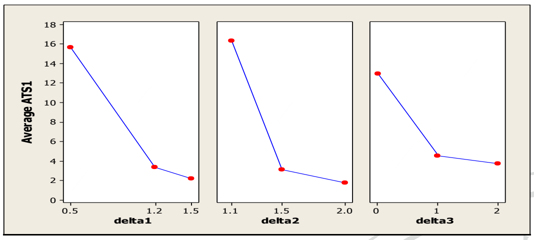

Figure 53. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.6

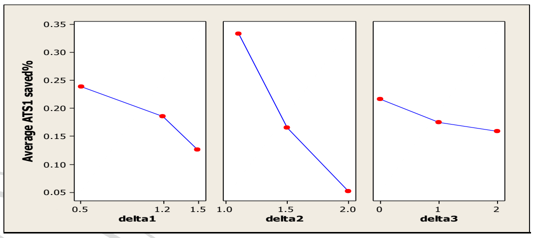

Figure 54. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.6

From Table 52, Table 53, Figure 53 and Figure 54, we found

a. ARL_1 saved % from 1.15% to 46.54%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.7 are listed in Table 54.

Table 54. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.7

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.62	1.70	2.17	0.13	90.77	109.54	17.14%
2	0.5	1.1	1	4.28	1.90	2.86	0.29	26.15	38.67	32.36%
3	0.5	1.1	2	3.89	2.19	4.68	0.93	18.70	29.36	36.32%
4	0.5	1.5	0	4.62	1.70	2.17	0.13	6.29	7.79	19.18%
5	0.5	1.5	1	4.28	1.90	2.86	0.29	4.78	5.90	18.94%
6	0.5	1.5	2	3.89	2.19	4.68	0.93	5.00	6.20	19.47%
7	0.5	2	0	4.62	1.70	2.17	0.13	2.19	2.36	7.20%
8	0.5	2	1	4.28	1.90	2.86	0.29	2.15	2.31	6.88%
9	0.5	2	2	3.89	2.19	4.68	0.93	2.46	2.66	7.66%
10	1.2	1.1	0	4.62	1.70	2.17	0.13	14.29	21.67	34.07%
11	1.2	1.1	1	4.28	1.90	2.86	0.29	3.62	5.10	28.99%
12	1.2	1.1	2	3.89	2.19	4.68	0.93	2.72	3.60	24.44%
13	1.2	1.5	0	4.62	1.70	2.17	0.13	3.45	4.14	16.77%
14	1.2	1.5	1	4.28	1.90	2.86	0.29	2.15	2.42	11.08%
15	1.2	1.5	2	3.89	2.19	4.68	0.93	1.99	2.21	9.81%
16	1.2	2	0	4.62	1.70	2.17	0.13	1.79	1.88	5.05%
17	1.2	2	1	4.28	1.90	2.86	0.29	1.54	1.59	3.20%
18	1.2	2	2	3.89	2.19	4.68	0.93	1.57	1.63	3.20%
19	1.5	1.1	0	4.62	1.70	2.17	0.13	6.55	9.86	33.57%
20	1.5	1.1	1	4.28	1.90	2.86	0.29	2.21	2.70	18.31%
21	1.5	1.1	2	3.89	2.19	4.68	0.93	1.79	2.04	12.08%
22	1.5	1.5	0	4.62	1.70	2.17	0.13	2.60	3.01	13.77%
23	1.5	1.5	1	4.28	1.90	2.86	0.29	1.68	1.79	6.52%
24	1.5	1.5	2	3.89	2.19	4.68	0.93	1.56	1.64	4.83%
25	1.5	2	0	4.62	1.70	2.17	0.13	1.61	1.67	3.83%
26	1.5	2	1	4.28	1.90	2.86	0.29	1.37	1.39	1.79%
27	1.5	2	2	3.89	2.19	4.68	0.93	1.37	1.39	1.51%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 55, Figure 55 and Figure 56).

Table 55. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.7

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	17.61	18.35%	1.1	18.53	26.36%	0	14.39	16.73%
1.2	3.68	15.18%	1.5	3.28	13.37%	1	5.07	14.23%
1.5	2.30	10.69%	2	1.78	4.48%	2	4.13	13.26%

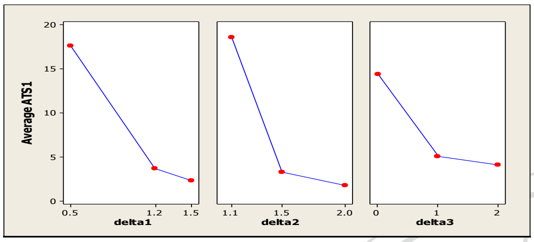

Figure 55. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.7

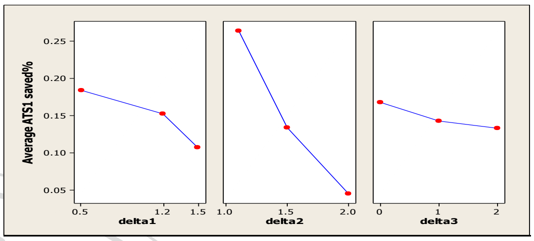

Figure 56. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.7

From Table 54, Table 55, Figure 55 and Figure 56, we found

a. ARL_1 saved % from 1.51% to 36.32%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.8 are listed in Table 56.

Table 56. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.8

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.71	1.61	2.37	0.10	98.84	109.54	9.77%
2	0.5	1.1	1	4.35	1.82	3.09	0.24	30.37	38.67	21.46%
3	0.5	1.1	2	3.95	2.13	4.99	0.82	22.05	29.36	24.90%
4	0.5	1.5	0	4.71	1.61	2.37	0.10	6.78	7.79	12.87%
5	0.5	1.5	1	4.35	1.82	3.09	0.24	5.13	5.90	13.08%
6	0.5	1.5	2	3.95	2.13	4.99	0.82	5.36	6.20	13.62%
7	0.5	2	0	4.71	1.61	2.37	0.10	2.24	2.36	5.13%
8	0.5	2	1	4.35	1.82	3.09	0.24	2.20	2.31	4.98%
9	0.5	2	2	3.95	2.13	4.99	0.82	2.51	2.66	5.63%
10	1.2	1.1	0	4.71	1.61	2.37	0.10	16.69	21.67	22.98%
11	1.2	1.1	1	4.35	1.82	3.09	0.24	4.03	5.10	21.11%
12	1.2	1.1	2	3.95	2.13	4.99	0.82	2.94	3.60	18.40%
13	1.2	1.5	0	4.71	1.61	2.37	0.10	3.66	4.14	11.68%
14	1.2	1.5	1	4.35	1.82	3.09	0.24	2.22	2.42	8.15%
15	1.2	1.5	2	3.95	2.13	4.99	0.82	2.05	2.21	7.46%
16	1.2	2	0	4.71	1.61	2.37	0.10	1.81	1.88	3.67%
17	1.2	2	1	4.35	1.82	3.09	0.24	1.55	1.59	2.51%
18	1.2	2	2	3.95	2.13	4.99	0.82	1.58	1.63	2.65%
19	1.5	1.1	0	4.71	1.61	2.37	0.10	7.55	9.86	23.47%
20	1.5	1.1	1	4.35	1.82	3.09	0.24	2.33	2.70	13.76%
21	1.5	1.1	2	3.95	2.13	4.99	0.82	1.84	2.04	9.52%
22	1.5	1.5	0	4.71	1.61	2.37	0.10	2.72	3.01	9.79%
23	1.5	1.5	1	4.35	1.82	3.09	0.24	1.71	1.79	4.96%
24	1.5	1.5	2	3.95	2.13	4.99	0.82	1.57	1.64	3.91%
25	1.5	2	0	4.71	1.61	2.37	0.10	1.62	1.67	2.88%
26	1.5	2	1	4.35	1.82	3.09	0.24	1.37	1.39	1.51%
27	1.5	2	2	3.95	2.13	4.99	0.82	1.37	1.39	1.37%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\,ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 57, Figure 57 and Figure 58).

Table 57. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.8

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	19.50	12.38%	1.1	20.74	18.37%	0	15.77	11.36%
1.2	4.06	10.96%	1.5	3.47	9.50%	1	5.66	10.17%
1.5	2.45	7.91%	2	1.81	3.37%	2	4.59	9.72%

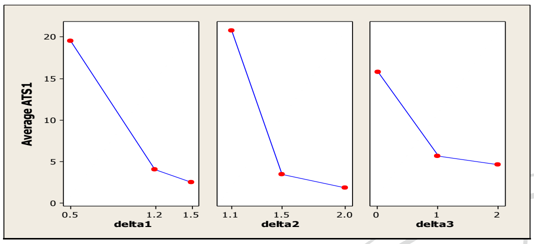

Figure 57. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.8

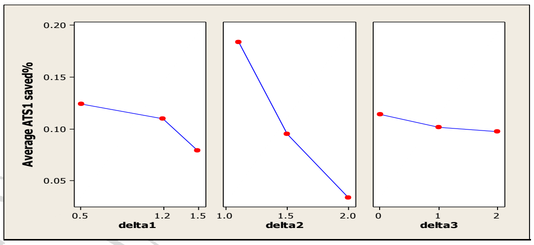

Figure 58. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.8

From Table 56, Table 57, Figure 57 and Figure 58, we found

a. ARL_1 saved % from 1.37% to 24.90%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

The ARL_1s of the 27combinations of δ_1, δ_2 and δ_3 under λ= 0.9 are listed in Table 58.

Table 58. ARL_1 for FP EWMA WL and ARL_1 Saved % Compared to FP WL Chart under λ=0.9

No.	δ_1	δ_2	δ_3	L1	L2	UCL	LCL	EWMA ARL_1	FP ARL_1	ARL_1 saved %
1	0.5	1.1	0	4.76	1.53	2.57	0.07	105.60	109.54	3.60%
2	0.5	1.1	1	4.39	1.75	3.33	0.18	34.60	38.67	10.52%
3	0.5	1.1	2	3.98	2.08	5.30	0.70	25.65	29.36	12.65%
4	0.5	1.5	0	4.76	1.53	2.57	0.07	7.29	7.79	6.40%
5	0.5	1.5	1	4.39	1.75	3.33	0.18	5.51	5.90	6.71%
6	0.5	1.5	2	3.98	2.08	5.30	0.70	5.77	6.20	7.08%
7	0.5	2	0	4.76	1.53	2.57	0.07	2.30	2.36	2.71%
8	0.5	2	1	4.39	1.75	3.33	0.18	2.25	2.31	2.68%
9	0.5	2	2	3.98	2.08	5.30	0.70	2.58	2.66	3.04%
10	1.2	1.1	0	4.76	1.53	2.57	0.07	19.19	21.67	11.46%
11	1.2	1.1	1	4.39	1.75	3.33	0.18	4.52	5.10	11.43%
12	1.2	1.1	2	3.98	2.08	5.30	0.70	3.23	3.60	10.34%
13	1.2	1.5	0	4.76	1.53	2.57	0.07	3.89	4.14	6.06%
14	1.2	1.5	1	4.39	1.75	3.33	0.18	2.31	2.42	4.43%
15	1.2	1.5	2	3.98	2.08	5.30	0.70	2.12	2.21	4.21%
16	1.2	2	0	4.76	1.53	2.57	0.07	1.85	1.88	1.97%
17	1.2	2	1	4.39	1.75	3.33	0.18	1.57	1.59	1.44%
18	1.2	2	2	3.98	2.08	5.30	0.70	1.60	1.63	1.54%
19	1.5	1.1	0	4.76	1.53	2.57	0.07	8.67	9.86	12.15%
20	1.5	1.1	1	4.39	1.75	3.33	0.18	2.50	2.70	7.70%
21	1.5	1.1	2	3.98	2.08	5.30	0.70	1.92	2.04	5.60%
22	1.5	1.5	0	4.76	1.53	2.57	0.07	2.86	3.01	5.18%
23	1.5	1.5	1	4.39	1.75	3.33	0.18	1.74	1.79	2.79%
24	1.5	1.5	2	3.98	2.08	5.30	0.70	1.60	1.64	2.32%
25	1.5	2	0	4.76	1.53	2.57	0.07	1.64	1.67	1.56%
26	1.5	2	1	4.39	1.75	3.33	0.18	1.38	1.39	0.86%
27	1.5	2	2	3.98	2.08	5.30	0.70	1.38	1.39	0.86%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - FP.EWMA\,ARL_1\right) / FP\ ARL_1\%$

The response table and response diagram show the results of the sensitivity analysis (Table 59, Figure 59 and Figure 60).

Table 59. FP EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 v.s. $\overline{ARL_1}$ and δ_1, δ_2, δ_3 v.s. $\overline{ARL_1 \text{ saved } \%}$ under λ=0.9

δ_1	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_2	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$	δ_3	$\overline{ARL_1}$	$\overline{ARL_1 \text{ saved } \%}$
0.5	21.28	6.15%	1.1	22.87	9.49%	0	17.03	5.68%
1.2	4.47	5.88%	1.5	3.68	5.02%	1	6.26	5.40%
1.5	2.63	4.34%	2	1.84	1.85%	2	5.09	5.29%

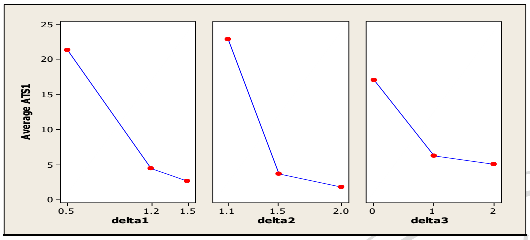

Figure 59. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1}$ under λ = 0.9

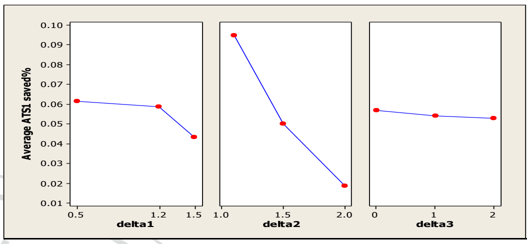

Figure 60. FP EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ARL_1 \text{ saved } \%}$ under λ = 0.9

From Table 58, Table 59, Figure 59 and Figure 60, we found

a. ARL_1 saved % from 0.86% to 12.65%.

b. the $\overline{ARL_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ARL_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ARL_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ARL_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

From the above results, when the value of λ is small (λ<0.5), the ARL_1 saved % may be negative when the δ_1, δ_2 and/or δ_3 are large. It is because the EWMA scheme is more effective under small λ when the shifts of mean and/or variance are small so that the performance compare to FP WL chart is not great under large shifts of mean and/or variance.

For different values of λ in Tables 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56 and 58, the choice of the λ with maximal ARL_1 saved % for each combination of δ_1, δ_2 and δ_3 in $L_{27}(3^{13})$ is listed in Table 62. That is, we may choose the λ for the given combination δ_1, δ_2 and δ_3 from Table 60.

Table 60. The Optimal λ to Maximize the ARL_1 Saved % Time

No.	δ_1	δ_2	δ_3	λ^*	ARL_1 saved %	No.	δ_1	δ_2	δ_3	λ^*	ARL_1 saved %
1	0.5	1.1	0	0.05	76.72%	16	1.2	2	0	0.5	6.43%
2	0.5	1.1	1	0.1	74.84%	17	1.2	2	1	0.6	3.52%
3	0.5	1.1	2	0.15	72.24%	18	1.2	2	2	0.6	3.26%
4	0.5	1.5	0	0.2	39.58%	19	1.5	1.1	0	0.2	58.90%
5	0.5	1.5	1	0.25	33.79%	20	1.5	1.1	1	0.4	23.68%
6	0.5	1.5	2	0.3	32.70%	21	1.5	1.1	2	0.6	13.30%
7	0.5	2	0	0.4	10.34%	22	1.5	1.5	0	0.4	20.80%
8	0.5	2	1	0.5	9.00%	23	1.5	1.5	1	0.5	7.41%
9	0.5	2	2	0.5	9.76%	24	1.5	1.5	2	0.6	4.95%
10	1.2	1.1	0	0.15	70.38%	25	1.5	2	0	0.5	4.49%
11	1.2	1.1	1	0.3	43.30%	26	1.5	2	1	0.7	1.79%
12	1.2	1.1	2	0.4	32.28%	27	1.5	2	2	0.7	1.51%
13	1.2	1.5	0	0.3	28.35%						
14	1.2	1.5	1	0.4	14.60%						
15	1.2	1.5	2	0.5	11.67%						

The results in Table 60 tell us that we have to choose the biggerλ when the values of δ_1, δ_2 and/or δ_3 are large. However, the ARL saved % of the combination #1 is 76.72% which is the maximum , that is, the FP EWMA WL chart has the best performance under λ=0.05 when the shifts of the mean and/or variance are much smaller.

(2) ATS_1 Analysis of VSI EWMA WL Chart and ATS_1 Comparison between VSI EWMA WL Chart and FP WL Chart

Under $\mu_0 = 0$, $\sigma_0^2 = 1$, $n_0 = 5$, $h_0 = 1$, $\alpha_0 = 0.0027$ (or $ARL_0 = 370.37$) and $a = 0.6$. Consider the levels of the parameters, δ_1 = (0.5, 1.2, 1.5), δ_2 = (1.1, 1.5, 2), δ_3 = (0, 1, 2) and (h_2, h_1) = (0.2, 2), (0.5,1.5), (0.8, 1.2). The 27 combinations are arranged by $L_{27}(3^{13})$.

Similar to Section 5.2, the values of L_1, L_2, W_1 and W_2 under the combinations of λ = 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.9, δ_3 = 0, 1, 2 and (h_2, h_1) = (0.2, 2), (0.5, 1.5), (0.8, 1.2) could be found.

Under λ = 0.05, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 61 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 62.

Table 61. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ = 0.05

				W_2=0.5	W_2=1	W_2=1.5	W_2=2
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1	W_1
0	(0.2,2.0)	2.74	2.27	0.63	0.16	-0.09	-0.18
1	(0.2,2.0)	2.68	2.32	0.62	0.17	-0.07	-0.16
2	(0.2,2.0)	2.63	2.38	0.62	0.18	-0.05	-0.15
0	(0.5,1.5)	2.74	2.27	0.82	0.30	0.05	-0.05
1	(0.5,1.5)	2.68	2.32	0.81	0.31	0.06	-0.03
2	(0.5,1.5)	2.63	2.38	0.80	0.32	0.08	-0.02
0	(0.8,1.2)	2.74	2.27	0.82	0.30	0.05	-0.05
1	(0.8,1.2)	2.68	2.32	0.81	0.31	0.06	-0.03
2	(0.8,1.2)	2.63	2.38	0.80	0.32	0.08	-0.02

Table 62. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under $\lambda = 0.05$

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	2.74	2.27	-0.18	2.00	0.52	0.54	0.67	0.87	10.53	109.54	90.39%
2	0.5	1.1	1	(0.5,1.5)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	6.70	38.67	82.66%
3	0.5	1.1	2	(0.8,1.2)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	7.71	29.36	73.73%
4	0.5	1.5	0	(0.2,2.0)	2.74	2.27	-0.18	2.00	0.52	0.54	0.67	0.87	2.74	7.79	64.87%
5	0.5	1.5	1	(0.5,1.5)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	3.37	5.90	42.92%
6	0.5	1.5	2	(0.8,1.2)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	4.66	6.20	24.95%
7	0.5	2	0	(0.2,2.0)	2.74	2.27	-0.18	2.00	0.52	0.54	0.67	0.87	1.62	2.36	31.27%
8	0.5	2	1	(0.5,1.5)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	2.11	2.31	8.66%
9	0.5	2	2	(0.8,1.2)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	3.01	2.66	-12.80%
10	1.2	1.1	0	(0.5,1.5)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	4.67	21.67	78.44%
11	1.2	1.1	1	(0.8,1.2)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	3.30	5.10	35.32%
12	1.2	1.1	2	(0.2,2.0)	2.63	2.38	-0.15	2.00	1.96	2.01	2.26	2.63	1.79	3.60	50.26%
13	1.2	1.5	0	(0.5,1.5)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	2.63	4.14	36.49%
14	1.2	1.5	1	(0.8,1.2)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	2.52	2.42	-4.38%
15	1.2	1.5	2	(0.2,2.0)	2.63	2.38	-0.15	2.00	1.96	2.01	2.26	2.63	1.61	2.21	27.18%
16	1.2	2	0	(0.5,1.5)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	1.77	1.88	6.06%
17	1.2	2	1	(0.8,1.2)	2.68	2.32	-0.03	2.00	0.87	0.90	1.08	1.32	1.93	1.59	-20.97%
18	1.2	2	2	(0.2,2.0)	2.63	2.38	-0.15	2.00	1.96	2.01	2.26	2.63	1.43	1.63	12.12%
19	1.5	1.1	0	(0.8,1.2)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	4.18	9.86	57.65%
20	1.5	1.1	1	(0.2,2.0)	2.68	2.32	-0.16	2.00	0.87	0.90	1.07	1.32	1.57	2.70	42.10%
21	1.5	1.1	2	(0.5,1.5)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	1.97	2.04	3.34%
22	1.5	1.5	0	(0.8,1.2)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	2.73	3.01	9.56%
23	1.5	1.5	1	(0.2,2.0)	2.68	2.32	-0.16	2.00	0.87	0.90	1.07	1.32	1.41	1.79	21.57%
24	1.5	1.5	2	(0.5,1.5)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	1.78	1.64	-8.99%
25	1.5	2	0	(0.8,1.2)	2.74	2.27	-0.05	2.00	0.52	0.54	0.68	0.87	1.91	1.67	-14.32%
26	1.5	2	1	(0.2,2.0)	2.68	2.32	-0.16	2.00	0.87	0.90	1.07	1.32	1.26	1.39	9.33%
27	1.5	2	2	(0.5,1.5)	2.63	2.38	-0.02	2.00	1.96	2.01	2.28	2.63	1.58	1.39	-13.60%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI.EWMA\ ARL_1\right) / FP\ ARL_1 \%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

63, Figure 61 and Figure 62).

Table 63. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under $\lambda = 0.05$

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	4.72	4518.00%	1.1	4.71	57.10%	0	3.64	40.05%
1.2	2.41	2450.00%	1.5	2.61	23.80%	1	2.69	24.13%
1.5	2.04	1185.00%	2	1.85	0.64%	2	2.84	17.35%

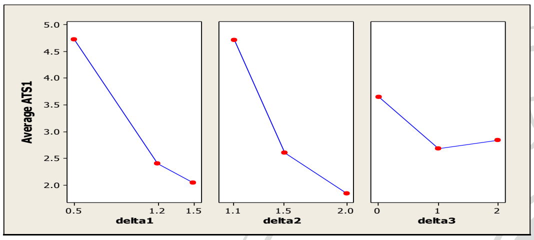

Figure 61. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under $\lambda = 0.05$

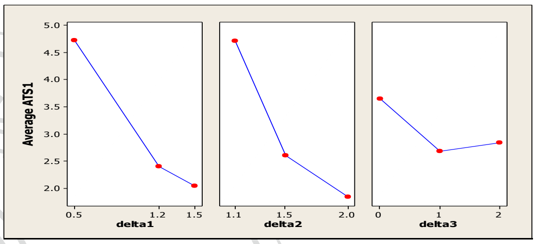

Figure 62. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under $\lambda = 0.05$

From Table 62, Table 63, Figure 61 and Figure 62, we found

a. ATS_1 saved % from -20.97% to 90.39%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is first decreasing then increasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is first decreasing then increasing as δ_3 increasing.

Under λ= 0.1, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 64 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 65.

Table 64. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.1

				W_2=0.5	W_2=1	W_2=1.5	W_2=2
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1	W_1
0	(0.2,2.0)	3.15	2.29	0.66	0.16	-0.10	-0.19
1	(0.2,2.0)	3.05	2.38	0.66	0.18	-0.08	-0.17
2	(0.2,2.0)	2.95	2.48	0.66	0.19	-0.05	-0.15
0	(0.5,1.5)	3.15	2.29	0.87	0.30	0.04	-0.06
1	(0.5,1.5)	3.05	2.38	0.86	0.32	0.06	-0.04
2	(0.5,1.5)	2.95	2.48	0.85	0.34	0.09	-0.02
0	(0.8,1.2)	3.15	2.29	0.87	0.30	0.04	-0.06
1	(0.8,1.2)	3.05	2.38	0.86	0.32	0.06	-0.04
2	(0.8,1.2)	2.95	2.48	0.85	0.34	0.09	-0.02

Table 65. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.1

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	3.15	2.29	-0.19	2.00	0.45	0.48	0.66	1.00	10.96	109.54	90.00%
2	0.5	1.1	1	(0.5,1.5)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	6.17	38.67	84.04%
3	0.5	1.1	2	(0.8,1.2)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	7.03	29.36	76.04%
4	0.5	1.5	0	(0.2,2.0)	3.15	2.29	-0.19	2.00	0.45	0.48	0.66	1.00	2.41	7.79	69.09%
5	0.5	1.5	1	(0.5,1.5)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	2.96	5.90	49.81%
6	0.5	1.5	2	(0.8,1.2)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	4.07	6.20	34.38%
7	0.5	2	0	(0.2,2.0)	3.15	2.29	-0.19	2.00	0.45	0.48	0.66	1.00	1.48	2.36	37.46%
8	0.5	2	1	(0.5,1.5)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	1.88	2.31	18.61%
9	0.5	2	2	(0.8,1.2)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	2.62	2.66	1.65%
10	1.2	1.1	0	(0.5,1.5)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	4.20	21.67	80.63%
11	1.2	1.1	1	(0.8,1.2)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	2.88	5.10	43.65%
12	1.2	1.1	2	(0.2,2.0)	2.95	2.48	-0.15	2.00	1.80	1.90	2.25	2.85	1.55	3.60	56.91%
13	1.2	1.5	0	(0.5,1.5)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	2.33	4.14	43.77%
14	1.2	1.5	1	(0.8,1.2)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	2.21	2.42	8.64%
15	1.2	1.5	2	(0.2,2.0)	2.95	2.48	-0.15	2.00	1.80	1.90	2.25	2.85	1.43	2.21	35.41%
16	1.2	2	0	(0.5,1.5)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	1.60	1.88	14.93%
17	1.2	2	1	(0.8,1.2)	3.05	2.38	-0.04	2.00	0.77	0.82	1.08	1.48	1.71	1.59	-7.60%
18	1.2	2	2	(0.2,2.0)	2.95	2.48	-0.15	2.00	1.80	1.90	2.25	2.85	1.30	1.63	19.88%
19	1.5	1.1	0	(0.8,1.2)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	3.72	9.86	62.33%
20	1.5	1.1	1	(0.2,2.0)	3.05	2.38	-0.17	2.00	0.77	0.82	1.06	1.48	1.39	2.70	48.65%
21	1.5	1.1	2	(0.5,1.5)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	1.71	2.04	16.15%
22	1.5	1.5	0	(0.8,1.2)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	2.41	3.01	20.01%
23	1.5	1.5	1	(0.2,2.0)	3.05	2.38	-0.17	2.00	0.77	0.82	1.06	1.48	1.28	1.79	28.60%
24	1.5	1.5	2	(0.5,1.5)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	1.57	1.64	4.03%
25	1.5	2	0	(0.8,1.2)	3.15	2.29	-0.06	2.00	0.45	0.48	0.67	1.00	1.71	1.67	-2.70%
26	1.5	2	1	(0.2,2.0)	3.05	2.38	-0.17	2.00	0.77	0.82	1.06	1.48	1.18	1.39	15.22%
27	1.5	2	2	(0.5,1.5)	2.95	2.48	-0.02	2.00	1.80	1.90	2.28	2.85	1.42	1.39	-1.87%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI.EWMA\ ARL_1\right) / FP\ ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

66, Figure 63 and Figure 64).

Table 66. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.1

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	4.40	51.23%	1.1	4.40	62.04%	0	3.42	46.17%
1.2	2.13	32.91%	1.5	2.30	32.64%	1	2.41	32.18%
1.5	1.82	21.16%	2	1.66	10.62%	2	2.52	26.95%

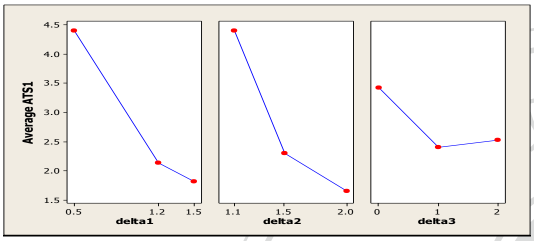

Figure 63. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.1

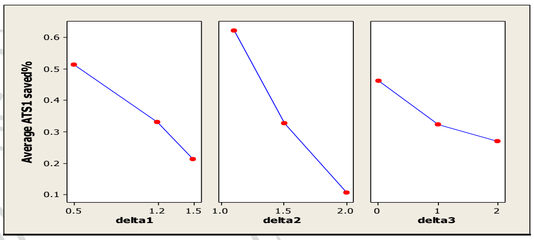

Figure 64. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.1

From Table 65, Table 66, Figure 63 and Figure 64, we found

a. ATS_1 saved % from -7.60% to 90.00%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is first decreasing then increasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

Under λ= 0.2, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 67 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 68.

Table 67. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.2

				W_2=0.5	W_2=1	W_2=1.5	W_2=2
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1	W_1
0	(0.2,2.0)	3.58	2.21	0.74	0.17	-0.10	-0.19
1	(0.2,2.0)	3.49	2.33	0.61	0.11	-0.15	-0.25
2	(0.2,2.0)	3.29	2.48	0.64	0.16	-0.09	-0.19
0	(0.5,1.5)	3.58	2.21	1.04	0.36	0.07	-0.03
1	(0.5,1.5)	3.49	2.33	0.76	0.21	-0.05	-0.15
2	(0.5,1.5)	3.29	2.48	0.81	0.28	0.02	-0.08
0	(0.8,1.2)	3.58	2.21	1.28	0.48	0.17	0.07
1	(0.8,1.2)	3.49	2.33	0.57	0.08	-0.18	-0.28
2	(0.8,1.2)	3.29	2.48	0.70	0.21	-0.05	-0.15

Table 68. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.2

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	3.58	2.21	-0.19	2.00	0.36	0.39	0.65	1.20	15.15	109.54	86.17%
2	0.5	1.1	1	(0.5,1.5)	3.49	2.33	-0.15	2.00	0.64	0.70	1.05	1.74	6.42	38.67	83.40%
3	0.5	1.1	2	(0.8,1.2)	3.29	2.48	-0.15	2.00	1.59	1.72	2.24	3.20	7.13	29.36	75.73%
4	0.5	1.5	0	(0.2,2.0)	3.58	2.21	-0.19	2.00	0.36	0.39	0.65	1.20	2.25	7.79	71.10%
5	0.5	1.5	1	(0.5,1.5)	3.49	2.33	-0.15	2.00	0.64	0.70	1.05	1.74	2.67	5.90	54.79%
6	0.5	1.5	2	(0.8,1.2)	3.29	2.48	-0.15	2.00	1.59	1.72	2.24	3.20	3.69	6.20	40.55%
7	0.5	2	0	(0.2,2.0)	3.58	2.21	-0.19	2.00	0.36	0.39	0.65	1.20	1.41	2.36	40.38%
8	0.5	2	1	(0.5,1.5)	3.49	2.33	-0.15	2.00	0.64	0.70	1.05	1.74	1.68	2.31	27.45%
9	0.5	2	2	(0.8,1.2)	3.29	2.48	-0.15	2.00	1.59	1.72	2.24	3.20	2.32	2.66	13.06%
10	1.2	1.1	0	(0.5,1.5)	3.58	2.21	-0.03	2.00	0.36	0.39	0.68	1.20	4.13	21.67	80.93%
11	1.2	1.1	1	(0.8,1.2)	3.49	2.33	-0.28	2.00	0.64	0.70	1.03	1.74	2.57	5.10	49.55%
12	1.2	1.1	2	(0.2,2.0)	3.29	2.48	-0.19	2.00	1.59	1.72	2.23	3.20	1.37	3.60	61.91%
13	1.2	1.5	0	(0.5,1.5)	3.58	2.21	-0.03	2.00	0.36	0.39	0.68	1.20	2.16	4.14	47.80%
14	1.2	1.5	1	(0.8,1.2)	3.49	2.33	-0.28	2.00	0.64	0.70	1.03	1.74	1.95	2.42	19.27%
15	1.2	1.5	2	(0.2,2.0)	3.29	2.48	-0.19	2.00	1.59	1.72	2.23	3.20	1.28	2.21	41.97%
16	1.2	2	0	(0.5,1.5)	3.58	2.21	-0.03	2.00	0.36	0.39	0.68	1.20	1.51	1.88	19.77%
17	1.2	2	1	(0.8,1.2)	3.49	2.33	-0.28	2.00	0.64	0.70	1.03	1.74	1.52	1.59	4.39%
18	1.2	2	2	(0.2,2.0)	3.29	2.48	-0.19	2.00	1.59	1.72	2.23	3.20	1.19	1.63	26.52%
19	1.5	1.1	0	(0.8,1.2)	3.58	2.21	0.07	2.00	0.36	0.39	0.69	1.20	3.52	9.86	64.35%
20	1.5	1.1	1	(0.2,2.0)	3.49	2.33	-0.25	2.00	0.64	0.70	1.03	1.74	1.24	2.70	54.09%
21	1.5	1.1	2	(0.5,1.5)	3.29	2.48	-0.08	2.00	1.59	1.72	2.26	3.20	1.50	2.04	26.26%
22	1.5	1.5	0	(0.8,1.2)	3.58	2.21	0.07	2.00	0.36	0.39	0.69	1.20	2.22	3.01	26.38%
23	1.5	1.5	1	(0.2,2.0)	3.49	2.33	-0.25	2.00	0.64	0.70	1.03	1.74	1.16	1.79	35.23%
24	1.5	1.5	2	(0.5,1.5)	3.29	2.48	-0.08	2.00	1.59	1.72	2.26	3.20	1.39	1.64	14.85%
25	1.5	2	0	(0.8,1.2)	3.58	2.21	0.07	2.00	0.36	0.39	0.69	1.20	1.60	1.67	4.31%
26	1.5	2	1	(0.2,2.0)	3.49	2.33	-0.25	2.00	0.64	0.70	1.03	1.74	1.09	1.39	21.75%
27	1.5	2	2	(0.5,1.5)	3.29	2.48	-0.08	2.00	1.59	1.72	2.26	3.20	1.28	1.39	8.06%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI.\ EWMA\ ARL_1\right) / \ FP\ ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

69, Figure 65 and Figure 66).

Table 69. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.2

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	4.74	54.74%	1.1	4.78	64.71%	0	3.77	49.02%
1.2	1.97	39.12%	1.5	2.09	39.10%	1	2.26	38.88%
1.5	1.67	28.36%	2	1.51	18.41%	2	2.35	34.32%

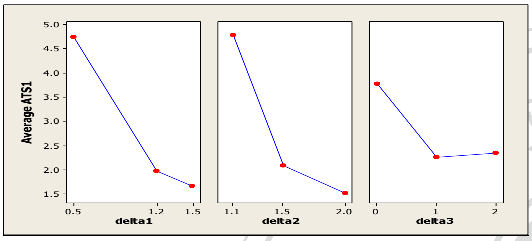

Figure 65. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.2

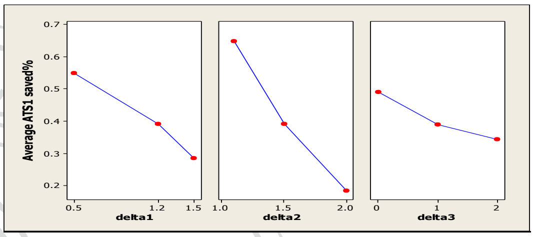

Figure 66. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.2

From Table 68, Table 69, Figure 65 and Figure 66, we found

a. ATS_1 saved % from 4.31% to 86.17%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is first decreasing then increasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

Under λ= 0.3, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 70 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 71.

Table 70. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.3

				W_2=0.5	W_2=1	W_2=1.5	W_2=2
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1	W_1
0	(0.2,2.0)	3.92	2.10	0.69	0.11	-0.17	-0.25
1	(0.2,2.0)	3.71	2.25	0.69	0.14	-0.13	-0.22
2	(0.2,2.0)	3.47	2.44	0.69	0.18	-0.08	-0.18
0	(0.5,1.5)	3.92	2.10	0.93	0.26	-0.04	-0.12
1	(0.5,1.5)	3.71	2.25	0.92	0.29	0.01	-0.09
2	(0.5,1.5)	3.47	2.44	0.90	0.33	0.06	-0.05
0	(0.8,1.2)	3.92	2.10	0.93	0.26	-0.04	-0.12
1	(0.8,1.2)	3.71	2.25	0.92	0.29	0.01	-0.09
2	(0.8,1.2)	3.47	2.44	0.90	0.33	0.06	-0.05

Table 71. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.3

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	3.92	2.10	-0.25	2.00	0.29	0.31	0.63	1.40	21.15	109.54	80.70%
2	0.5	1.1	1	(0.5,1.5)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	7.40	38.67	80.86%
3	0.5	1.1	2	(0.8,1.2)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	7.93	29.36	72.99%
4	0.5	1.5	0	(0.2,2.0)	3.92	2.10	-0.25	2.00	0.29	0.31	0.63	1.40	2.17	7.79	72.10%
5	0.5	1.5	1	(0.5,1.5)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	2.67	5.90	54.85%
6	0.5	1.5	2	(0.8,1.2)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	3.64	6.20	41.26%
7	0.5	2	0	(0.2,2.0)	3.92	2.10	-0.25	2.00	0.29	0.31	0.63	1.40	1.34	2.36	43.39%
8	0.5	2	1	(0.5,1.5)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	1.65	2.31	28.48%
9	0.5	2	2	(0.8,1.2)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	2.23	2.66	16.40%
10	1.2	1.1	0	(0.5,1.5)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	4.43	21.67	79.54%
11	1.2	1.1	1	(0.8,1.2)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	2.53	5.10	50.35%
12	1.2	1.1	2	(0.2,2.0)	3.47	2.44	-0.18	2.00	1.42	1.58	2.22	3.50	1.33	3.60	62.91%
13	1.2	1.5	0	(0.5,1.5)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	2.09	4.14	49.69%
14	1.2	1.5	1	(0.8,1.2)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	1.89	2.42	21.67%
15	1.2	1.5	2	(0.2,2.0)	3.47	2.44	-0.18	2.00	1.42	1.58	2.22	3.50	1.26	2.21	43.19%
16	1.2	2	0	(0.5,1.5)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	1.44	1.88	23.65%
17	1.2	2	1	(0.8,1.2)	3.71	2.25	-0.09	2.00	0.55	0.60	1.06	1.96	1.49	1.59	6.59%
18	1.2	2	2	(0.2,2.0)	3.47	2.44	-0.18	2.00	1.42	1.58	2.22	3.50	1.18	1.63	27.38%
19	1.5	1.1	0	(0.8,1.2)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	3.59	9.86	63.61%
20	1.5	1.1	1	(0.2,2.0)	3.71	2.25	-0.22	2.00	0.55	0.60	1.03	1.96	1.24	2.70	54.24%
21	1.5	1.1	2	(0.5,1.5)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	1.44	2.04	29.26%
22	1.5	1.5	0	(0.8,1.2)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	2.13	3.01	29.26%
23	1.5	1.5	1	(0.2,2.0)	3.71	2.25	-0.22	2.00	0.55	0.60	1.03	1.96	1.17	1.79	35.01%
24	1.5	1.5	2	(0.5,1.5)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	1.34	1.64	17.85%
25	1.5	2	0	(0.8,1.2)	3.92	2.10	-0.12	2.00	0.29	0.31	0.66	1.40	1.52	1.67	9.11%
26	1.5	2	1	(0.2,2.0)	3.71	2.25	-0.22	2.00	0.55	0.60	1.03	1.96	1.11	1.39	20.67%
27	1.5	2	2	(0.5,1.5)	3.47	2.44	-0.05	2.00	1.42	1.58	2.26	3.50	1.25	1.39	10.36%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI.EWMA\,ARL_1\right) / \,FP\,ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

72, Figure 67 and Figure 68).

Table 72. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.3

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	5.58	54.56%	1.1	5.67	63.83%	0	4.43	50.12%
1.2	1.96	40.55%	1.5	2.04	40.54%	1	2.35	39.19%
1.5	1.64	29.93%	2	1.47	20.67%	2	2.40	35.73%

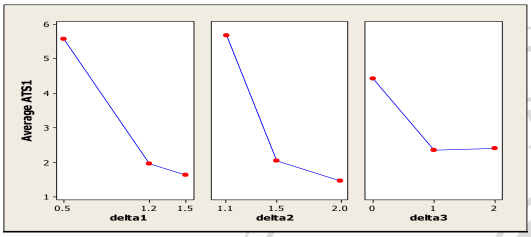

Figure 67. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.3

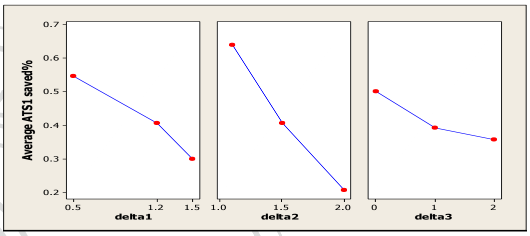

Figure 68. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.3

From Table 71, Table 72, Figure 67 and Figure 68, we found

a. ATS_1 saved % from 6.59% to 80.86%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is first decreasing then increasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

Under λ= 0.5, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 73 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 74.

Table 73. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.5

				W_2=0.5	W_2=1	W_2=1.5
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1
0	(0.2,2.0)	4.35	1.89	0.71	0.05	-0.23
1	(0.2,2.0)	4.06	2.07	0.71	0.10	-0.18
2	(0.2,2.0)	3.73	2.31	0.70	0.15	-0.12
0	(0.5,1.5)	4.35	1.89	0.97	0.20	-0.10
1	(0.5,1.5)	4.06	2.07	0.95	0.25	-0.05
2	(0.5,1.5)	3.73	2.31	0.93	0.31	0.02
0	(0.8,1.2)	4.35	1.89	0.97	0.20	-0.10
1	(0.8,1.2)	4.06	2.07	0.95	0.25	-0.05
2	(0.8,1.2)	3.73	2.31	0.93	0.31	0.02

Table 74. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.5

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	4.35	1.89	-0.23	1.50	0.20	0.30	0.62	1.78	38.23	109.54	65.10%
2	0.5	1.1	1	(0.5,1.5)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	11.07	38.67	71.38%
3	0.5	1.1	2	(0.8,1.2)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	11.04	29.36	62.39%
4	0.5	1.5	0	(0.2,2.0)	4.35	1.89	-0.23	1.50	0.20	0.30	0.62	1.78	2.34	7.79	69.97%
5	0.5	1.5	1	(0.5,1.5)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	2.83	5.90	52.05%
6	0.5	1.5	2	(0.8,1.2)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	3.87	6.20	37.60%
7	0.5	2	0	(0.2,2.0)	4.35	1.89	-0.23	1.50	0.20	0.30	0.62	1.78	1.32	2.36	44.11%
8	0.5	2	1	(0.5,1.5)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	1.63	2.31	29.61%
9	0.5	2	2	(0.8,1.2)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	2.18	2.66	18.21%
10	1.2	1.1	0	(0.5,1.5)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	5.89	21.67	72.84%
11	1.2	1.1	1	(0.8,1.2)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	2.68	5.10	47.43%
12	1.2	1.1	2	(0.2,2.0)	3.73	2.31	-0.12	1.50	1.16	1.55	2.22	4.09	1.32	3.60	63.33%
13	1.2	1.5	0	(0.5,1.5)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	2.15	4.14	48.21%
14	1.2	1.5	1	(0.8,1.2)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	1.87	2.42	22.83%
15	1.2	1.5	2	(0.2,2.0)	3.73	2.31	-0.12	1.50	1.16	1.55	2.22	4.09	1.23	2.21	44.46%
16	1.2	2	0	(0.5,1.5)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	1.41	1.88	24.97%
17	1.2	2	1	(0.8,1.2)	4.06	2.07	-0.05	1.50	0.40	0.59	1.07	2.41	1.44	1.59	9.48%
18	1.2	2	2	(0.2,2.0)	3.73	2.31	-0.12	1.50	1.16	1.55	2.22	4.09	1.16	1.63	28.80%
19	1.5	1.1	0	(0.8,1.2)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	4.25	9.86	56.88%
20	1.5	1.1	1	(0.2,2.0)	4.06	2.07	-0.18	1.50	0.40	0.59	1.02	2.41	1.22	2.70	54.75%
21	1.5	1.1	2	(0.5,1.5)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	1.39	2.04	31.91%
22	1.5	1.5	0	(0.8,1.2)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	2.16	3.01	28.37%
23	1.5	1.5	1	(0.2,2.0)	4.06	2.07	-0.18	1.50	0.40	0.59	1.02	2.41	1.15	1.79	36.18%
24	1.5	1.5	2	(0.5,1.5)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	1.29	1.64	20.97%
25	1.5	2	0	(0.8,1.2)	4.35	1.89	-0.10	1.50	0.20	0.30	0.65	1.78	1.48	1.67	11.14%
26	1.5	2	1	(0.2,2.0)	4.06	2.07	-0.18	1.50	0.40	0.59	1.02	2.41	1.09	1.39	21.82%
27	1.5	2	2	(0.5,1.5)	3.73	2.31	0.02	1.50	1.16	1.55	2.29	4.09	1.21	1.39	13.24%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI\ EWMA\ ARL_1\right) / \ FP\ ARL_1 \%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

75, Figure 69 and Figure 70).

Table 75. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.5

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	8.28	50.05%	1.1	8.57	58.45%	0	6.58	46.84%
1.2	2.13	40.26%	1.5	2.10	40.07%	1	2.77	38.39%
1.5	1.69	30.58%	2	1.44	22.38%	2	2.74	35.66%

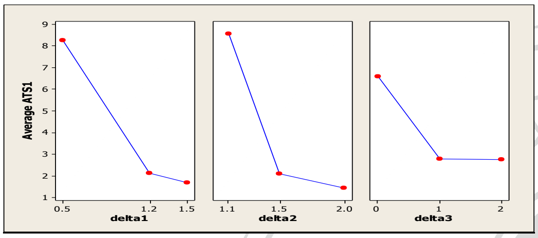

Figure 69. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.5

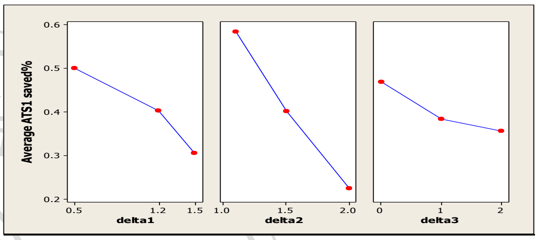

Figure 70. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.5

From Table 62, Table 63, Figure 69 and Figure 70, we found

a. ATS_1 saved % from 9.48% to 72.84%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

Under λ= 0.7, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are lisedt in Table 76 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 77.

Table 76. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.7

				W_2=0.5	W_2=1	W_2=1.5
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1
0	(0.2,2.0)	4.62	1.70	0.72	-0.01	-0.29
1	(0.2,2.0)	4.28	1.90	0.72	0.06	-0.23
2	(0.2,2.0)	3.89	2.19	0.71	0.13	-0.15
0	(0.5,1.5)	4.62	1.70	1.01	0.14	-0.16
1	(0.5,1.5)	4.28	1.90	0.99	0.21	-0.10
2	(0.5,1.5)	3.89	2.19	0.95	0.28	-0.01
0	(0.8,1.2)	4.62	1.70	1.01	0.14	-0.16
1	(0.8,1.2)	4.28	1.90	0.99	0.21	-0.10
2	(0.8,1.2)	3.89	2.19	0.95	0.28	-0.01

Table 77. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.7

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	4.62	1.70	-0.29	1.50	0.13	0.20	0.59	2.17	54.53	109.54	50.22%
2	0.5	1.1	1	(0.5,1.5)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	16.41	38.67	57.56%
3	0.5	1.1	2	(0.8,1.2)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	15.87	29.36	45.94%
4	0.5	1.5	0	(0.2,2.0)	4.62	1.70	-0.29	1.50	0.13	0.20	0.59	2.17	2.68	7.79	65.60%
5	0.5	1.5	1	(0.5,1.5)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	3.17	5.90	46.26%
6	0.5	1.5	2	(0.8,1.2)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	4.36	6.20	29.75%
7	0.5	2	0	(0.2,2.0)	4.62	1.70	-0.29	1.50	0.13	0.20	0.59	2.17	1.33	2.36	43.77%
8	0.5	2	1	(0.5,1.5)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	1.65	2.31	28.53%
9	0.5	2	2	(0.8,1.2)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	2.23	2.66	16.33%
10	1.2	1.1	0	(0.5,1.5)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	8.24	21.67	61.99%
11	1.2	1.1	1	(0.8,1.2)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	3.11	5.10	39.08%
12	1.2	1.1	2	(0.2,2.0)	3.89	2.19	-0.15	1.50	0.93	1.36	2.19	4.68	1.36	3.60	62.19%
13	1.2	1.5	0	(0.5,1.5)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	2.31	4.14	44.21%
14	1.2	1.5	1	(0.8,1.2)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	1.93	2.42	20.22%
15	1.2	1.5	2	(0.2,2.0)	3.89	2.19	-0.15	1.50	0.93	1.36	2.19	4.68	1.23	2.21	44.37%
16	1.2	2	0	(0.5,1.5)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	1.42	1.88	24.50%
17	1.2	2	1	(0.8,1.2)	4.28	1.90	-0.10	1.50	0.29	0.46	1.04	2.86	1.44	1.59	9.60%
18	1.2	2	2	(0.2,2.0)	3.89	2.19	-0.15	1.50	0.93	1.36	2.19	4.68	1.15	1.63	29.17%
19	1.5	1.1	0	(0.8,1.2)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	5.48	9.86	44.49%
20	1.5	1.1	1	(0.2,2.0)	4.28	1.90	-0.23	1.50	0.29	0.46	0.99	2.86	1.25	2.70	53.90%
21	1.5	1.1	2	(0.5,1.5)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	1.40	2.04	31.37%
22	1.5	1.5	0	(0.8,1.2)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	2.29	3.01	23.92%
23	1.5	1.5	1	(0.2,2.0)	4.28	1.90	-0.23	1.50	0.29	0.46	0.99	2.86	1.14	1.79	36.23%
24	1.5	1.5	2	(0.5,1.5)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	1.29	1.64	21.45%
25	1.5	2	0	(0.8,1.2)	4.62	1.70	-0.16	1.50	0.13	0.20	0.63	2.17	1.49	1.67	10.72%
26	1.5	2	1	(0.2,2.0)	4.28	1.90	-0.23	1.50	0.29	0.46	0.99	2.86	1.08	1.39	22.18%
27	1.5	2	2	(0.5,1.5)	3.89	2.19	-0.01	1.50	0.93	1.36	2.27	4.68	1.20	1.39	14.03%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI\ EWMA\ ARL_1\right) / \ FP\ ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

78, Figure 71 and Figure 72).

Table 78. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.7

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	11.36	42.66%	1.1	11.96	49.64%	0	8.86	41.05%
1.2	2.47	37.26%	1.5	2.27	36.89%	1	3.47	34.84%
1.5	1.85	28.70%	2	1.44	22.09%	2	3.34	32.73%

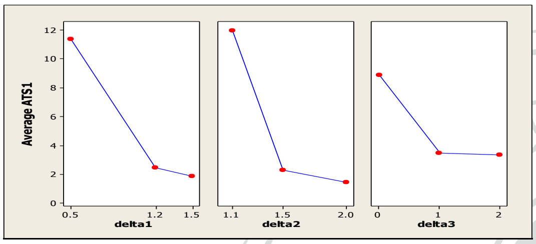

Figure 71. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.7

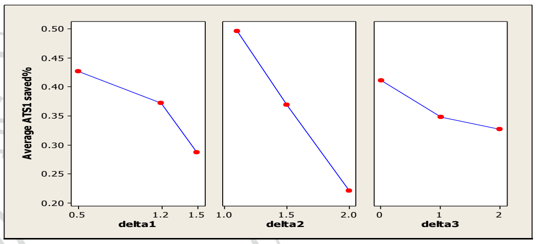

Figure 72. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.7

From Table 77, Table 78, Figure 71 and Figure 72, we found

a. ATS_1 saved % from 9.60% to 62.19%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

Under λ= 0.9, The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) are listed in Table 79 and the ATS_1s of the 27combinations of δ_1, δ_2 and δ_3 are calculated. We choose the combination of L_1, L_2, W_1 and W_2 which minimizes the ATS_1 and the results are listed in Table 80.

Table 79. The values of W_1 for various values of W_2, δ_3 and (h_2, h_1) under λ=0.9

				W_2=0.5	W_2=1	W_2=1.5
δ_3	(h_2,h_1)	L_1	L_2	W_1	W_1	W_1
0	(0.2,2.0)	4.76	1.53	0.75	-0.05	-0.33
1	(0.2,2.0)	4.39	1.75	0.74	0.04	-0.26
2	(0.2,2.0)	3.98	2.08	0.72	0.12	-0.17
0	(0.5,1.5)	4.76	1.53	1.06	0.10	-0.21
1	(0.5,1.5)	4.39	1.75	1.02	0.19	-0.13
2	(0.5,1.5)	3.98	2.08	0.97	0.27	-0.03
0	(0.8,1.2)	4.76	1.53	1.06	0.10	-0.21
1	(0.8,1.2)	4.39	1.75	1.02	0.19	-0.13
2	(0.8,1.2)	3.98	2.08	0.97	0.27	-0.03

Table 80. ATS_1 for VSI EWMA WL Chart and ATS_1 Saved % Compared to FP WL Chart under λ=0.9

No.	δ_1	δ_2	δ_3	(h_2, h_1)	L_1	L_2	W_1	W_2	LCL	LWL	UWL	UCL	VSI EWMA ATS_1	FP ARL_1	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	4.76	1.53	-0.33	1.50	0.07	0.08	0.55	2.57	70.07	109.54	36.04%
2	0.5	1.1	1	(0.5,1.5)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	23.10	38.67	40.27%
3	0.5	1.1	2	(0.8,1.2)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	22.14	29.36	24.59%
4	0.5	1.5	0	(0.2,2.0)	4.76	1.53	-0.33	1.50	0.07	0.08	0.55	2.57	3.31	7.79	57.51%
5	0.5	1.5	1	(0.5,1.5)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	3.70	5.90	37.29%
6	0.5	1.5	2	(0.8,1.2)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	5.06	6.20	18.50%
7	0.5	2	0	(0.2,2.0)	4.76	1.53	-0.33	1.50	0.07	0.08	0.55	2.57	1.37	2.36	41.95%
8	0.5	2	1	(0.5,1.5)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	1.72	2.31	25.63%
9	0.5	2	2	(0.8,1.2)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	2.34	2.66	12.09%
10	1.2	1.1	0	(0.5,1.5)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	11.46	21.67	47.12%
11	1.2	1.1	1	(0.8,1.2)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	3.83	5.10	24.93%
12	1.2	1.1	2	(0.2,2.0)	3.98	2.08	-0.17	1.50	0.70	1.14	2.15	5.30	1.47	3.60	59.24%
13	1.2	1.5	0	(0.5,1.5)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	2.58	4.14	37.69%
14	1.2	1.5	1	(0.8,1.2)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	2.06	2.42	14.81%
15	1.2	1.5	2	(0.2,2.0)	3.98	2.08	-0.17	1.50	0.70	1.14	2.15	5.30	1.26	2.21	42.97%
16	1.2	2	0	(0.5,1.5)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	1.45	1.88	22.79%
17	1.2	2	1	(0.8,1.2)	4.39	1.75	-0.13	1.50	0.18	0.31	1.01	3.33	1.46	1.59	8.16%
18	1.2	2	2	(0.2,2.0)	3.98	2.08	-0.17	1.50	0.70	1.14	2.15	5.30	1.16	1.63	28.62%
19	1.5	1.1	0	(0.8,1.2)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	7.21	9.86	26.92%
20	1.5	1.1	1	(0.2,2.0)	4.39	1.75	-0.26	1.50	0.18	0.31	0.95	3.33	1.30	2.70	51.79%
21	1.5	1.1	2	(0.5,1.5)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	1.46	2.04	28.18%
22	1.5	1.5	0	(0.8,1.2)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	2.51	3.01	16.89%
23	1.5	1.5	1	(0.2,2.0)	4.39	1.75	-0.26	1.50	0.18	0.31	0.95	3.33	1.16	1.79	35.51%
24	1.5	1.5	2	(0.5,1.5)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	1.31	1.64	20.23%
25	1.5	2	0	(0.8,1.2)	4.76	1.53	-0.21	1.50	0.07	0.08	0.60	2.57	1.52	1.67	8.87%
26	1.5	2	1	(0.2,2.0)	4.39	1.75	-0.26	1.50	0.18	0.31	0.95	3.33	1.09	1.39	22.04%
27	1.5	2	2	(0.5,1.5)	3.98	2.08	-0.03	1.50	0.70	1.14	2.26	5.30	1.20	1.39	13.67%

Note: ARL_1 saved %: $\left(FP\text{-}ARL_1 - VSI\ EWMA\ ARL_1\right) / FP\ ARL_1\%$

To investigate the effect of δ_1, δ_2 and δ_3 on the ATS_1 and ATS_1 saved %, the response table and response diagram show the results of the sensitivity analysis (Table

81, Figure 73 and Figure 74).

Table 81. VSI EWMA WL Chart –Response Table of δ_1, δ_2, δ_3 vs. $\overline{ATS_1}$ and δ_1, δ_2, δ_3 vs. $\overline{ATS_1 \text{ saved } \%}$ under λ=0.9

δ_1	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_2	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$	δ_3	$\overline{ATS_1}$	$\overline{ATS_1 \text{ saved } \%}$
0.5	14.76	32.65%	1.1	15.78	37.68%	0	11.28	32.86%
1.2	2.97	31.81%	1.5	2.55	31.27%	1	4.38	28.94%
1.5	2.08	24.90%	2	1.48	20.42%	2	4.16	27.57%

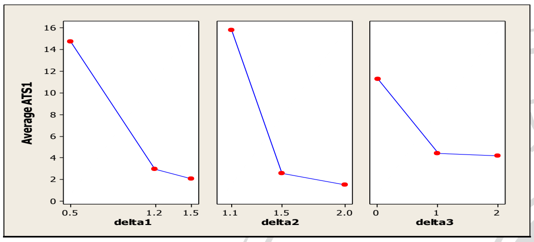

Figure 73. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1}$ under λ = 0.9

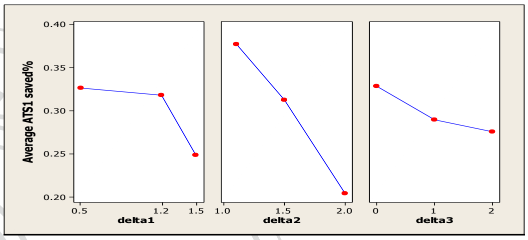

Figure 74. VSI EWMA WL Chart – Response Diagram of δ_1, δ_2, δ_3 and $\overline{ATS_1 \text{ saved } \%}$ under λ = 0.9

From Table 80, Table 81, Figure 73 and Figure 74, we found

a. ATS_1 saved % from 8.16% to 59.24%.

b. the $\overline{ATS_1}$ is decreasing as δ_1 increasing.

c. the $\overline{ATS_1}$ is decreasing as δ_2 increasing.

d. the $\overline{ATS_1}$ is decreasing as δ_3 increasing.

e. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_1 increasing.

f. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_2 increasing.

g. the $\overline{ATS_1 \text{ saved } \%}$ is decreasing as δ_3 increasing.

From the above results, same as the FP EWMA WL chart, when the value of λ is small (λ<0.2), the ATS_1 saved % may be negative when the δ_1, δ_2 and/or δ_3 are large.

For different values of λ in Tables 62, 65, 68, 71, 74, 77 and 80, the choice of the λ with maximal ATS_1 saved % for each combination of δ_1, δ_2, δ_3 and (h_2, h_1) in $L_{27}(3^{13})$ is listed in see Table 82.That is, we may choose the λ for the given combination δ_1, δ_2, δ_3 and (h_2, h_1) from Table 82.

Table 82. The Optimal λ to Maximize the ATS_1 Saved % Time

No.	δ_1	δ_2	δ_3	(h_2, h_1)	λ^*	ATS_1 saved %	No.	δ_1	δ_2	δ_3	(h_2, h_1)	λ^*	ATS_1 saved %
1	0.5	1.1	0	(0.2,2.0)	0.05	90.39%	16	1.2	2	0	(0.5,1.5)	0.5	24.97%
2	0.5	1.1	1	(0.5,1.5)	0.1	84.04%	17	1.2	2	1	(0.8,1.2)	0.7	9.60%
3	0.5	1.1	2	(0.8,1.2)	0.1	76.04%	18	1.2	2	2	(0.2,2.0)	0.7	29.17%
4	0.5	1.5	0	(0.2,2.0)	0.3	72.10%	19	1.5	1.1	0	(0.8,1.2)	0.2	64.35%
5	0.5	1.5	1	(0.5,1.5)	0.3	54.85%	20	1.5	1.1	1	(0.2,2.0)	0.5	54.75%
6	0.5	1.5	2	(0.8,1.2)	0.3	41.26%	21	1.5	1.1	2	(0.5,1.5)	0.5	31.91%
7	0.5	2	0	(0.2,2.0)	0.5	44.11%	22	1.5	1.5	0	(0.8,1.2)	0.3	29.26%
8	0.5	2	1	(0.5,1.5)	0.5	29.61%	23	1.5	1.5	1	(0.2,2.0)	0.7	36.23%
9	0.5	2	2	(0.8,1.2)	0.5	18.21%	24	1.5	1.5	2	(0.5,1.5)	0.7	21.45%
10	1.2	1.1	0	(0.5,1.5)	0.2	80.93%	25	1.5	2	0	(0.8,1.2)	0.5	11.14%
11	1.2	1.1	1	(0.8,1.2)	0.3	50.35%	26	1.5	2	1	(0.2,2.0)	0.7	22.18%
12	1.2	1.1	2	(0.2,2.0)	0.5	63.33%	27	1.5	2	2	(0.5,1.5)	0.7	14.03%
13	1.2	1.5	0	(0.5,1.5)	0.3	49.69%							
14	1.2	1.5	1	(0.8,1.2)	0.5	22.83%							
15	1.2	1.5	2	(0.2,2.0)	0.5	44.46%							

It can find that we have to choose the bigger λ when the values of δ_1, δ_2 and/or δ_3 are large. However, the ATS saved % of the combination #1 is 90.39% which is the maximum , that is, the VSI EWMA WL chart has the best performance under λ=0.05 when the shifts of the mean and/or variance are much smaller.

5.4 ATS$_1$ Comparison among the EWMA- T*, EWMA-T and One-Sided FP EWMA WL Control Charts

(1) One-sided FP EWMA WL chart vs. EWMA- T* Chart

Chen, Cheng and Xie (2004) proposed EWMA T* chart to monitor mean and variance simultaneously. They let $T = \mu_0 = 0\ (\delta_3 = 0)$, $\sigma_0^2 = 1$, n = 5, $ARL_0 = 370$. The statistic of this chart is

$$T_i^* = \frac{n(\bar{X}_i - \mu_0)^2}{\sigma_0^2} + \frac{(n-1)S_i^2}{\sigma_0^2}.$$

The EWMA of the T_i^* is

$$Q_i = (1-\lambda)Q_{i-1} + \lambda T_i^*,\ i = 1, 2, ...$$

where $0 < \lambda \le 1$.

Because $T_i^* \sim \chi_n^2$, the mean and the variance of Q_i are

$$E(Q_i) = E(T_i^*) = n$$

$$Var(Q_i) = \frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]Var(T_i^*) \overset{i\to\infty}{=} \frac{2n\lambda}{(2-\lambda)}.$$

Then the control limits of EWMA T* chart are

$$UCL = n + L\sqrt{\frac{2n\lambda}{(2-\lambda)}} \text{ and } LCL = 0$$

where $0 < \lambda \le 1$ and L is the control factor.

The control limits of our one-sided FP EWMA WL chart are

$$UCL = E(WL) + L\sqrt{\frac{\lambda}{(2-\lambda)}Var(WL)} \text{ and } LCL = 0$$

where $0 < \lambda \le 1$ and L is the control factor.

To compare the ARL in the two chart, we let the FP EWMA WL chart have the same λ value as with the EWMA T* chart. We would try to find the optimal a* that minimizes the ARL$_1$ of the one-sided FP EWMA chart. That is

$$\text{Min. } ARL_1$$

$$\text{s.t. } 0 < a < 1$$

The ARL_1s of the one-sided FP EWMA WL chart and ARL_1s of EWMA T^* chart for various δ_1 and δ_2 are listed in Table 85.

Table 83. Values of ARL of the FP One-sided EWMA-WL Chart with Optimal Weight and Optimal EWMA T^* Chart.

			δ_1							
δ_2			0	0.25	0.5	1	1.5	2	2.5	3
1		λ^*	0.05	0.05	0.05	0.05	0.365	0.655	0.9	0.98
	ARL	EWMA T^*	369.99	78.75	23.66	5.33	2.26	1.30	1.04	1.00
		EWMA WL	370.00 a*=0.2	73.54 a*=0.2	15.33 a*=0.2	3.72 a*=0.2	1.54 a*=0.2	1.07 a*=0.2	1.00 a*=0.2	1.00 a*=0.2
1.25		λ	0.05	0.05	0.05	0.265	0.465	0.675	0.885	0.975
	ARL	EWMA T^*	9.80	8.80	6.73	3.47	1.89	1.27	1.06	1.01
		EWMA WL	11.80 a*=0.45	10.40 a*=0.35	7.25 a*=0.25	2.77 a*=0.2	1.52 a*=0.2	1.11 a*=0.2	1.01 a*=0.2	1.00 a*=0.2
1.5		λ	0.05	0.05	0.275	0.365	0.555	0.7	0.87	0.95
	ARL	EWMA T^*	4.48	4.28	3.71	2.44	1.64	1.24	1.07	1.01
		EWMA WL	5.24 a*=0.45	5.00 a*=0.4	3.66 a*=0.4	2.26 a*=0.3	1.47 a*=0.3	1.13 a*=0.2	1.02 a*=0.2	1.00 a*=0.2
2		λ	0.465	0.465	0.485	0.555	0.655	0.785	0.88	0.95
	ARL	EWMA T^*	1.98	1.93	1.84	1.58	1.33	1.16	1.07	1.03
		EWMA WL	1.97 a*=0.45	1.94 a*=0.45	1.85 a*=0.4	1.57 a*=0.4	1.30 a*=0.35	1.12 a*=0.3	1.03 a*=0.3	1.01 a*=0.25
2.5		λ	0.655	0.655	0.655	0.675	0.755	0.845	0.88	0.98
	ARL	EWMA T^*	1.38	1.35	1.37	1.27	1.18	1.11	1.06	1.02
		EWMA WL	1.38 a*=0.45	1.38 a*=0.45	1.35 a*=0.45	1.27 a*=0.4	1.17 a*=0.4	1.09 a*=0.35	1.04 a*=0.35	1.01 a*=0.3
3		λ	0.775	0.785	0.785	0.815	0.845	0.88	0.93	0.93
	ARL	EWMA T^*	1.18	1.17	1.17	1.14	1.10	1.07	1.04	1.02
		EWMA WL	1.18 a*=0.45	1.18 a*=0.45	1.17 a*=0.45	1.14 a*=0.45	1.10 a*=0.4	1.06 a*=0.35	1.03 a*=0.35	1.01 a*=0.35

From Table 83, most ARL_1 of optimal FP WL chart are smaller than or close to the ARL_1 of EWMA T* chart under $\delta_1 > 0.5$ and $\delta_2 > 1.25$, but when $\delta_1 \le 0.5$ and $1 < \delta_2 \le 1.25$, FP EWMA WL chart performs poorer than the EWMA T* chart.

(2) One-sided FP EWMA WL Control Chart vs. EWMA T

Costa and Rahim (2006) proposed the EWMA T chart to monitor mean and variance simultaneously. They let $\mu_0 = 0$, $\sigma_0^2 = 1$, n = 5, $ARL_0 = 370$. The statistic of the EWMA T chart is

$$T_i = \sum_{j=1}^{n} (X_{ij} - \mu_0 + \zeta_i \sigma_0)^2 .$$

They also define

$$e_i = \bar{X}_i - \mu_0 \quad \text{and}$$

$$\zeta_i = \begin{cases} d \text{ , } if \;\; e_i \ge 0 \; (\bar{X}_i \ge \mu_0) \\ -d \text{ , } if \;\; e_i < 0 \; (\bar{X}_i < \mu_0) \end{cases}$$

when d = 0,

$$\frac{T_i}{(\delta_2 \sigma_0)^2} \sim \chi^2_{n,\lambda} \text{ , } \lambda = \frac{n\delta_1^2}{\delta_2^2}$$

when d > 0,

$$\frac{T_i}{(\delta_2 \sigma_0)^2} \sim \chi^2_{n,\lambda^*} \text{ , } \lambda^* = \begin{cases} \dfrac{n(\delta_1 + d)^2}{\delta_2^2} \text{ , } if \;\; \bar{X}_i \ge \mu_0 \\ \dfrac{n(\delta_1 - d)^2}{\delta_2^2} \text{ , } if \;\; \bar{X}_i < \mu_0 \end{cases}$$

where d is equal to δ_3.

The EWMA of the T_i is

$$W_i = (1-\lambda) W_{i-1} + \lambda T_i \;\; , i = 1, 2, ...$$

where $0 < \lambda \le 1$.

The control limits of EWMA T chart are

$$UCL = k\sigma_0^2 \quad \text{and} \quad LCL = 0$$

Let $\alpha = P(W_i > UCL) = 0.0027$ and UCL could be computed by Markov chain approach.

To compare the ARL_1 of the EWMA T chart and the one-sided FP EWMA WL chart with optimal *a*, we have to determine optimal *a* by

$$\text{Min. } ARL_1$$
$$\text{s.t. } 0 < a < 1$$

The ARL_1s of the one-sided FP EWMA WL chart, the EWMA T chart and $\bar{X} - R$ chart are listed in Table 84 and Table 85 under $\lambda = 0.1$ and $\lambda = 0.2$. Here, the control limits of $\bar{X} - R$ chart with control limits of $\bar{X}$ at $\mu_0 \pm 3.190\sigma_0 / \sqrt{5}$ and upper control limit of R chart at $5.397\sigma_0$.

Table 84. Values of ARL of the One-sided FP EWMA WL Chart with Optimal Weight, EWMA T Chart and $\bar{X}-R$ Chart under $\lambda=0.1$

			$\delta_3=0\ (d=0)$			$\delta_3=1\ (d=1)$		
			EWMA T	EWMA WL		EWMA T	EWMA WL	
δ_1	δ_2	$\bar{X}-R$	k=6.954		a^*	k=16.5737		a^*
0.0	1.0	370.4	370.4	370.4		370.4	370.4	
	1.25	33.6	11.6	11.5	0.45	14.2	15.5	0.6
	1.5	8.9	4.9	4.8	0.45	5.9	6.2	0.6
	2.0	2.6	2.3	2.3	0.45	2.8	2.8	0.6
0.25	1.0	178.0	147.3	83.0	0.2	80.3	21.8	0.25
	1.25	26.4	10.3	10.0	0.4	11.3	8.9	0.6
	1.5	8.1	4.7	4.5	0.4	5.5	4.8	0.6
	2.0	2.5	2.3	2.2	0.4	2.7	2.5	0.6
0.5	1.0	49.0	33.3	15.7	0.2	16.3	7.5	0.25
	1.25	14.9	7.6	6.8	0.3	7.2	5.5	0.45
	1.5	6.3	4.1	3.9	0.35	4.4	3.8	0.6
	2.0	2.3	2.2	2.1	0.4	2.5	2.3	0.6
0.75	1.0	15.1	11.6	6.1	0.2	6.8	4.3	0.25
	1.25	7.7	5.4	4.3	0.2	4.7	3.7	0.4
	1.5	4.4	3.5	3.2	0.3	3.5	3.1	0.5
	2.0	2.1	2.1	2.0	0.4	2.3	2.1	0.6
1.0	1.0	5.8	6.0	3.4	0.2	4.0	2.9	0.25
	1.25	4.2	3.9	2.9	0.2	3.3	2.8	0.35
	1.5	3.1	2.9	2.5	0.25	2.8	2.5	0.45
	2.0	1.9	1.9	1.8	0.35	2.1	1.9	0.6
1.5	1.0	1.8	2.7	1.7	0.2	2.2	1.6	0.2
	1.25	1.8	2.3	1.7	0.20	2.0	1.8	0.3
	1.5	1.7	2.0	1.6	0.20	1.9	1.7	0.4
	2.0	1.5	1.6	1.5	0.30	1.6	1.5	0.5

Table 85. Values of ARL of the One-sided FP EWMA WL Chart with Optimal Weight, EWMA T Chart and $\bar{X}-R$ Chart under $\lambda=0.2$

			$\delta_3=0\ (d=0)$			$\delta_3=1\ (d=1)$		
			EWMA T	EWMA WL		EWMA T	EWMA WL	
δ_1	δ_2	$\bar{X}-R$	k=8.337		a^*	K=18.644		a^*
0.0	1.0	370.4	370.4	370.4		370.4	370.4	
	1.25	33.6	12.2	12.3	0.45	15.2	16.8	0.6
	1.5	8.9	4.6	4.5	0.45	5.5	5.9	0.6
	2.0	2.6	2.1	2.1	0.45	2.4	2.5	0.6
0.25	1.0	178.0	174.5	96.9	0.2	96.8	28.7	0.25
	1.25	26.4	10.6	10.5	0.4	11.5	9.1	0.6
	1.5	8.1	4.3	4.3	0.4	5.0	4.5	0.6
	2.0	2.5	2.1	2.0	0.45	2.4	2.3	0.6
0.5	1.0	49.0	41.6	18.1	0.2	17.8	8.0	0.25
	1.25	14.9	7.5	6.8	0.3	6.8	5.4	0.5
	1.5	6.3	3.8	3.7	0.35	4.0	3.5	0.6
	2.0	2.3	2.0	2.0	0.4	2.2	2.1	0.6
0.75	1.0	15.1	12.6	6.3	0.2	6.4	4.1	0.25
	1.25	7.7	5.1	4.2	0.25	4.2	3.5	0.4
	1.5	4.4	3.1	3.0	0.3	3.1	2.8	0.55
	2.0	2.1	1.8	1.8	0.4	2.0	1.9	0.6
1.0	1.0	5.8	5.8	3.3	0.2	3.6	2.7	0.25
	1.25	4.2	3.5	2.8	0.2	2.9	2.5	0.35
	1.5	3.1	2.6	2.3	0.3	2.4	2.2	0.5
	2.0	1.9	1.7	1.7	0.35	1.8	1.7	0.6
1.5	1.0	1.8	2.4	1.6	0.2	1.8	1.5	0.2
	1.25	1.8	2.0	1.6	0.2	1.7	1.6	0.3
	1.5	1.7	1.8	1.5	0.2	1.6	1.6	0.4
	2.0	1.5	1.4	1.4	0.3	1.5	1.4	0.55

From Table 84 and Table 85, we found the EWMA WL chart has smaller ARL_1s than the EWMA T chart and $\bar{X}-R$ chart under $\delta_3=0$ for $\lambda=0.1$ and $\lambda=0.2$. However, the EWMA WL chart has smaller ARL_1s than the EWMA T chart when $\delta_1\neq 0$ under $\delta_3=1$ for $\lambda=0.1$ and $\lambda=0.2$.

CHAPTER 6. AN EXAMPLE

An example is provided to illustrate the application of the FP EWMA WL, the VSI EWMA, the FP WL, the optimal VSI WL and the optimal VP WL control charts. We also construct the $\bar{Y}-S^2$ and $EWMA\,\bar{Y}-EWMA\ln(S^2)$ charts to compare with our charts.

Consider the thickness data of gold films form Yang, Lin and Hung (2008).

25 samples are used to construct the control charts. First, we define the design parameters:

Thickness target = 18 μm, sample size n = 4, ARL_0 (ATS_0) = 370.37, $a = 0.5$, $\lambda = 0.05$ and Y = T – X, where X is the thickness.

First, we use the normal probability plot to check the normality. From Figure 75, it shows the p-value is 0.261, larger than 0.05. So X seems to fit a normal distribution.

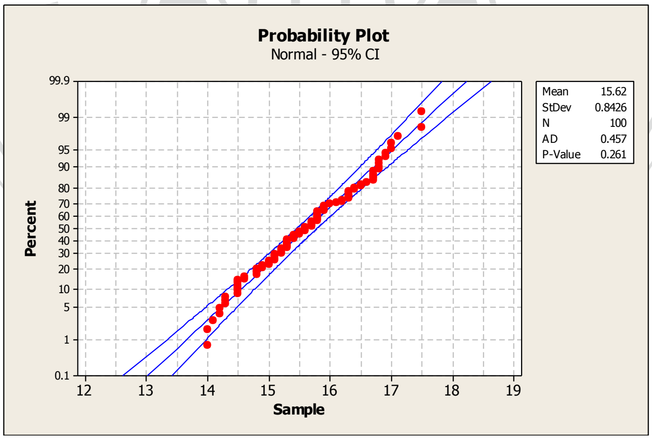

Figure 75. The Probability Plot for X

We then construct the FP WEMA WL, the VSI EWMA WL, the FP WL, the optimal VSI WL, the optimal VP WL, the $\bar{Y}-S^2$ and the $EWMA\,\bar{Y}-EWMA\ln(S^2)$ charts and compare their performances. The values of plotting statistics of these charts are evaluated and listed in Table 86.

Table 86. Thickness and Various Plotting Statistics

No.	X_1	X_2	X_3	X_4	$\bar{X}$	S	S^2	$\bar{Y}$	WL	EWMA WL	EWMA $\bar{Y}$	EWMA $\ln S^2$
1	15.8	15.8	16.7	16.2	16.13	0.43	0.18	1.88	1.85	3.09	2.36	-1.05
2	15.3	15.9	16.8	16.8	16.20	0.74	0.54	1.80	1.89	3.03	2.33	-1.03
3	17	15.6	15.7	17.5	16.45	0.95	0.90	1.55	1.65	2.96	2.29	-0.98
4	17.1	16.3	15.4	16.7	16.38	0.73	0.53	1.63	1.59	2.89	2.26	-0.96
5	16.2	16.5	15.8	15.7	16.05	0.37	0.14	1.95	1.97	2.85	2.24	-1.02
6	15.5	15.3	16.1	16.9	15.95	0.72	0.52	2.05	2.36	2.82	2.23	-1.00
7	15.4	15.7	14.5	15.2	15.20	0.51	0.26	2.80	4.05	2.88	2.26	-1.02
8	16.3	15.8	16.7	16	16.20	0.39	0.15	1.80	1.70	2.83	2.24	-1.06
9	14.8	16.3	15	15.9	15.50	0.72	0.51	2.50	3.38	2.85	2.25	-1.04
10	15.7	16.8	15.3	16.7	16.13	0.74	0.55	1.88	2.03	2.81	2.23	-1.02
11	15.1	15.8	15.4	15.7	15.50	0.32	0.10	2.50	3.18	2.83	2.25	-1.08
12	14.5	14.8	16.9	14.2	15.10	1.23	1.50	2.90	4.96	2.94	2.28	-1.01
13	15	14.5	14.5	14.9	14.73	0.26	0.07	3.28	5.40	3.06	2.33	-1.09
14	14.6	15.5	15.8	15.3	15.30	0.51	0.26	2.70	3.78	3.10	2.35	-1.10
15	15.4	14.9	15.1	14.1	14.88	0.56	0.31	3.13	5.04	3.19	2.39	-1.11
16	15.9	14.3	14.6	15.2	15.00	0.71	0.50	3.00	4.75	3.27	2.42	-1.09
17	14.5	16.3	15.1	14	14.98	0.99	0.98	3.03	5.07	3.36	2.45	-1.03
18	14	14.3	15	15.2	14.63	0.57	0.32	3.38	5.86	3.49	2.49	-1.04
19	15.2	14.2	15.9	15.5	15.20	0.73	0.53	2.80	4.18	3.52	2.51	-1.02
20	15.6	16.3	17.5	14.8	16.05	1.15	1.31	1.95	2.56	3.47	2.48	-0.95
21	15.1	16.3	16.4	16.6	16.10	0.68	0.46	1.90	2.04	3.40	2.45	-0.94
22	15.9	16.4	16.8	15.8	16.23	0.47	0.22	1.78	1.68	3.31	2.42	-0.97
23	15.1	15.3	15.8	17	15.80	0.85	0.73	2.20	2.78	3.29	2.41	-0.94
24	16.7	14.8	16.5	15.6	15.90	0.88	0.77	2.10	2.59	3.25	2.39	-0.91
25	14.3	14.5	15.3	15.3	14.85	0.53	0.28	3.15	5.10	3.35	2.43	-0.93
					$\bar{\bar{X}}_0 =$ 15.62	$\bar{S}_0 =$ 0.67						

Use $\bar{\bar{X}}_0 = 15.62$, $\frac{\bar{S}_0}{c_4}$=0.73 to estimate μ_0 and σ_0 where $c_4 = 0.9213$ is the factor of S chart.

(1) FP EWMA WL Chart

From data, we obtain

$$\hat{\tau}_0 = \frac{n(\hat{\mu}_0 - T)^2}{\hat{\sigma}_0^2} = 43.31$$

$$E(\hat{WL}) = a\hat{\sigma}_0^2 + (1-a)\frac{(1+\hat{\tau}_0)\hat{\sigma}_0^2}{n} = 3.17$$

$$Var(\hat{WL}) = a^2\frac{2\hat{\sigma}_0^4}{n-1} + (1-a)^2\frac{2(1+2\hat{\tau}_0)\hat{\sigma}_0^4}{n^2} = 0.80$$

$$T - \hat{\mu}_0 = \hat{\delta}_3\hat{\sigma}_0 \Rightarrow \hat{\delta}_3 = \frac{T-\hat{\mu}_0}{\hat{\sigma}_0} = 3.29$$

We can get $L_1 = 2.60$ and $L_2 = 2.41$ under $ARL_0 = 370.37$ based on section 5.1. The FP EWMA WL control limits are:

$$UCL = E(\hat{WL}) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 3.54,$$

$$LCL = E(\hat{WL}) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 2.82.$$

Then we plot the values of EWMA WL on the FP EWMA WL chart (Figure 76).

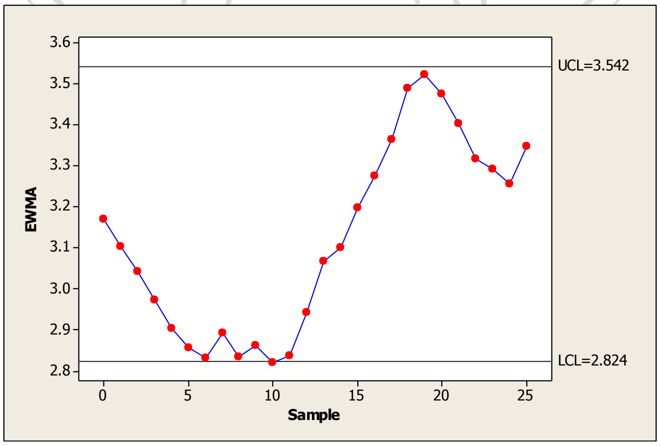

Figure 76. FP EWMA WL Control Chart

Sample #10 is out-of-control, so we delete the sample and reconstruct the FP

EWMA-WL chart. The data and the EWMA plotting statistics for WL are listed in Table 87.

Table 87. Thickness Data after Deleting Sample #10 and Their EWMA WL Values

No.	X_1	X_2	X_3	X_4	$\bar{X}$	S	EWMA
1	15.8	15.8	16.7	16.2	16.13	0.43	3.15
2	15.3	15.9	16.8	16.8	16.20	0.74	3.09
3	17	15.6	15.7	17.5	16.45	0.95	3.01
4	17.1	16.3	15.4	16.7	16.38	0.73	2.94
5	16.2	16.5	15.8	15.7	16.05	0.37	2.89
6	15.5	15.3	16.1	16.9	15.95	0.72	2.87
7	15.4	15.7	14.5	15.2	15.20	0.51	2.93
8	16.3	15.8	16.7	16	16.20	0.39	2.87
9	14.8	16.3	15	15.9	15.50	0.72	2.89
11	15.1	15.8	15.4	15.7	15.50	0.32	2.91
12	14.5	14.8	16.9	14.2	15.10	1.23	3.01
13	15	14.5	14.5	14.9	14.73	0.26	3.13
14	14.6	15.5	15.8	15.3	15.30	0.51	3.16
15	15.4	14.9	15.1	14.1	14.88	0.56	3.25
16	15.9	14.3	14.6	15.2	15.00	0.71	3.33
17	14.5	16.3	15.1	14	14.98	0.99	3.42
18	14	14.3	15	15.2	14.63	0.57	3.54
19	15.2	14.2	15.9	15.5	15.20	0.73	3.57
20	15.6	16.3	17.5	14.8	16.05	1.15	3.52
21	15.1	16.3	16.4	16.6	16.10	0.68	3.45
22	15.9	16.4	16.8	15.8	16.23	0.47	3.36
23	15.1	15.3	15.8	17	15.80	0.85	3.33
24	16.7	14.8	16.5	15.6	15.90	0.88	3.29
25	14.3	14.5	15.3	15.3	14.85	0.53	3.38
					$\bar{\bar{X}}_0 =$ 15.60	$\bar{S}_0 =$ 0.66	

The estimates of μ_0 and σ_0 are $\bar{\bar{X}}_0 = 15.60$ and $\frac{\bar{S}_0}{c_4}$ =0.72.

From data, we obtain

$$\hat{\tau}_0 = \frac{n(\hat{\mu}_0 - T)^2}{\hat{\sigma}_0^2} = 44.49$$

$$E(\hat{WL}) = a\hat{\sigma}_0^2 + (1-a)\frac{(1+\hat{\tau}_0)\hat{\sigma}_0^2}{n} = 3.22$$

$$Var(\hat{WL}) = a^2\frac{2\hat{\sigma}_0^4}{n-1} + (1-a)^2\frac{2(1+2\hat{\tau}_0)\hat{\sigma}_0^4}{n^2} = 0.81$$

$$T - \hat{\mu}_0 = \hat{\delta}_3\hat{\sigma}_0 \Rightarrow \hat{\delta}_3 = \frac{T-\hat{\mu}_0}{\hat{\sigma}_0} = 3.34$$

We calculated $L_1 = 2.59$ and $L_2 = 2.42$ under $ARL_0 = 370.37$ using previous approach. The FP EWMA WL control limits are

$$UCL = E(\hat{WL}) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 3.59,$$

$$LCL = E(\hat{WL}) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 2.87.$$

Then we plotted the values of EWMA WL on the FP EWMA WL chart (Figure 77).

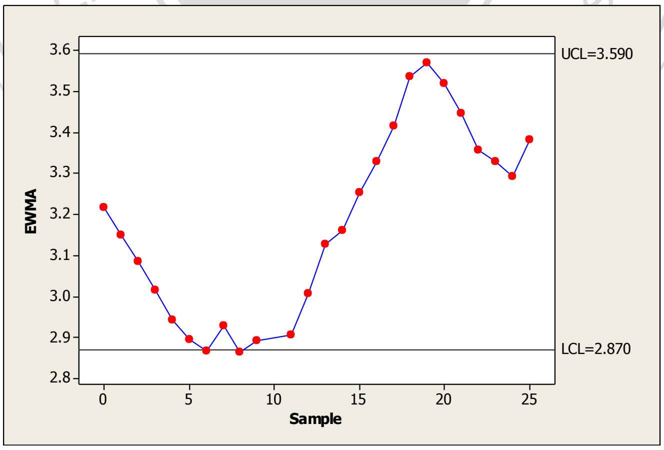

Figure 77. FP EWMA WL Chart

Samples #6 and #8 are out-of-control, so we delete the both samples and reconstruct the FP EWMA-WL chart. The data and the EWMA plotting statistics for WL are listed in Table 88.

Table 88. Thickness Data after Deleting sample #6 and #8 and Their EWMA WL Values

No.	X_1	X_2	X_3	X_4	$\bar{X}$	S	EWMA WL
1	15.8	15.8	16.7	16.2	16.13	0.43	3.26
2	15.3	15.9	16.8	16.8	16.20	0.74	3.19
3	17	15.6	15.7	17.5	16.45	0.95	3.11
4	17.1	16.3	15.4	16.7	16.38	0.73	3.04
5	16.2	16.5	15.8	15.7	16.05	0.37	2.98
7	15.4	15.7	14.5	15.2	15.20	0.51	3.04
9	14.8	16.3	15	15.9	15.50	0.72	3.05
11	15.1	15.8	15.4	15.7	15.50	0.32	3.06
12	14.5	14.8	16.9	14.2	15.10	1.23	3.16
13	15	14.5	14.5	14.9	14.73	0.26	3.27
14	14.6	15.5	15.8	15.3	15.30	0.51	3.29
15	15.4	14.9	15.1	14.1	14.88	0.56	3.38
16	15.9	14.3	14.6	15.2	15.00	0.71	3.45
17	14.5	16.3	15.1	14	14.98	0.99	3.53
18	14	14.3	15	15.2	14.63	0.57	3.65
19	15.2	14.2	15.9	15.5	15.20	0.73	3.67
20	15.6	16.3	17.5	14.8	16.05	1.15	3.62
21	15.1	16.3	16.4	16.6	16.10	0.68	3.54
22	15.9	16.4	16.8	15.8	16.23	0.47	3.45
23	15.1	15.3	15.8	17	15.80	0.85	3.41
24	16.7	14.8	16.5	15.6	15.90	0.88	3.37
25	14.3	14.5	15.3	15.3	14.85	0.53	3.46
					$\bar{\bar{X}}_0 = 15.55$	$\bar{S}_0 = 0.67$	

The estimates of μ_0 and σ_0 are $\bar{\bar{X}}_0 = 15.55$ and $\frac{\bar{S}_0}{c_4} = 0.67$.

From data, we obtain

$$\hat{\tau}_0 = \frac{n(\hat{\mu}_0 - T)^2}{\hat{\sigma}_0^2} = 44.77$$

$$E(\hat{WL}) = a\hat{\sigma}_0^2 + (1-a)\frac{(1+\hat{\tau}_0)\hat{\sigma}_0^2}{n} = 3.33$$

$$Var(\hat{WL}) = a^2\frac{2\hat{\sigma}_0^4}{n-1} + (1-a)^2\frac{2(1+2\hat{\tau}_0)\hat{\sigma}_0^4}{n^2} = 0.86$$

$$T-\hat{\mu}_0=\hat{\delta}_3\hat{\sigma}_0 \Rightarrow \hat{\delta}_3=\frac{T-\hat{\mu}_0}{\hat{\sigma}_0}=3.35$$

We calculated $L_1=2.60$ and $L_2=2.42$ under $ARL_0=370.37$ using previous approach. The FP EWMA WL control limits are

$$UCL=E(\hat{WL})+L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})}=3.72,$$

$$LCL=E(\hat{WL})-L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})}=2.98.$$

Then we plotted the values of EWMA WL on the FP EWMA WL chart (Figure 78).

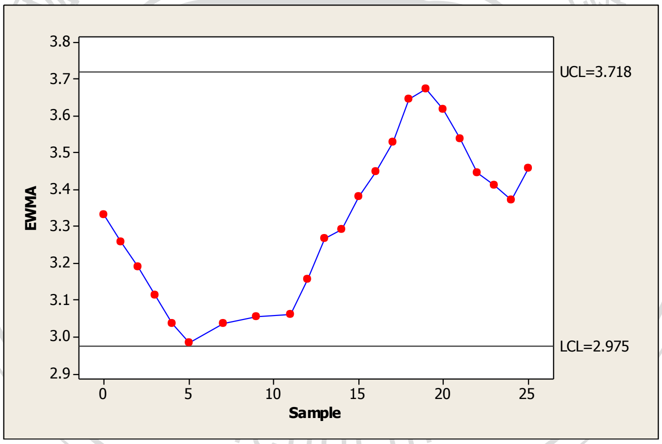

Figure 78. FP EWMA WL Chart

From Figure 78, there is no signal, hence we would use this chart to monitor mean and variance simultaneously from now on.

In order to compute the ATS_1, we use sample #6, #8 and #10 to derive the out-of-control distribution, and compute the shift parameters δ_1 and δ_2 of the mean and variance.

Table 89. The Three out-of-control Points

No.	X_1	X_2	X_3	X_4	$\bar{X}$	S
6	15.5	15.3	16.1	16.9	15.95	0.72
8	16.3	15.8	16.7	16	16.20	0.39
10	15.7	16.8	15.3	16.7	16.13	0.74
					$\bar{\bar{X}}_1 = 16.09$	$\bar{S}_1 = 0.62$

For out-of-control process, the estimates of μ_1 and σ_1 are $\bar{\bar{x}}_1 = 16.09$, and $\frac{\bar{S}_1}{c_4} = 0.67$ by Table 89. So we could compute the values of $\hat{\delta}_1$ and $\hat{\delta}_2$:

$$\hat{\mu}_1 = \hat{\mu}_0 - \hat{\delta}_1 \hat{\sigma}_0 \Rightarrow \hat{\delta}_1 = \frac{\hat{\mu}_0 - \hat{\mu}_1}{\hat{\sigma}_0} = \text{-}0.74$$

$$\hat{\sigma}_1^2 = \hat{\delta}_2^2 \hat{\sigma}_0^2 \Rightarrow \hat{\delta}_2 = \sqrt{\frac{\hat{\sigma}_1^2}{\hat{\sigma}_0^2}} = 0.92$$

Hence, the ATS_1 of the FP EWMA WL chart is $ATS_1 = 7.22$.

(2) VSI EWMA-WL Chart

Use the data in Table 88 where the original samples #6, #8 and #10 were deleted to construct the VSI EWMA WL control chart. The values of $\hat{\tau}_0$, $E(\hat{WL})$, $Var(\hat{WL})$, $\hat{\delta}_1$, $\hat{\delta}_2$ and $\hat{\delta}_3$ are the same as the FP EWMA WL control chart.

Adopt ARL_0 = 370.37, $L_1 = 2.59$ and $L_2 = 2.42$ from the FP EWMA WL chart. We could obtain W_1 under W_2 = 0.5, 1, 1.5, 2 and (h_2, h_1) = (0.2, 2.0), (0.5, 1.5), (0.8, 1.2) based on the procedure in Section 5.2.

Table 90. The Values of W_1 and ATS_1 for Various W_2 and (h_2, h_1)

(h_2, h_1)	(W_1, W_2)	ATS_1
(0.2,2.0)	(0.62,0.5)	4.20
(0.5,1.5)	(0.80,0.5)	5.27
(0.8,1.2)	(0.80,0.5)	6.44
(0.2,2.0)	(0.19 ,1)	5.42
(0.5,1.5)	(0.33,1)	5.98
(0.8,1.2)	(0.33,1)	6.72
(0.2,2.0)	(-0.04 ,1.5)	7.24
(0.5,1.5)	(0.09,1.5)	7.01
(0.8,1.2)	(0.09,1.5)	7.14
(0.2,2.0)	(-0.14,2)	9.65
(0.5,1.5)	(-0.01,2)	8.37
(0.8,1.2)	(-0.01,2)	7.68

We use the combinations of (h_2, h_1) = (0.2, 2.0) and $(W_1, W_2) = (0.62, 0.5)$ to construct the chart. In Table 90, the combinations (h_2, h_1) = (0.2,2.0) and (W_1, W_2) = (0.62, 0.5) gave the minimal ATS_1 , so we use them to construct the VSI EWMA WL chart. The control limits are

$$UCL = E(\hat{WL}) + L_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 3.72,$$

$$UWL = E(\hat{WL}) + W_1\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 3.43,$$

$$LWL = E(\hat{WL}) - W_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 3.26,$$

$$LCL = E(\hat{WL}) - L_2\sqrt{\frac{\lambda}{(2-\lambda)}Var(\hat{WL})} = 2.98.$$

We plot the values of the EWMA WL on the VSI EWMA WL chart (Figure 79).

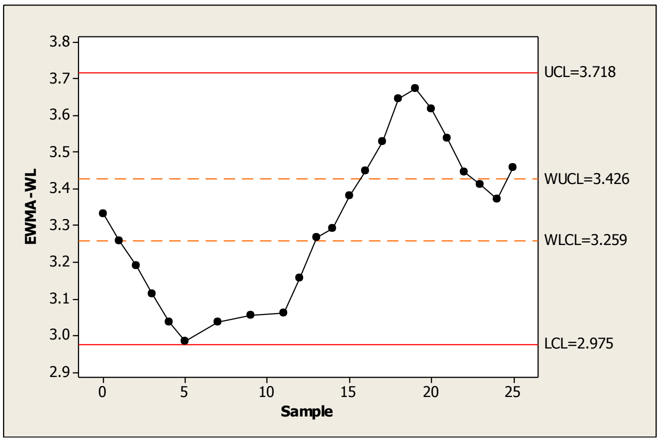

Figure 79. VSI EWMA WL Chart

From Figure 79, all samples fall inside the control limits, hence we would use the chart to monitor the mean and variance from now on. The ATS_1 of the VSI EWMA WL chart is $ATS_1 = 4.20$.

Use the data in Table 86, the FP WL, the VSI WL, the VP WL, the $\bar{Y} - S^2$ and the $EWMA\,\bar{Y} - EWMA\,\ln(S^2)$ chart are constructed. The estimates of μ_0 and σ_0 are $\bar{\bar{X}}_0 = 15.62$ and $\dfrac{\bar{S}_0}{c_4} = 0.73$.

(3) FP WL Chart

The control limits of the FP WL chart are UCL = 6.54 and LCL = 1.14. We plot the values of WL on the FP WL chart (Figure 80).

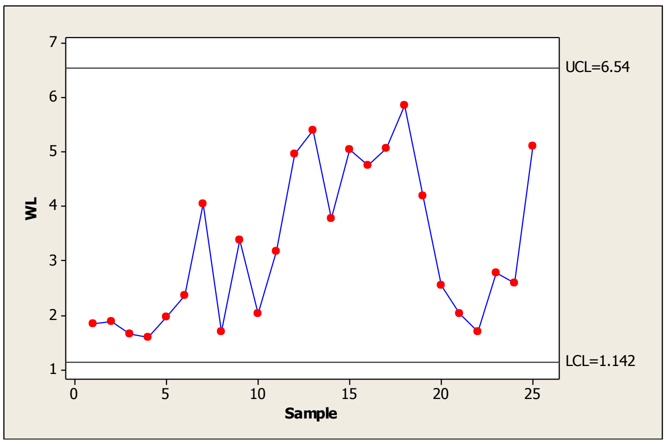

Figure 80. FP WL Chart

From Figure 80, no samples fall out of the control limits, hence we would use this chart to monitor the mean and variance from now on.

(4) Specified VSI WL Chart

We give the values of the optimal parameters, $(h_2^*, h_1^*) = (0.2, 2)$ and $p^* = 0.5$ under $h_0 = 1$, $n_0 = 4$, and $\delta_3 = 3.29$. The control limits and warning limits of the optimal VSI WL chart are UCL = 6.540, LCL = 1.14, UWL = 3.63 and LWL = 2.60. Then we plot the values of WL on the specified VSI WL chart (Figure 81).

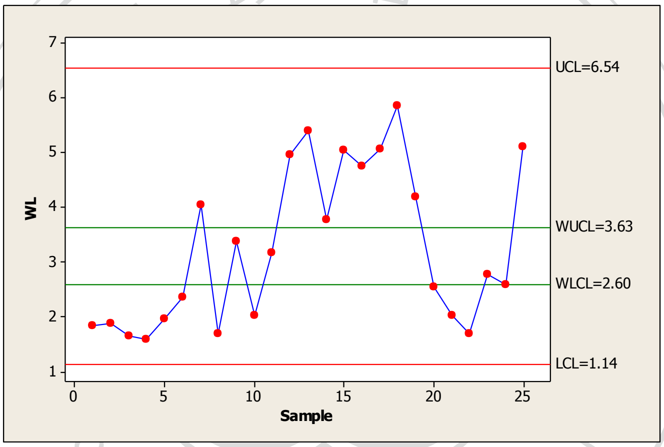

Figure 81. VSI WL Chart

From Figure 81, all samples fall inside the control limits, hence we would use the control chart to monitor the mean and variance from now on.

(5) Specified VP WL Chart

We give the values of the optimal parameters, $(h_1, n_1, \alpha_1) = (1.09, 2, 0.0027)$, $(h_2, n_2, \alpha_2) = (0.1, 25, 0.0027)$ and $p^* = 0.5$ under $h_0 = 1$, $n_0 = 4$ and $\delta_3 = 3.29$.

The control limits and warning limits of the optimal VP WL chart are UCL_1 = 8.49, LCL_1 = 0.67, UWL_1 = 5.71, LWL_1 = 1.40, UCL_2 = 4.40, LCL_2 = 2.16, UWL_2 = 3.74, LWL_2 = 2.54. Then we plot the values of WL on the specified VP WL chart (Figure 82).

And the sample points are listed in Table 91.

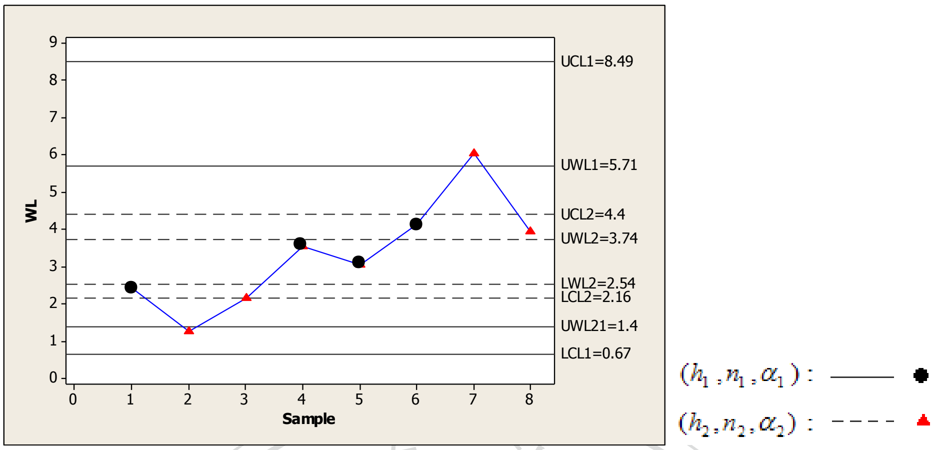

Figure 82. Optimal VP WL Chart

From Figure 82, all samples fall inside the control limits, hence we would use the control chart to monitor the mean and variance from now on.

Table 91. The Plotting Statistics for Optimal VP WL Chart

No.	(h_i, n_i)	WL	Which region	WL chart signal?
1	$(h_1, n_1, \alpha_1) = (1.09, 2, 0.0027)$	2.42	Central region	No
2	$(h_2, n_2, \alpha_2) = (0.1, 25, 0.0027)$	1.26	Warn region	No
3	$(h_2, n_2, \alpha_2) = (0.1, 25, 0.0027)$	2.17	Warn region	No
4	$(h_1, n_1, \alpha_1) = (1.09, 2, 0.0027)$	3.56	Central region	No
5	$(h_1, n_1, \alpha_1) = (1.09, 2, 0.0027)$	3.06	Central region	No
6	$(h_1, n_1, \alpha_1) = (1.09, 2, 0.0027)$	4.12	Central region	No
7	$(h_2, n_2, \alpha_2) = (0.1, 25, 0.0027)$	6.03	Warn region	No
8	$(h_2, n_2, \alpha_2) = (0.1, 25, 0.0027)$	3.94	Warn region	No

(6) $\bar{Y} - S^2$ Chart

The false alarm rate for the $\bar{Y} - S^2$ chart is set at $\alpha = 0.0027$, so we give $\alpha_{\bar{Y}} = 0.00135$ for $\bar{Y}$ chart and $\alpha_{S^2} = 0.00135$ for S^2 chart respectively. When the $\alpha_{\bar{Y}} = 0.00135$, the factor of the control limits of $\bar{Y}$ chart is 3.205.

The control limits of S^2 chart are

$$UCL_{S^2} = \frac{\hat{\sigma}_0^2}{n-1}\chi^2_{0.00135/2,n-1} = 2.00$$
$$LCL_{S^2} = \frac{\hat{\sigma}_0^2}{n-1}\chi^2_{1-0.00135/2,n-1} = 0.00$$

The control limits of $\overline{Y}$ chart are

$$UCL_{\overline{Y}} = T - \hat{\mu}_0 + 3.205\frac{\hat{\sigma}_0}{\sqrt{n}} = 3.55$$
$$CL_{\overline{Y}} = T - \hat{\mu}_0 = 2.38$$
$$LCL_{\overline{Y}} = T - \hat{\mu}_0 - 3.205\frac{\hat{\sigma}_0}{\sqrt{n}} = 1.22$$

Plot the values of S^2 and $\overline{Y}$ on the two charts (Figure 84).

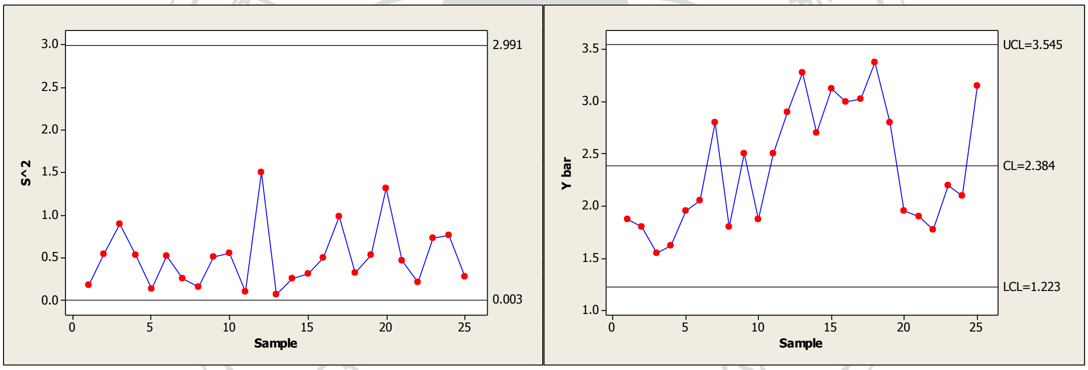

Figure 83. $\overline{Y} - S^2$ Chart

From Figure 83, all samples fall inside the control limits, hence we would use these charts to monitor the mean and variance from now on.

(7) $EWMA\,\overline{Y} - EWMA\ln(S^2)$ **Chart**

The control limits for the EWMA $\ln(S^2)$ chart are

$$UCL = \mu_2 + L_{\ln(S^2)}\sqrt{\frac{\lambda}{(2-\lambda)}}\sigma_2^2$$
$$LCL = \mu_2 - L_{\ln(S^2)}\sqrt{\frac{\lambda}{(2-\lambda)}}\sigma_2^2$$

where

$$\mu_2 \approx \ln(\sigma_0^2) - \frac{1}{n-1} - \frac{1}{3(n-1)^2} + \frac{1}{15(n-1)^5}$$

$$\sigma_2^2 \approx \frac{2}{n-1} + \frac{2}{(n-1)^2} + \frac{4}{3(n-1)^3} - \frac{16}{15(n-1)^5}$$

The EWMA plotting statistics of $\ln(S^2)$ are

$$R_i = \lambda_{S^2} \ln(S_i^2) + (1-\lambda_{S^2}) R_{i-1} \text{ , } i = 1,2,3\ldots \text{ with } R_0 = \mu_2 .$$

The control limits for EWMA $\bar{Y}$ chart is

$$UCL = T - \hat{\mu}_0 + L_{\bar{Y}} \sqrt{\frac{\lambda}{(2-\lambda)} \frac{\hat{\sigma}_0^2}{n}}$$

$$UCL = T - \hat{\mu}_0 - L_{\bar{Y}} \sqrt{\frac{\lambda}{(2-\lambda)} \frac{\hat{\sigma}_0^2}{n}}$$

The EWMA plotting statistics of $\bar{Y}$ are

$$Q_i = \lambda_Y \bar{Y}_i + (1-\lambda_Y) Q_{i-1} \text{ , } i = 1,2,3\ldots \text{with } Q_0 = T - \hat{\mu}_0 .$$

We could use the Markov chain (Lucas and Saccucci (1990)) approach to calculate the ARL for the EWMA chart. Under $ARL_0 = 740.74$ and $\lambda = 0.05$, using Zero(s) in Fortran IMSL subroutine to obtain $L_{\ln S^2}$ and L_Y. So $L_{\ln S^2}$ = 2.78 and L_Y =2.77. The control limits of these two charts are

EWMA $\ln S^2$ chart	EWMA $\bar{Y}$ chart
UCL = -0.58	UCL = 2.55
LCL = -1.44	LCL = 2.22

Plot the values of EWMA $\ln S^2$ and EWMA $\bar{Y}$ on these two charts (Figure 84).

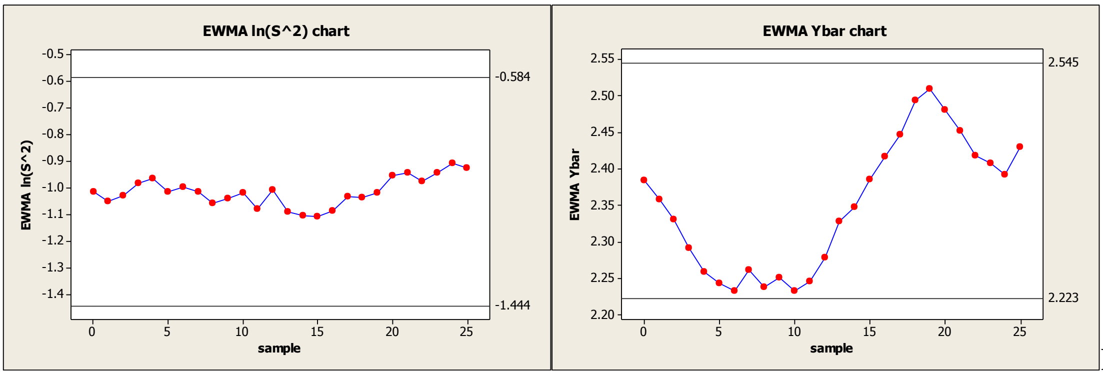

Figure 84. $EWMA\,\bar{Y} - EWMA\,\ln(S^2)$ Chart

From Figure 84, all samples fall inside the control limits, hence we would use these charts to monitor the mean and variance from now on.

From the above results, the FP EWMA WL and the VSI EWMA WL charts have three out-of-control samples but the other charts have none. It indicates that the FP EWMA WL and the VSI EWMA WL charts are more effective than other charts. However, we are concerned with the mean shift and/or the variance shift which is the cause of the three signals. So we could look into the statistic WL. Since WL combines the statistic S^2 and $(\bar{X}-T)^2$, we could compute the deviations of S^2 from $\hat{\sigma}_0^2$ and $(\bar{X}-T)^2$ form $(\hat{\mu}_0 - T)^2$. Then the bigger one of the two deviations can determine the main cause of the signal.

CHAPTER 7. CONCLUSION AND FUTURE RESEARCH

The traditional approach to monitor the mean and the variance of a process is using two control charts. But it is laborious and time-consuming. In this project, we proposed a single chart for monitoring the mean and the variance simultaneously. The main advantage of this chart is that the users could control a process by looking at only one chart.

We proposed the FP, VSI, VSSI, VP, optimal FP, optimal VSI, optimal VSSI optimal VP, FP EWMA and VSI EWMA WL charts to monitor the mean and variance simultaneously when the in-control mean may not equal to the target of a process. Comparing to the $\bar{X}-S$ chart, the WL chart is not only more powerful but also simpler to design and implement. One advantage of the WL chart is that it could vary the weight *a* to change the weights between the statistics S^2 and $(\bar{X}-T)^2$ such that the performance is more effective under various shifts of the mean and the variance. However, from the performance comparison, the adaptive WL charts have smaller ATS_1 than the FP WL chart and the optimal WL charts outperform the specified WL charts. Especially the optimal WL charts with the optimal weight *a*, their ATS_1 (ARL_1) are smaller than the ATS_1 (ARL_1) of the specified WL charts when the shifts of mean and variance are small. We have shown that the weight *a* is an important element which could affect the performance of the WL charts. Furthermore, the design is facilitated by the design table provided in this project. The users could find the proper weight *a* and control limits from table according to the specified δ_1, δ_2 and δ_3.

Comparing with the FP WL chart, we found the optimal VSSI and the optimal VP WL charts with the optimal *a* could save more ATS_1. Most of the ATS_1 saved % of the two charts are bigger than 90%. And when the shifts of the mean and the variance are small, they also have large ANOS saved % than the other charts. About the specified FP EWMA and VSI EWMA WL charts, they have great performance when λ is small for small mean and variance shifts. The $ATS_1(ARL_1)$ saved % of the two EWMA charts is larger than the $ATS_1(ARL_1)$ saved % of the specified adaptive WL charts and the optimal adaptive WL charts with the optimal *a* under small λ when the mean and the variance shifts are small. As a result, the optimal adaptive WL charts with the optimal weight *a* have the best performance among these charts proposed in the project.

The design of the WL charts could be easily found. So it makes the WL chart a better choice for the further development of more advanced charts, for example, to construct a chart by CUSUM scheme. And we could also construct the VSSI EWMA or the VP EWMA WL charts. The effectiveness of these charts may be sensitive to small shift of a process.

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APPENDIX:

Calculation of the Cumulative Probability Function (c.d.f.) of a Linear Combination of the Central Chi-square Random Variables

Moschopoulos and Canada (1984) proposed the method of the calculation of the approximate distribution of a finite linear combination of independent central chi-square random variables. Let $c = (c_1, \ldots\ldots, c_p)'$ be a vector of real non-zero constants and $n = (n_1, \ldots\ldots, n_p)'$ a vector of integers where $n_i \geq 1$, I = 1, • • •, p. Then the linear combination of central chi-square random variables is

$$Q(c,n) = \sum_{i=1}^{p} c_i \chi^2(n_i)$$

where $\chi^2(n_i), i = 1, \ldots, p$ are independent chi-square random variables with n_i degrees of freedom. Let

$$b_i = (c_1 / c_i)^{m_i} \quad \text{and} \quad A(c_i, r) = \frac{(m_i)_r (1 - c_1 / c_i)^r}{r!}$$

where $m_i = n_i / 2$ and $(m_i)_r = m_i(m_i + 1) \cdots\cdots (m_i + r - 1)$.

The approximate c.d.f. of Q could be found by

$$F(w) = P(Q \leq w) = (\prod_{i=2}^{p} b_i) \sum_{j=0}^{\infty} a_j \int_0^w g_j(y) dy \qquad (1)$$

where

$40<k<100$, $a_j = A_j^{(p)}, A_j^{(i)} = \sum_{k=0}^{j} A_k^{(i-1)} A(c_i, j-k)$, $i = 3,4,\ldots,p$, $j = 0,1,2,\ldots,$

$r = 0,1,2,\ldots,$ $A_r^{(2)} = A(c_2, r)$ and $g_j(y) = \dfrac{y^{s+j-1} e^{-y/2c_1}}{(2c_1)^{s+j} \Gamma(s+j)}$ is a p.d.f. of Gamma(s+j, $2c_1$) where $s = \sum_{i=1}^{p} m_i$.