國立政治大學統計學系

碩士學位論文

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最大利潤下規格上限與 EWMA 管制圖之設計

Design of Upper Specification and EWMA Control Chart with Maximal Profit

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中華民國一百零一年六月

謝辭

研究生生活即將結束，這兩年我過得很充實，學到了許多寶貴的知識以及經驗，讓我成長不少，這一切必須感謝很多人這些日子以來的幫助與支持。

首先最要感謝的是我的指導老師 楊素芬教授。謝謝楊老師這兩年對我的辛苦栽培，讓我學習到許多統計品管上的知識，更重要的是了解做人處世的道理。老師嚴格的指導，讓我能更加積極地去面對任何事情。也謝謝老師對我碩士論文的協助，讓英文不優的我，能順利完成此篇論文。

接著要感謝我的口試委員，曾勝滄教授與 葉小蓁教授；感謝兩位教授在百忙之中抽出時間閱讀論文以及前來政大擔任口試委員。更要謝謝兩位教授對論文提出寶貴的建議與想法，讓我能加以改進，使本篇論文更加完善。也要謝謝兩位教授在口試後所給予的勉勵與祝福。

碩士兩年期間，要感謝我的同學們為我打氣以及給予我的幫助。有大家的陪伴，讓我能度過許多艱辛的日子。有大家一起玩樂，也讓我更有動力去完成許多事情。特別要感謝瑋倫和依潔，謝謝你們在研究上給予我的協助，有幸能與你們討論，讓我解決許多困難的問題。

最後一定要感謝的是我的家人，謝謝你們給我的鼓勵與支持，更要謝謝你們讓我能無後顧之憂的專心處理課業上的事情。你們永遠是我最大的支柱，往後的日子我也將更加努力，並且希望能對這個家有所貢獻。

本研究承蒙行政院國家科學委員會，計畫編號NSC100-2118-M-004-003-MY2補助，謹此致謝。

此項成果使用國家高速網路與計算中心之計算資源，特此感謝。

蔡佳宏 謹致

中華民國一百零一年六月

ABSTRACT

The determination of economic control charts and the determination of specification limits with minimum cost are two different research topics. In this study, we first combine the design of economic control charts and the determination of specification limits to maximize the expected profit per unit time for the smaller the better quality variable following the gamma distribution. Because of the asymmetric distribution, we design the EWMA control chart with asymmetric control limits. We simultaneously determine the economic EWMA control chart and upper specification limit with maximum expected profit per unit time. Then, extend the approach to determine the economic variable sampling interval EWMA control chart and upper specification limit with maximum expected profit per unit time.

In all our numerical examples of the two profit models, the optimum expected profit per unit time under inspection is higher than that of no inspection. The detection ability of the EWMA chart with an appropriate weight is always better than the X-bar probability chart. The detection ability of the VSI EWMA chart is also superior to that of the fixed sampling interval EWMA chart. Sensitivity analyses are provided to determine the significant parameters for the optimal design parameters and the optimal expected profit per unit time.

Keywords: Economic design; Specification; EWMA control chart; VSI control chart; Markov chain; Gamma distribution; Optimization technique

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1. INTRODUCTION

1.1 Research Motivation

Control charts are widely used in statistical process control, and their design parameters must be pre-determined for their use, such as sample size, sampling time, and control limits. Economic design of control charts have been widely used to determine these parameters from economic viewpoint. However, to reduce product quality loss, the 100% inspection with specification limits is always performed. To determine the specification limits, a widely use method is to minimize products cost. In the technology industry, like the product's insulation property, its loss of heat is the smaller the better. In the service industry, the service time is also the smaller the better. How to determine their specification limits and control chart to monitor the quality variable with maximum profit is an important issue. In this project, we simultaneously determine the specification limits and the control chart parameters with maximize expected profit per unit time. Most articles on economic design of specification and control charts consider normal distribution of quality variable, but in this article, we consider gamma distribution for the smaller the better quality variable; hence, we determine only an upper specification limit in the gamma distribution. For wide use of different shift scales and because of the asymmetric gamma distribution, we design the asymmetric economic $EWMA_{X\text{-bar}}$ control chart and the asymmetric economic VSI $EWMA_{X\text{-bar}}$ control chart. Hence, we combine product cost and control chart cost in a profit model and then we maximize this profit per unit time to determine the optimum upper specification limit and the design parameters of the $EWMA_{X\text{-bar}}$ control chart and the VSI $EWMA_{X\text{-bar}}$ control chart.

1.2 Literature Review

Economic design of control charts have been widely used to determine control chart parameters from economic viewpoint. Duncan (1956) first proposed the concept of an economic design for the X-bar control chart. He considered a process that does not shut down when the assignable cause is searched, and developed a process cost model that includes the cost of sampling and finding the assignable cause when it exists or when none exists. He also demonstrated how to determine control chart parameters. Montgomery (1980) presented a review and literature survey in the economic design of control charts. Panagos, Heikes, and Montgomery (1985) described two continuous and discontinuous manufacturing process models, where the continuous process model is consistent with that developed by Duncan. They showed that the wrong choice of a process model would have a potentially serious economic result.

Control charts typically take a fixed number of samples with a fixed sampling time and plot them on the control chart with a fixed control limit. To improve control chart performance, the adaptive control chart has been developed such as variable sampling interval control chart. Reynolds et al. (1988) proposed the VSI X-bar control chart. Bai and Lee (1998) considered the economic design of the VSI X-bar control chart. They preferred to use only two sampling interval lengths with two sub-regions between the two control limits.

The EWMA chart is widely used for detecting small process shifts. Roberts (1959) first introduced the exponentially weighted moving average chart. Crowder (1987) and Saccucci and Lucas (1990) discussed the *ARL* calculation of the EWMA control chart. Montgomery et al. (1995) presented a statistically constrained economic design of the EWMA control chart. They minimized the cost model, subject to statistical constraints on average run length or average time to signal, to determine the design parameters of the EWMA control chart. Chou et al. (2006) proposed an economic design of VSI EWMA charts. They considered two sampling interval lengths and derived the cost model to determine the parameters of VSI EWMA control charts using the genetic algorithm.

To reduce product quality loss, the 100% inspection with specification limits is necessary. Kapur (1988) considered three types of quality characteristics; the smaller the better, the larger the better, and the nominal the best and used three types of loss function to evaluate the loss of three types of quality characteristics with normal distribution. Phillips and Cho (1998a) used the truncated quadratic loss function on the smaller the better quality characteristic, which follows gamma distribution. Phillips and Cho (1998b) used linear empirical loss function and quadratic empirical loss function for the quality variable, which follows normal distribution. Feng and Kapur (2006) considered asymmetric quadratic loss function and asymmetric piecewise linear loss function for the quality variable with normal distribution. These four articles minimize the expected cost per product to determine the specification limits. Hong et al. (2006) considered the larger the better quality characteristics of normal distribution. They used two types of profit models, unconformable items that are reprocessed and unconformable items that are sold at a discount price, to maximize the profit model to determine the optimum process mean and specification limit. Hong and Cho (2007) considered several available markets with different price structures. They derived the model of expected profit per item and maximized it to determine the process mean and tolerance. They also investigated the effects of measurement errors on the process mean and tolerance.

1.3 Research Method

This study simultaneously determines the upper specification limit and the design parameters of $EWMA_{X\text{-bar}}$ control chart with maximal profit. In Chapter 2, we consider the smaller the better quality variable with in-control and out-of-control gamma distributions. To measure the performance of the proposed $EWMA_{X\text{-bar}}$ chart, we let in-control average run length (ARL_0) equal to 370 by using the Markov chain approach and then find the best reference value of λ, which is a weight of $EWMA_{X\text{-bar}}$ statistic, and factors of control limits of $EWMA_{X\text{-bar}}$ chart to minimizing out-of-control average run length (ARL_1). In Chapter 3, we derive the profit model per unit time with only $EWMA_{X\text{-bar}}$ chart but without producer tolerance. We then give an example to determine the optimum design parameters of the $EWMA_{X\text{-bar}}$ chart by maximizing the expected profit per unit time and present a sensitivity analysis to find the significant parameters. We also compare the performance of $EWMA_{X\text{-bar}}$ chart with λ=1 which is equivalent to the X-bar probability chart. In Chapter 4, we derive the profit model per unit time with $EWMA_{X\text{-bar}}$ chart and producer tolerance. We give an example to determine the optimum upper specification limit and the design parameters of the $EWMA_{X\text{-bar}}$ chart by maximizing the expected profit per unit time and compare the performance to $EWMA_{X\text{-bar}}$ chart without producer tolerance. Finally, we present a sensitivity analysis and compare the performance to $EWMA_{X\text{-bar}}$ chart with λ=1. In Chapter 5, we determine the best λ in the $EWMA_{X\text{-bar}}$ chart under six different shift scales by maximizing the expected profit per unit time and conclude the better λ for different shift scales in the mean and variance. In Chapter 6, we consider the VSI $EWMA_{X\text{-bar}}$ chart and calculate average time to signal (*ATS*) to measure the performance of this chart. We also derive the profit model and then give two examples to determine the upper specification limit and the design parameters of the VSI $EWMA_{X\text{-bar}}$ chart by maximizing the expected profit per unit time and conducting sensitivity analysis. We also compare the performance with the FSI $EWMA_{X\text{-bar}}$ chart. Finally, we summarize the results in Chapter 7. In this study, we use the R program to perform all calculations, including using the “uniroot” command to solve the one-dimensional root and using the “DEoptim” command to find the global optimum value of the expected profit per unit time using the differential evolution algorithm. We also use the R program to plot all of the figures.

CHAPTER 2. THE SMALLER THE BETTER QUALITY VARIABLE WITH GAMMA DISTRIBUTION

2.1 In-control Sampling Distribution of X-bar under Gamma Distribution

Construct the $EWMA_{X\text{-bar}}$ control chart based on the sample mean, in this section, we need to derive the X-bar distribution.

We take the in-control gamma distribution as follows:

$$X_I \sim \Gamma(a_I, b_I),\ E(X_I) = a_I b_I,\ Var(X_I) = a_I b_I^2$$

$$f_I(x) = \frac{1}{\Gamma(a_I)(b_I)^{a_I}}\, x^{(a_I-1)}\, e^{\frac{-x}{b_I}},\ 0 < x < \infty\ \ a_I, b_I > 0 \qquad (2\text{-}1)$$

We choose sample size n, s.t. $\bar{X}_I = \frac{1}{n}\sum_{i=1}^{n} X_{I,i}$, where $X_{I,i} \overset{iid}{\sim} \Gamma(a_I, b_I),\ i = 1,2,\ldots,n$, and then obtain the distribution of $\bar{X}_I$ as follows:

$$\bar{X}_I = \frac{1}{n}\sum_{i=1}^{n} X_{I,i} \sim \frac{1}{n}\sum_{i=1}^{n}\Gamma(a_I, b_I) \equiv \sum_{i=1}^{n}\Gamma\left(a_I, \frac{b_I}{n}\right) \equiv \Gamma\left(na_I, \frac{b_I}{n}\right)$$

and $E(\bar{X}_I) = a_I b_I,\ Var(\bar{X}_I) = \frac{a_I b_I^2}{n}$.

2.2 Out-of-control Sampling Distribution of X-bar under Gamma Distribution

To choose out-of-control distribution, we first compare different gamma distributions.

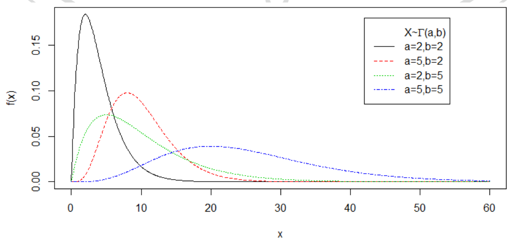

Figure 2-1. The p.d.f Comparison of Different Gamma Distributions

According to Figure 2-1, if *a* or *b* increases, the p.d.f. shifts right and both mean and variance of the distribution increase. Because the considered quality variable is the smaller the better, we assume that both *a* and *b* of the out-of-control distribution are bigger than those of the in-control distribution.

We assume the out-of-control gamma distribution as follows:

$$a_O = a_I + \delta_1,\ b_O = b_I + \delta_2,\ \ \delta_1, \delta_2 > 0$$

$$X_O \sim \Gamma(a_O, b_O),\ E(X_O) = a_O b_O,\ Var(X_O) = a_O b_O^2$$

$$f_O(x) = \frac{1}{\Gamma(a_O)(b_O)^{a_O}}\, x^{(a_O\text{-}1)}\, e^{\frac{-x}{b_O}},\ 0 < x < \infty\ \ a_O, b_O > 0 \qquad (2\text{-}2)$$

We choose sample size n, s.t. $\bar{X}_O = \frac{1}{n}\sum_{i=1}^{n} X_{O,i}$, where $X_{O,i} \overset{iid}{\sim} \Gamma(a_O, b_O)$, $\ i = 1,2,\ldots,n$

and then obtain $\bar{X}_O \sim \Gamma(na_O, \frac{b_O}{n}), E(\bar{X}_O) = a_O b_O\ , Var(\bar{X}_I) = \frac{a_O b_O^2}{n}$.

Because *a* and *b* both exist in the mean and variance, when the mean changes, the variance also changes. Hence, the EWMA$_{\text{X-bar}}$ control chart detects both mean and variance.

2.3 Construction of the EWMA$_{\text{X-bar}}$ Control Chart Based on X-bar Sampling Distribution

The EWMA$_{\text{X-bar}}$ statistic is based on the X-bar sampling distribution. It is expressed as

$$Z_t = \lambda \bar{X}_t + (1-\lambda) Z_{t-1}\ ,\ t = 1,2,\ldots\ ,\ 0 < \lambda \le 1$$

where $\bar{X}_t = \frac{1}{n}\sum_{i=1}^{n} X_i$ (at time *t*), λ is a weight of the EWMA$_{\text{X-bar}}$ statistic. Let $Z_0 = E(\bar{X}_t) = a_I b_I$, then $E(Z_t) = a_I b_I$ and $Var(Z_t) \cong \frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}$, as t→∞, when $\bar{X}_t$ is the in-control distribution.

As t→∞, the approximate control limits of the EWMA$_{\text{X-bar}}$ control chart is

$$UCL = E(Z_t) + L_1\sqrt{Var(Z_t)} = a_I b_I + L_1\sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

$$CL = E(Z_t) = a_I b_I$$

$$LCL = E(Z_t) - L_2\sqrt{Var(Z_t)} = a_I b_I - L_2\sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

where *UCL* is upper control limit, *LCL* is lower control limit, *CL* is central limit, L_1 is the coefficient of *UCL* and L_2 is the coefficient of *LCL*.

2.4 Calculation of Average Run Length for the EWMA$_{\text{X-bar}}$ Chart

We used the Markov chain to calculate *ARL*, referring to Saccucci and Lucas (1990).

The procedure is as follows:

Step1. Divide the interval between the upper and lower control limits into g=2m+1, the number of states, sub-intervals of width 2δ, where $\delta = \frac{UCL - LCL}{2g}$.

Step2. Define state j=($S_j - \delta$, $S_j + \delta$), j=-m,…-1,0,1,…,m, and S_j as the midpoint for the j-th interval.

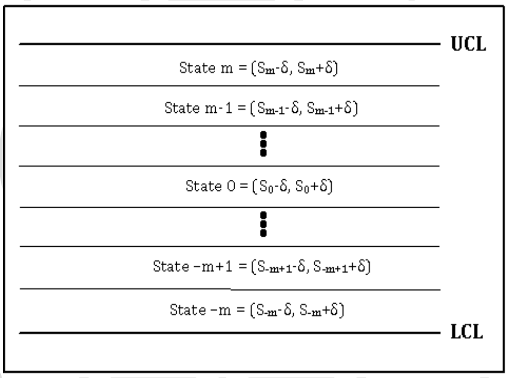

Figure 2-2. States between the Control Limits of EWMA$_{\text{X-bar}}$ Chart

Step3. The statistic $Z_{t,j}$ is in transient state j at time t, if

$$S_j - \delta < Z_{t,j} \le S_j + \delta \ \ for \ -m \le j \le m$$

Step4. The transition probability matrix for the transient state is

$$R = [p_{t-1,t}(jk)], \quad j, k=\text{-m,…, -1, 0, 1,…, m}$$

$$\begin{aligned}
\text{where } p_{t-1,t}(jk) &= P(S_k - \delta < Z_{t,k} \le S_k + \delta \mid S_j - \delta < Z_{t-1,j} \le S_j + \delta) \\
&= P(S_k - \delta < Z_{t,k} \le S_k + \delta \mid Z_{t-1,j} = S_j) \\
&= P(S_k - \delta < \lambda \overline{X}_{t,k} + (1-\lambda) Z_{t-1,k} \le S_k + \delta \mid Z_{t-1,j} = S_j) \\
&= P\left(\frac{(S_k - \delta) - (1-\lambda) S_j}{\lambda} < \overline{X}_t \le \frac{(S_k + \delta) - (1-\lambda) S_j}{\lambda}\right)
\end{aligned}$$

Step5. Assume that the process begins from state 0; thus, $p_{zs} = (\overbrace{0,0,...,0}^{m},1,\overbrace{0,...,0,0}^{m})$.

Step6.

(1) Calculate zero-state ARL_0

$$ARL_0 = p_{zs}^T (I - R_I)^{-1} \vec{1} \qquad (2\text{-}3)$$

where R_I is the transition probability matrix calculated by in-control gamma distribution, I is the $g*g$ dimension identity matrix and $\vec{1}$ is the $g*1$ dimension vector with all components are 1.

(2) Calculate zero-state ARL_1

$$ARL_1 = p_{zs}^T (I - R_O)^{-1} \vec{1} \qquad (2\text{-}4)$$

where R_O is the transition probability matrix calculated by out-of-control gamma distribution, I is the $g*g$ dimension identity matrix and $\vec{1}$ is the $g*1$ dimension vector with all components are 1.

2.5 Determining Control Limit Coefficient on EWMA$_{\text{X-bar}}$ Control Chart under Different *n* and *λ*

We determine the control limit coefficient by using the following step:

Step1. Determine the *UCL* coefficient (L_1) of the EWMA$_{\text{X-bar}}$ control chart. With n, a_I, b_I, and λ, let LCL=0 and ARL_0=740 to solve L_1 using the routine "uniroot" in the R program. Hence, *UCL* is determined.

Step2. Determine the *LCL* coefficient (L_2) of the EWMA$_{\text{X-bar}}$ control chart. With *UCL*, let ARL_0=370 to solve L_2 by using the routine "uniroot" in the R program. Hence, the economic EWMA$_{\text{X-bar}}$ control chart is constructed.

We then obtain a combination (λ, L_1, L_2) for given n, a_I, b_I, and λ with ARL_0=370.

Table 2-1. The solved L_1 and L_2 under various combinations of λ and n for a_I=1.5, b_I=2, ARL_0=370 and g=301

λ \ n (L_1,L_2)	2		3		4		5		6		7		8		9		10	
0.05	2.666	2.300	2.623	2.346	2.597	2.373	2.580	2.392	2.567	2.406	2.557	2.417	2.549	2.425	2.542	2.433	2.537	2.439
0.1	3.075	2.339	3.001	2.407	2.957	2.449	2.927	2.477	2.905	2.498	2.888	2.515	2.874	2.528	2.863	2.539	2.853	2.549
0.2	3.506	2.280	3.383	2.379	3.310	2.441	3.260	2.484	3.224	2.516	3.195	2.541	3.172	2.561	3.153	2.578	3.138	2.592
0.3	3.785	2.185	3.622	2.308	3.526	2.384	3.460	2.438	3.411	2.478	3.374	2.509	3.344	2.535	3.319	2.557	3.298	2.575
0.4	3.996	2.087	3.801	2.228	3.685	2.316	3.606	2.378	3.548	2.425	3.503	2.462	3.466	2.493	3.436	2.518	3.411	2.539
0.5	4.165	1.991	3.943	2.147	3.811	2.245	3.721	2.315	3.654	2.368	3.603	2.410	3.561	2.444	3.527	2.473	3.498	2.497
0.6	4.300	1.899	4.057	2.067	3.912	2.175	3.813	2.252	3.739	2.310	3.682	2.356	3.637	2.394	3.599	2.426	3.567	2.453
0.7	4.404	1.811	4.146	1.991	3.990	2.108	3.884	2.191	3.806	2.254	3.745	2.304	3.696	2.346	3.655	2.380	3.620	2.410
0.8	4.480	1.728	4.210	1.920	4.048	2.045	3.937	2.135	3.855	2.203	3.791	2.257	3.739	2.302	3.696	2.339	3.660	2.371
0.9	4.527	1.653	4.250	1.859	4.084	1.993	3.969	2.090	3.885	2.163	3.819	2.221	3.766	2.268	3.722	2.308	3.685	2.342

We plot L_1 or L_2 at various n, which is shown in Figs. 2-3 and 2-4, respectively. The value of L_1 decrease and L_2 almost increase as n increase or λ decrease.

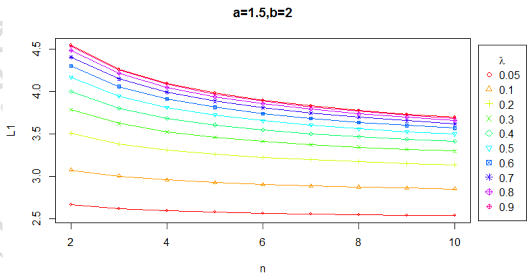

Figure 2-3. The Value of L_1 under Various n at ARL_0=370, a_I =1.5, b_I =2 and g=301

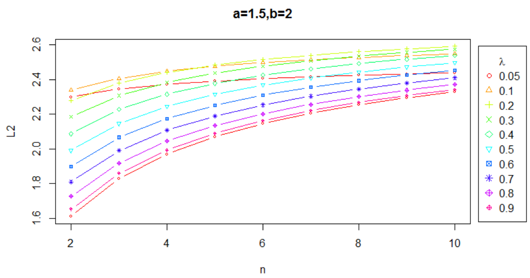

Figure 2-4. The Value of L_2 under Various n at ARL_0=370, a_I =1.5, b_I =2 and g=301

Table 2-2. The solved L_1 and L_2 under various combinations of λ and n for a_I=24.349, b_I=0.205, ARL_0=370 and g=301

n / (L_1,L_2) / λ	2		3		4		5		6		7		8		9		10	
0.05	2.496	2.483	2.491	2.488	2.491	2.488	2.494	2.486	2.497	2.482	2.502	2.478	2.507	2.473	2.512	2.468	2.518	2.462
0.1	2.775	2.626	2.758	2.643	2.749	2.653	2.743	2.659	2.739	2.662	2.737	2.665	2.735	2.667	2.734	2.668	2.734	2.669
0.2	3.007	2.713	2.978	2.741	2.961	2.758	2.949	2.770	2.941	2.778	2.934	2.784	2.929	2.789	2.925	2.794	2.921	2.797
0.3	3.126	2.729	3.088	2.766	3.065	2.787	3.049	2.802	3.038	2.813	3.029	2.822	3.022	2.829	3.016	2.834	3.011	2.839
0.4	3.205	2.722	3.158	2.765	3.130	2.791	3.112	2.809	3.098	2.822	3.087	2.833	3.078	2.841	3.071	2.848	3.065	2.854
0.5	3.262	2.705	3.208	2.755	3.176	2.784	3.155	2.805	3.139	2.820	3.127	2.832	3.117	2.841	3.108	2.849	3.101	2.856
0.6	3.305	2.684	3.246	2.739	3.211	2.772	3.187	2.795	3.169	2.812	3.155	2.825	3.144	2.835	3.135	2.844	3.127	2.852
0.7	3.338	2.663	3.274	2.722	3.236	2.758	3.210	2.783	3.191	2.801	3.176	2.816	3.164	2.827	3.154	2.837	3.146	2.845
0.8	3.363	2.643	3.295	2.707	3.255	2.746	3.227	2.772	3.207	2.791	3.191	2.807	3.179	2.819	3.168	2.829	3.159	2.838
0.9	3.378	2.628	3.308	2.695	3.266	2.736	3.238	2.763	3.217	2.784	3.201	2.800	3.188	2.813	3.177	2.823	3.168	2.832

We plot L_1 or L_2 at various n, which is shown in Figs. 2-5 and 2-6, respectively. The value of L_1 decrease and L_2 almost increase as n increase or λ decrease.

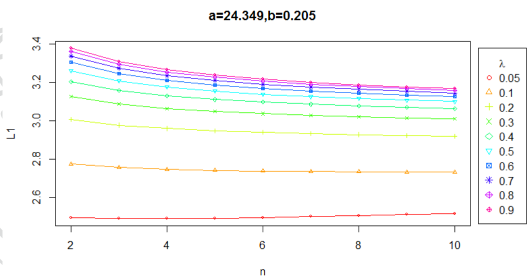

Figure 2-5. The Value of L_1 under Various n at ARL_0=370, a_I=24.349, b_I=0.205 and g=301

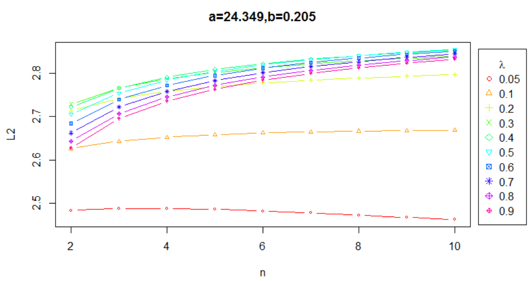

Figure 2-6. The Value of L_2 under Various n at ARL_0=370, a_I=24.349, b_I=0.205 and g=301

Table 2-3. The solved L_1 and L_2 under various combinations of λ and n for a_I=1, b_I=0.202, ARL_0=370 and g=301

n / (L_1,L_2) / λ	2		3		4		5		6		7		8		9		10	
0.05	2.718	2.246	2.666	2.300	2.635	2.333	2.613	2.356	2.597	2.373	2.585	2.386	2.575	2.397	2.567	2.406	2.560	2.413
0.1	3.165	2.259	3.075	2.339	3.021	2.389	2.985	2.423	2.957	2.449	2.936	2.468	2.919	2.485	2.905	2.498	2.893	2.510
0.2	3.657	2.163	3.506	2.280	3.416	2.352	3.355	2.403	3.310	2.441	3.275	2.471	3.247	2.495	3.224	2.516	3.204	2.533
0.3	3.984	2.044	3.785	2.185	3.666	2.274	3.585	2.337	3.526	2.384	3.479	2.422	3.442	2.452	3.411	2.478	3.385	2.500
0.4	4.234	1.928	3.996	2.087	3.854	2.189	3.757	2.261	3.685	2.316	3.630	2.360	3.585	2.395	3.548	2.425	3.517	2.451
0.5	4.434	1.818	4.165	1.991	4.003	2.104	3.893	2.184	3.811	2.245	3.748	2.294	3.697	2.334	3.654	2.368	3.619	2.397
0.6	4.594	1.714	4.300	1.899	4.123	2.020	4.001	2.108	3.912	2.175	3.842	2.229	3.786	2.273	3.739	2.310	3.700	2.342
0.7	4.717	1.616	4.404	1.811	4.216	1.941	4.086	2.035	3.990	2.108	3.916	2.166	3.856	2.214	3.806	2.254	3.764	2.289
0.8	4.806	1.523	4.480	1.728	4.283	1.866	4.148	1.967	4.048	2.045	3.970	2.108	3.907	2.159	3.855	2.203	3.810	2.241
0.9	4.860	1.437	4.527	1.653	4.325	1.801	4.186	1.909	4.084	1.993	4.003	2.061	3.939	2.116	3.885	2.163	3.839	2.203

We plot L_1 or L_2 at various n, which is shown in Figs. 2-7 and 2-8, respectively. The value of L_1 decrease and L_2 almost increase as n increase or λ decrease.

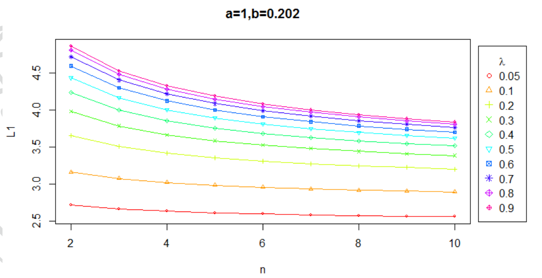

Figure 2-7. The Value of L_1 under Various n at ARL_0=370, a_I=1, b_I=0.202 and g=301

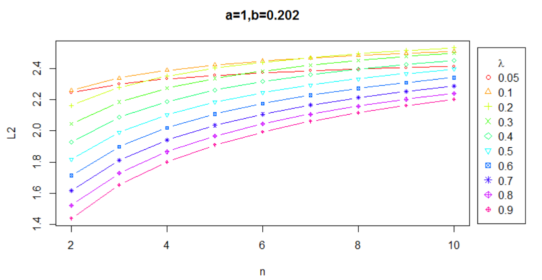

Figure 2-8. The Value of L_2 under Various n at ARL_0=370, a_I=1, b_I=0.202 and g=301

2.6 Determining the Best λ in the EWMA$_{X\text{-}bar}$ Chart under Different δ_1, δ_2 and n

We use L_1 and L_2 in Table 2-1, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-4 at a_I=1.5, b_I=2, δ_1=0.1, δ_2=0.05, and g=301.

Table 2-4. The Value of ARL_1 under Various n and λ at a_I=1.5, b_I=2, δ_1=0.1, δ_2=0.05, g=301 and ARL_0=370

λ \ n (ARL_1)	2	3	4	5	6	7	8	9	10
0.05	**134.33**	**102.74**	**83.83**	**71.23**	**62.22**	**55.46**	**50.18**	**45.96**	**42.49**
0.1	173.51	133.99	108.98	91.77	79.25	69.75	62.31	56.33	51.43
0.2	228.61	184.38	153.69	131.23	114.14	100.73	89.95	81.12	73.76
0.3	265.27	222.07	190.04	165.42	145.97	130.24	117.28	106.44	97.25
0.4	291.10	250.95	219.65	194.64	174.24	157.33	143.07	130.93	120.47
0.5	309.69	273.30	243.80	219.45	199.06	181.78	166.92	154.06	142.82
0.6	323.11	290.50	263.27	240.26	220.54	203.48	188.59	175.50	163.88
0.7	332.53	303.46	278.69	257.32	238.71	222.34	207.83	194.89	183.28
0.8	338.65	312.69	290.33	270.79	253.54	238.16	224.36	211.91	200.61
0.9	341.63	318.19	298.07	280.41	264.68	250.56	237.74	226.05	215.36

According to Table 2-4, to minimize ARL_1, 0.05 is the best λ for n=2,…, 10.

We use L_1 and L_2 in Table 2-2, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-5 at a_I =24.349, b_I =0.205, δ_1=0.919, δ_2=0.06, and g=301.

Table 2-5. The Value of ARL_1 under Various n and λ at a_I =24.349, b_I =0.205, δ_1=0.919, δ_2=0.06, g=301 and ARL_0=370

λ \ n (ARL_1)	2	3	4	5	6	7	8	9	10
0.05	4.23	3.47	3.03	2.74	2.52	2.35	2.23	2.13	2.06
0.1	3.56	2.89	2.52	2.28	2.12	2.00	1.91	1.83	1.76
0.2	3.07	2.45	2.12	1.91	1.75	1.63	1.52	1.43	1.34
0.3	2.89	2.25	1.92	1.70	1.54	1.42	1.32	1.24	1.18
0.4	**2.85**	2.15	1.80	1.58	1.43	1.31	1.23	1.16	1.11
0.5	2.89	**2.11**	1.73	1.51	1.36	1.25	1.17	1.12	1.08
0.6	3.01	2.12	**1.70**	1.47	1.32	1.21	1.14	1.10	1.06
0.7	3.23	2.17	1.71	**1.45**	1.30	1.20	1.13	1.08	**1.05**
0.8	3.54	2.27	1.74	1.46	**1.29**	**1.19**	**1.12**	1.08	**1.05**
0.9	3.97	2.44	1.81	1.49	1.30	**1.19**	**1.12**	**1.07**	**1.05**

According to Table 2-5, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-6.

Table 2-6. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3	4	5	6	7	
λ	0.4	0.5	0.6	0.7	0.8	0.8	0.9
L_1	3.205	3.208	3.211	3.21	3.207	3.191	3.201
L_2	2.722	2.755	2.772	2.783	2.791	2.807	2.8
n	8		9	10			
λ	0.8	0.9	0.9	0.7	0.8	0.9	
L_1	3.179	3.188	3.177	3.146	3.159	3.168	
L_2	2.819	2.813	2.823	2.845	2.838	2.832	

We use L_1 and L_2 in Table 2-2, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-7 at a_I=24.349, b_I=0.205, δ_1=16.983, δ_2=-0.045, and g=301.

Table 2-7. The Value of ARL_1 under Various n and λ at a_I=24.349, b_I=0.205, δ_1=16.983, δ_2=-0.045, g=301 and ARL_0=370

λ \ ARL_1 \ n	2	3	4	5	6	7	8	9	10
0.05	4.42	3.63	3.17	2.87	2.64	2.45	2.29	2.17	2.09
0.1	3.72	3.02	2.62	2.36	2.18	2.07	1.99	1.93	1.88
0.2	3.22	2.55	2.21	2.00	1.85	1.73	1.62	1.51	1.41
0.3	**3.07**	2.36	2.01	1.79	1.62	1.49	1.37	1.27	1.19
0.4	3.08	2.27	1.88	1.65	1.47	1.34	1.24	1.16	1.10
0.5	3.22	**2.24**	1.81	1.56	1.38	1.26	1.17	1.11	1.06
0.6	3.51	2.28	**1.78**	1.51	1.33	1.21	1.13	1.08	1.05
0.7	3.98	2.40	1.80	**1.49**	**1.30**	1.19	1.11	1.07	1.04
0.8	4.73	2.61	1.86	1.50	**1.30**	**1.18**	**1.10**	**1.06**	**1.03**
0.9	5.85	2.96	1.98	1.54	1.31	**1.18**	**1.10**	**1.06**	**1.03**

According to Table 2-7, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-8.

Table 2-8. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3	4	5	6		7	
λ	0.3	0.5	0.6	0.7	0.7	0.8	0.8	0.9
L_1	3.126	3.208	3.211	3.21	3.191	3.207	3.191	3.201
L_2	2.729	2.755	2.772	2.783	2.801	2.791	2.807	2.8
n	8		9		10			
λ	0.8	0.9	0.8	0.9	0.8	0.9		
L_1	3.179	3.188	3.168	3.177	3.159	3.168		
L_2	2.819	2.813	2.829	2.823	2.838	2.832		

We use L_1 and L_2 in Table 2-2, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-9 at a_I=24.349, b_I=0.205, δ_1=-8.741, δ_2=0.123, and g=301.

Table 2-9. The Value of ARL_1 under Various n and λ at a_I=24.349, b_I=0.205, δ_1=-8.741, δ_2=0.123, g=301 and ARL_0=370

λ \ ARL_1 \ n	2	3	4	5	6	7	8	9	10
0.05	68.69	57.75	50.29	44.84	40.66	37.35	34.65	32.40	30.50
0.1	64.82	55.51	48.76	**43.62**	**39.58**	**36.30**	**33.60**	**31.32**	**29.38**
0.2	60.84	53.73	48.18	43.71	40.04	36.98	34.37	32.12	30.18
0.3	58.58	52.85	**48.17**	44.27	40.96	38.12	35.66	33.51	31.60
0.4	57.15	52.38	48.36	44.92	41.93	39.32	37.02	34.96	33.13
0.5	56.21	52.17	48.68	45.63	42.93	40.53	38.38	36.44	34.68
0.6	55.61	**52.15**	49.11	46.39	43.96	41.76	39.76	37.94	36.27
0.7	55.23	52.26	49.60	47.20	45.00	43.00	41.16	39.46	37.89
0.8	54.98	52.46	50.15	48.02	46.06	44.25	42.57	41.00	39.54
0.9	**54.83**	52.70	50.71	48.86	47.12	45.50	43.98	42.54	41.20

According to Table 2-9, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-10.

Table 2-10. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3	4	5	6	7	8	9	10
λ	0.9	0.6	0.3	0.1	0.1	0.1	0.1	0.1	0.1
L_1	3.378	3.246	3.065	2.494	2.497	2.502	2.507	2.512	2.518
L_2	2.628	2.739	2.787	2.486	2.482	2.478	2.473	2.468	2.462

We use L_1 and L_2 in Table 2-2, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-11 at a_I=24.349, b_I=0.205, δ_1=9.452, δ_2=-0.097, and g=301.

Table 2-11. The Value of ARL_1 under Various n and λ at a_I=24.349, b_I=0.205, δ_1=9.452, δ_2=-0.097, g=301 and ARL_0=370

λ \ n (ARL_1)	2	3	4	5	6	7	8	9	10
0.05	5.26	4.29	3.73	3.34	3.09	2.95	2.80	2.62	2.42
0.1	4.38	3.53	3.09	2.79	2.52	2.30	2.14	2.06	2.02
0.2	3.70	2.94	2.50	2.23	2.08	2.01	1.96	1.90	1.82
0.3	3.44	2.65	2.27	2.06	1.94	1.83	1.70	1.56	1.43
0.4	**3.35**	2.52	2.14	1.93	1.75	1.59	1.44	1.30	1.20
0.5	3.41	2.47	2.05	1.79	1.58	1.41	1.27	1.17	1.10
0.6	3.62	**2.46**	1.97	1.67	1.45	1.30	1.18	1.11	1.06
0.7	4.04	2.52	**1.92**	1.58	1.37	1.22	1.13	1.07	1.04
0.8	4.83	2.69	**1.92**	1.54	1.32	1.18	1.10	1.05	1.03
0.9	6.33	3.05	2.00	**1.53**	**1.29**	**1.16**	**1.08**	**1.04**	**1.02**

According to Table 2-11, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-12.

Table 2-12. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3	4		5	6	7	8	9	10
λ	0.4	0.6	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9
L_1	3.205	3.246	3.236	3.255	3.238	3.217	3.201	3.188	3.177	3.168
L_2	2.722	2.739	2.758	2.746	2.763	2.784	2.8	2.813	2.823	2.832

We use L_1 and L_2 in Table 2-3, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-13 at a_I =1, b_I =0.202, δ_1=0, δ_2=0.077, and g=301.

Table 2-13. The Value of ARL_1 under Various n and λ at a_I =1, b_I =0.202, δ_1=0, δ_2=0.077, g=301 and ARL_0=370

ARL_1 (λ \ n)	2	3	4	5	6	7	8	9	10
0.05	**24.93**	**18.87**	15.60	13.52	12.06	10.97	10.12	9.44	8.87
0.1	27.11	19.52	**15.58**	**13.15**	**11.49**	**10.29**	**9.37**	**8.64**	**8.05**
0.2	34.26	23.62	18.08	14.72	12.48	10.89	9.70	8.78	**8.05**
0.3	42.05	28.87	21.78	17.43	14.52	12.45	10.92	9.74	8.80
0.4	49.62	34.51	26.06	20.75	17.15	14.57	12.65	11.17	10.00
0.5	56.67	40.21	30.64	24.47	20.21	17.12	14.79	12.99	11.56
0.6	63.06	45.76	35.34	28.44	23.58	20.00	17.28	15.15	13.45
0.7	68.68	51.00	39.99	32.52	27.16	23.15	20.06	17.61	15.64
0.8	73.50	55.80	44.46	36.58	30.83	26.45	23.03	20.30	18.07
0.9	77.43	60.02	48.58	40.48	34.45	29.80	26.12	23.14	20.70

According to Table 2-13, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-14.

Table 2-14. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3	4	5	6	7	8	9	10	
λ	0.05	0.05	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
L_1	2.718	2.666	3.021	2.985	2.957	2.936	2.919	2.905	2.983	3.204
L_2	2.246	2.3	2.389	2.423	2.449	2.468	2.485	2.498	2.51	2.533

We use L_1 and L_2 in Table 2-3, to ensure that ARL_0=370, The ARL_1 under various λ and n are illustrated in Table 2-15 at a_I=1, b_I=0.202, δ_1=0, δ_2=0.442, and g=301.

Table 2-15. The Value of ARL_1 under Various n and λ at a_I=1, b_I=0.202, δ_1=0, δ_2=0.442, g=301 and ARL_0=370

λ \ ARL_1 \ n	2	3	4	5	6	7	8	9	10
0.05	3.76	3.05	2.65	2.38	2.19	2.05	1.94	1.84	1.76
0.1	3.31	2.64	2.28	2.04	1.88	1.75	1.65	1.56	1.49
0.2	3.04	2.37	2.01	1.79	1.63	1.52	1.43	1.36	1.30
0.3	**3.02**	**2.29**	1.92	1.69	1.54	1.43	1.34	1.28	1.23
0.4	3.10	**2.29**	**1.89**	1.66	1.50	1.39	1.30	1.24	1.19
0.5	3.23	2.34	1.91	**1.65**	**1.49**	**1.37**	1.29	**1.22**	**1.17**
0.6	3.40	2.42	1.94	1.67	**1.49**	**1.37**	**1.28**	**1.22**	**1.17**
0.7	3.60	2.52	2.00	1.70	1.51	1.38	1.29	**1.22**	**1.17**
0.8	3.82	2.65	2.08	1.75	1.54	1.40	1.30	1.23	**1.17**
0.9	4.05	2.79	2.17	1.81	1.58	1.43	1.32	1.24	1.18

According to Table 2-15, we find the best combination of λ and n with minimum ARL_1. They are summarized in Table 2-16.

Table 2-16. Combination of λ, n, L_1 and L_2 with Minimum ARL_1

n	2	3		4	5	6		7	
λ	0.3	0.3	0.4	0.4	0.5	0.5	0.6	0.5	0.6
L_1	3.984	3.785	3.996	3.854	3.893	3.811	3.912	3.748	3.842
L_2	2.044	2.185	2.087	2.189	2.184	2.245	2.175	2.294	2.229

n	8	9			10				
λ	0.6	0.5	0.6	0.7	0.5	0.6	0.7	0.8	
L_1	3.786	3.654	3.739	3.806	3.619	3.7	3.764	3.81	
L_2	2.273	2.368	2.31	2.254	2.397	2.342	2.289	2.241	

We summarize all the best λ under various n in Tables 2-4, 2-5,…, 2-16 as follows:

Table 2-17. The Best λ in Tables 2-4, 2-5,…, 2-16.

a_I	b_I	δ_1	δ_2	mean shift scale	s.d. shift scale	n: 2	3	4	5	6	7	8	9	10
						The best λ								
1.5	2	0.1	0.05	0.114	1.058	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
24.349	0.205	0.919	0.06	1.685	1.317	0.4	0.5	0.6	0.7	0.8	0.8, 0.9	0.8, 0.9	0.9	0.7, 0.8, 0.9
24.349	0.205	16.983	-0.045	1.603	1.017	0.3	0.5	0.6	0.7	0.7, 0.8	0.8, 0.9	0.8, 0.9	0.8, 0.9	0.8, 0.9
24.349	0.205	-8.741	0.123	0.126	1.281	0.9	0.6	0.3	0.1	0.1	0.1	0.1	0.1	0.1
24.349	0.205	9.452	-0.097	-1.326	0.621	0.4	0.6	0.7, 0.8	0.9	0.9	0.9	0.9	0.9	0.9
1	0.202	0	0.077	0.381	1.381	0.05	0.05	0.1	0.1	0.1	0.1	0.1	0.1	0.1, 0.2
1	0.202	0	0.442	2.188	3.188	0.3	0.3, 0.4	0.4	0.5	0.5, 0.6	0.5, 0.6	0.6	0.5, 0.6, 0.7	0.5, 0.6, 0.7, 0.8

In Table 2-17, the value of the mean shift scale and the value of the s.d. shift scale are calculated as follow:

$$\text{mean shift scale} = \frac{a_O b_O - a_I b_I}{\sqrt{a_I b_I^2}} \quad \text{and s.d. shift scale} = \frac{\sqrt{a_O b_O^2}}{\sqrt{a_I b_I^2}}$$

According to Table 2-17, n significantly affects the value of the best λ when both the mean shift scale and the s.d. shift scale have a large value. In addition, the larger the mean shift scale or s.d. shift scale, the larger the value of the best λ.

CHAPTER 3. DERIVATION OF THE PROFIT MODEL WITHOUT PRODUCER INSPECTION

3.1 Derivation of Expected Cycle Time

In Sections 3.1 and 3.2, we derive the profit model, referring to Panagos, Heikes, and Montgomery (1985).

We begin with the following assumptions:

(1) The process has a single assignable cause. The time until occurrences of the assignable cause is the exponential distribution with θ mean per unit time.

(2) The process starts in the in-control state.

(3) When the assignable cause occurs, both parameters of gamma distribution a and b shift to $a+\delta_1$ and $b+\delta_2$, respectively.

(4) For every h unit time, a sample size n is taken, and its average is plotted on the $\text{EWMA}_{\text{X-bar}}$ control chart.

(5) The manufacturing continues when the assignable cause is searched.

Similar to Figure 3-1, the cycle starts in the in-control state, and then the assignable cause occurs, becoming an out-of-control state, an $\text{EWMA}_{\text{X-bar}}$ statistic falls outside the control limits, the result is tested and interpreted, and an assignable cause is then found and repaired.

Because the time until an assignable cause occurrence is the exponential distribution with θ mean per unit time, the expected time of the in-control state is $1/\theta$. The expected time of shift occurrence in the sampling time h is $\tau \cong \frac{h}{2} - \frac{\theta h^2}{12}$. The expected time of the out-of-control state is $h/(1-\beta)$, where $1-\beta$ is the power of the control chart. The time to test and interpret the results is equal to $e*n$ and the time to find and repair an assignable cause is equal to D.

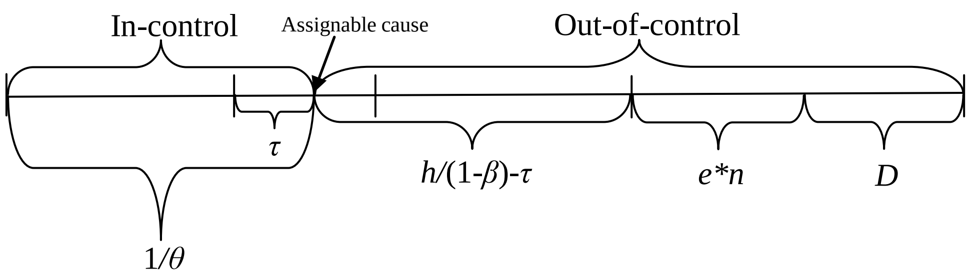

Figure 3-1. Continuous Process Cycle.

Because we use the EWMA$_{X\text{-bar}}$ control chart, we calculate α and β as follows:

$$\alpha = \frac{1}{ARL_0} \tag{3-1}$$

$$\beta = 1 - \frac{1}{ARL_1} \tag{3-2}$$

where ARL_0 and ARL_1 are calculated using Equations 2-3 and 2-4, respectively.

Hence, the expected cycle time is

$$ET = \frac{1}{\theta} + h(ARL_1 - \frac{1}{2} + \frac{\theta h}{12}) + en + D \tag{3-3}$$

3.2 Derivation of the Expected Cycle Profit

The quality variable, which we consider, is the smaller the better; therefore, we use the quadratic Taguchi loss function, as follows:

$$L = k_c X^2 \tag{3-4}$$

where $X > 0$ is the quality variable and k_c is the coefficient of loss function.

If the producer decides not to inspect products, then the expected cost per unit item using the quadratic Taguchi loss function is

$$E(L) = k_c\left[E(X)^2 + Var(X)\right] \tag{3-5}$$

Hence, in the in-control state, the expected cost per unit item is

$$\ell_I = k_c\left[E(X_I)^2 + Var(X_I)\right] = k_c\left(a_I^2 b_I^2 + a_I b_I^2\right) \tag{3-6}$$

and, in the out-of-control state, the expected cost per unit item is

$$\ell_O = k_c\left[E(X_O)^2 + Var(X_O)\right] = k_c\left[(a_I+\delta_1)^2(b_I+\delta_2)^2 + (a_I+\delta_1)(b_I+\delta_2)^2\right] \tag{3-7}$$

Assume that the sale price for conformable product is P_C, but the sale price for unconformable product is P_U, and $P_C > P_U$. We let the sale price for product without inspection is

$$P_W = P_C * P(X_I < USL) + P_U * P(X_I > USL) \tag{3-8}$$

Hence, in the in-control state, the expected net profit per unit time is

$$EP_I = (P_W - \ell_I) * R \tag{3-9}$$

where R is the number of products per unit time.

Similarly, in the out-of-control state, the expected net profit per unit time is

$$EP_O = (P_W - \ell_O) * R \tag{3-10}$$

We include the cost of investigating a false alarm which is T, the cost of taking a sample, is s_0+s_1*n, and the cost of finding and repairing an assignable cause which is W.

Hence, the expected cycle profit is

$$EP = EP_I \frac{1}{\theta} - \frac{T}{\theta h ARL_0} + EP_O \left[h(ARL_1 - \frac{1}{2} + \frac{\theta h}{12}) + en + D \right] - \frac{(s_0 + s_1 n)ET}{h} - W \tag{3-11}$$

Therefore, the expected profit per unit time is

$$EAP = \frac{EP}{ET} \tag{3-12}$$

3.3 Determining Optimum Design Parameters of the Economic EWMA$_{\text{X-bar}}$ Control Chart

The procedure to determine n^*, h^*, and (λ, L_1, L_2) of the economic EWMA$_{\text{X-bar}}$ control chart without producer inspection is as follows:

Step1. Let n=2.

Step2. Determine the UCL coefficient (L_1) of the EWMA$_{\text{X-bar}}$ control chart. With a, b, and λ, let LCL=0 and ARL_0=740 to solve L_1 by using the routine "uniroot" in the R program. Hence, UCL is determined.

Step3. Determine the LCL coefficient (L_2) of the EWMA$_{\text{X-bar}}$ control chart. With UCL, let ARL_0=370 to solve L_2 using the routine "uniroot" in the R program. Hence, the economic EWMA$_{\text{X-bar}}$ control chart is constructed.

Step4. We use the routine "DEoptim" of the R program for global optimization by differential evolution to maximize *EAP*, subject to $0.5 \leqq h \leqq 8$. Hence, *h** is determined. If *EAP(n+1)* is greater than *EAP(n)*, then we choose *EAP(n+1)* to become *EAP**.

Step5. Let $n=n+1$, $3 \leqq n \leqq 25$. Proceed to Step2.

3.4 An Example

For the gamma distribution parameters, we let $a_I=1.5$, $b_I=2$, $\delta_1=0.1$, and $\delta_2=0.05$ which are same as that in Table 2-4. For cycle time and profit parameters, we let $\theta=0.01$, $e=0.05$, $D=20$, $T=250$, $s_0=5$, $s_1=0.1$, and $W=500$. We let $P_C=300$, $P_U=150$, and $R=200$. We also set $USL=8.66$; thus, $P_W=294.875$.

According to Table 2-4, with minimize ARL_1, $\lambda=0.05$ at $n=2,3,\ldots,10$. Because it takes a significant time to conduct optimization by the differential evolution, we choose $\lambda=0.05$, 0.5 and 1 and $g=101$, where the $EWMA_{X\text{-bar}}$ chart with $\lambda=1$ is the same as the X-bar probability chart. We compare the optimum results for profit model with $\lambda=1$, 0.5, and 0.05 as follows:

Table 3-1. Optimum Results of Profit Model under Three Different λ

λ	1	0.5	0.05
L_1	3.438	3.303	2.604
L_2	2.571	2.668	2.387
n^*	25	25	25
h^*	0.5	0.5	0.5
EAP^*	26621.1	27246.47	**27738.33**
ARL_1	135.66	63.02	**22.94**
UCL^*	4.684	3.934	3.204
LCL^*	1.741	2.245	2.813

According to Table 3-1, n^* and h^* are the same in three types of optimum results, but we have the largest EAP^*, the smallest ARL_1, and the narrowest chart when we use the economic $EWMA_{X\text{-bar}}$ chart with $\lambda=0.05$. The table shows that λ affects the optimum result significantly. Therefore, we suggest that the producer use the economic $EWMA_{X\text{-bar}}$ chart with $\lambda=0.05$ and take 25 samples every 0.5 unit time to obtain 27738.3 profits per unit time.

3.5 Sensitivity Analysis and Comparing the Results with λ=1

In sensitivity analysis, we choose two levels of parameters in orthogonal arrays $L_{20}(2^{19})$, which is designed by Plackett and Burman (1946), as follows:

Table 3-2. Level of Parameters

	k_c, A, IC, R	δ_1	δ_2	θ	e	D	s_0	s_1	W	T	P_C, P_U
level 1	10,600,0.1,200	6.5	0.03	0.05	0.5	20	5	1	500	250	500,200
level 2	5,100,0.05,1000	3.5	-0.01	0.01	0.05	3	0.5	0.1	50	35	300,150

Table 3-3. Parameters for Each Experiment

Exp.	k_c, A, IC, R	δ_1	δ_2	θ	e	D	s_0	s_1	W	T	P_C, P_U
1	10,600,0.1,200	6.5	0.03	0.05	0.05	3	5	1	50	250	500,200
2	10,600,0.1,200	6.5	0.03	0.01	0.05	20	5	0.1	500	250	300,150
3	10,600,0.1,200	6.5	-0.01	0.05	0.5	3	0.5	0.1	50	250	300,150
4	10,600,0.1,200	6.5	-0.01	0.01	0.5	20	0.5	1	500	35	300,150
5	10,600,0.1,200	6.5	-0.01	0.01	0.05	3	5	0.1	500	35	500,200
6	10,600,0.1,200	3.5	0.03	0.05	0.5	20	0.5	0.1	500	250	300,150
7	10,600,0.1,200	3.5	0.03	0.05	0.05	3	0.5	0.1	500	35	500,200
8	10,600,0.1,200	3.5	0.03	0.01	0.5	20	5	1	50	35	500,200
9	10,600,0.1,200	3.5	-0.01	0.05	0.5	3	5	1	50	35	300,150
10	10,600,0.1,200	3.5	-0.01	0.01	0.05	20	0.5	1	50	250	500,200
11	5,100,0.05,1000	6.5	0.03	0.05	0.5	3	0.5	1	500	35	500,200
12	5,100,0.05,1000	6.5	0.03	0.01	0.5	20	0.5	0.1	50	35	500,200
13	5,100,0.05,1000	6.5	0.03	0.01	0.05	3	0.5	1	50	250	300,150
14	5,100,0.05,1000	6.5	-0.01	0.05	0.5	20	5	0.1	50	250	500,200
15	5,100,0.05,1000	6.5	-0.01	0.05	0.05	20	5	1	500	35	300,150
16	5,100,0.05,1000	3.5	0.03	0.05	0.05	20	5	0.1	50	35	300,150
17	5,100,0.05,1000	3.5	0.03	0.01	0.5	3	5	1	500	250	300,150
18	5,100,0.05,1000	3.5	-0.01	0.05	0.05	20	0.5	1	500	250	500,200
19	5,100,0.05,1000	3.5	-0.01	0.01	0.5	3	5	0.1	500	250	500,200
20	5,100,0.05,1000	3.5	-0.01	0.01	0.05	3	0.5	0.1	50	35	300,150

In Table 3-4, we let a_I=25 and b_I=0.2 to maximize EAP and determine optimum n^* and h^* at each experiment, subject to $2 \leqq n \leqq 25$ and $0.5 \leqq h \leqq 8$. The optimum results are solved as follows:

Table 3-4. Optimum Results in Each Experiment

	Optimum results for profit model with λ=1						Optimum results for profit model with λ=0.05					
Exp.	*EAP**	ARL_1	*n**	*h**	*UCL**	*LCL**	*EAP**	ARL_1	*n**	*h**	*UCL**	*LCL**
1	39491.38	1.09	5	0.5	6.449	3.764	38425.31	2.06	6	0.5	5.185	4.852
2	-1643.75	1.09	5	0.5	6.449	3.764	-1850.61	2.06	6	0.5	5.185	4.852
3	**1699.77**	4.51	6	0.5	6.314	3.864	**2114.75**	6.07	3	0.5	5.249	4.784
4	**3568.72**	4.51	6	0.5	6.314	3.864	**3676.62**	6.07	3	0.5	5.249	4.784
5	47096.10	1.45	14	0.5	5.840	4.236	46958.37	2.72	15	0.5	5.109	4.902
6	-11767.50	2.29	4	0.5	6.634	3.632	-11836.72	4.64	2	0.5	5.298	4.732
7	42136.44	1.19	8	0.5	6.128	4.006	41358.06	2.21	10	0.5	5.143	4.885
8	41089.50	2.29	4	0.5	6.634	3.632	41050.95	4.64	2	0.5	5.298	4.732
9	**3892.49**	10.99	18	0.5	5.737	4.322	**4772.07**	11.46	6	0.5	5.185	4.852
10	**46241.96**	**6.47**	25	0.5	5.621	4.421	**46298.41**	**5.67**	18	0.5	5.092	4.904
11	341940.72	2.38	2	0.5	7.390	3.142	340013.93	3.24	2	0.5	5.298	4.732
12	344515.55	2.38	2	0.5	7.390	3.142	344106.96	3.24	2	0.5	5.298	4.732
13	160898.13	1.09	5	0.5	6.449	3.764	160196.33	2.06	6	0.5	5.185	4.852
14	**339392.19**	4.51	6	0.5	6.314	3.864	**339790.00**	6.07	3	0.5	5.249	4.784
15	137503.35	1.45	14	0.5	5.840	4.236	137066.81	2.82	14	0.5	5.115	4.900
16	118678.91	1.19	8	0.5	6.128	4.006	118010.50	2.21	10	0.5	5.143	4.885
17	160562.21	2.29	4	0.5	6.634	3.632	160422.99	4.64	2	0.5	5.298	4.732
18	**357863.35**	**6.47**	25	0.5	5.621	4.421	**358052.43**	**5.50**	19	0.5	5.087	4.902
19	**366626.34**	10.99	18	0.5	5.737	4.322	**367588.80**	10.45	7	0.5	5.172	4.863
20	**164254.35**	**6.47**	25	0.5	5.621	4.421	**164405.92**	**5.39**	20	0.5	5.082	4.899

According to Table 3-4, at Experiment 3, 4, 9, 10, 14, 18, 19, and 20 the profit model with λ=0.05 has larger *EAP** than the profit model with λ=1. We find δ_1=3.5 and δ_2=-0.01 are very small shift at Experiment 9, 10, 18, 19, and 20. The δ_1=6.5, δ_2=-0.01 and e=0.5 are small shift, but e is large at Experiments 3, 4, and 14.

At Experiment 10, 18, and 20, the profit model with λ=0.05 has larger *EAP** and smaller ARL_1 than the profit model with λ=1. For these three experiments with larger *EAP**, we find δ_1=3.5 and δ_2=-0.01 are very small, also e=0.05 and s_0=0.5, are small.

We use the optimum results for profit model with λ=0.05 in Table 3-4 to plot the response figures (from Figure 3-2. to 3-7.) and determine the parameters that affects optimum value significantly.

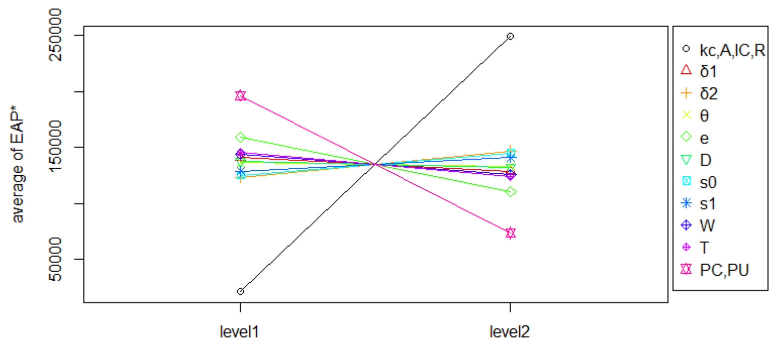

Figure 3-2. Response Figure of $\overline{EAP}*$

According to Figure 3-2, (k_c, A, IC, R) and (P_C ,P_U) are the most significant. The smaller the (k_c, A, IC, R), the larger the $EAP*$, and the larger the (P_C ,P_U), the larger the $EAP*$. The smaller the cost, the larger the profit is, and the larger the selling price, the larger the profit is.

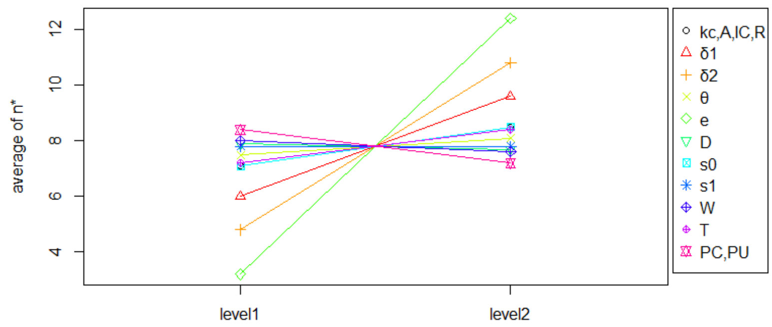

Figure 3-3. Response Figure of $\overline{n}*$

According to Figure 3-3, δ_1, δ_2, and e are the most significant. The smaller the δ_1, δ_2 or e, the larger the $n*$ is. The smaller shift in products necessitates more samples for testing. The term $e*n$ causes the parameter e to affect the optimum value $n*$ significantly.

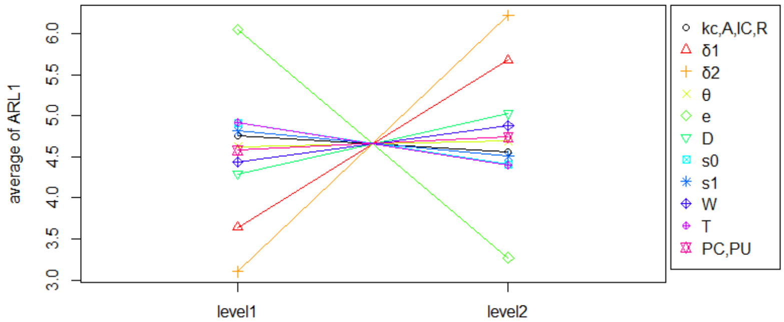

Figure 3-4. Response Figure of $\overline{ARL_1}$

According to Figure 3-4, δ_1, δ_2, and e are the most significant. The larger the δ_1 *or* δ_2, the smaller the ARL_1 is, and also the smaller the e, the smaller the ARL_1 is. A larger shift results in larger power; hence, the smaller the ARL_1. The smaller the e, the larger the n^* is; hence, the smaller the ARL_1.

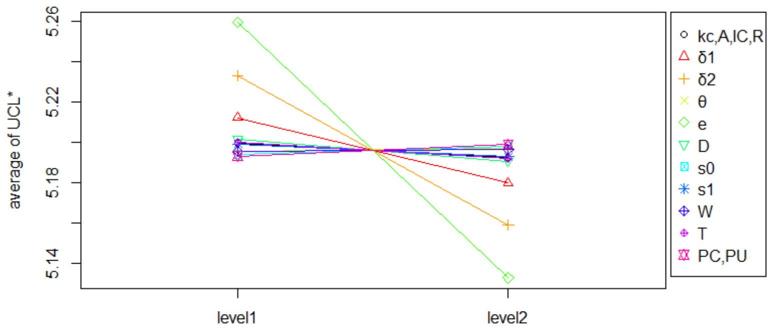

Figure 3-5. Response Figure of $\overline{UCL^*}$

According to Figure 3-5, δ_1, δ_2, and e are the most significant. The smaller the δ_1, δ_2 or e, the smaller the UCL^* is. The smaller shift in products necessitates a narrower chart to test; hence, the smaller the UCL^*. The smaller the e, the larger the n^* is; hence, the smaller the UCL^*.

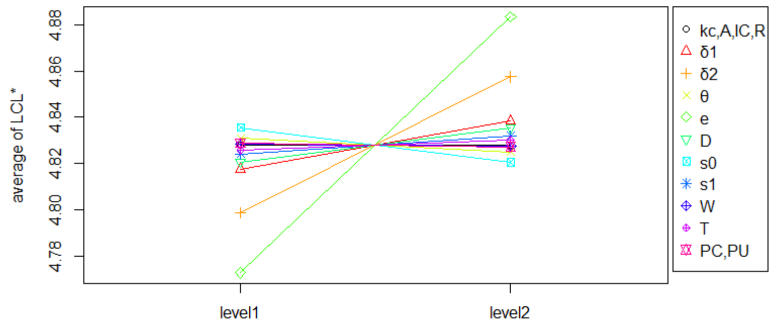

Figure 3-6. Response Figure of $\overline{LCL*}$

According to Figure 3-6, δ_2 and *e* are the most significant. The smaller the δ_2 or *e*, the larger the *LCL** is. The smaller shift in products necessitates a narrower chart to test; hence, the larger the *LCL**. The smaller the *e*, the larger the *n** is; hence, the larger the *LCL**.

We use the value of *EAP** of the EWMA$_{X\text{-bar}}$ chart with λ=0.05 minus the *EAP** of the EWMA$_{X\text{-bar}}$ chart with λ=1 to plot the response figure and to determine the significant parameters.

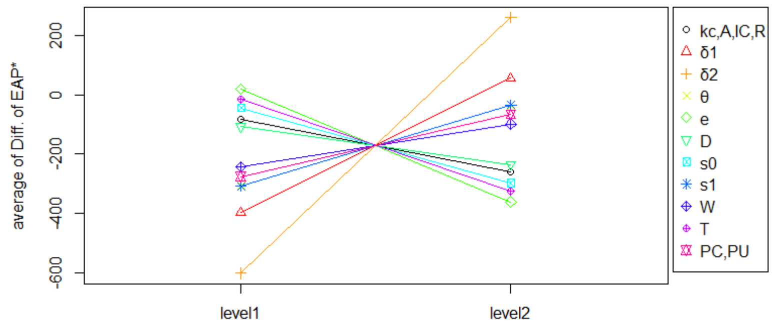

Figure 3-7. Response Figure of $\overline{\text{Difference of } EAP*}$

According to Figure 3-7, δ_2 is the most significant. The smaller the δ_2, the larger the difference of *EAP** is. The smaller the shift in *b*, the better performance of the EWMA$_{X\text{-bar}}$ chart with λ=0.05. This is because the smaller the shift, the smaller λ we need.

CHAPTER 4. DERIVATION OF THE PROFIT MODEL WITH PRODUCER INSPECTION

4.1 Derivation of the Expected Cycle Time

We use the same assumptions in Section 3.1 and with or without inspection, we have the same formula of expected cycle time.

Hence, the same as Equation 3-3, the expected cycle time is

$$ET = \frac{1}{\theta} + h(ARL_1 - \frac{1}{2} + \frac{\theta h}{12}) + en + D \quad (4\text{-}1)$$

4.2 Derivation of the Expected Cycle Profit

Because the quality variable is the smaller the better, we have only the upper specification limit if the producer decides to inspect products. Hence, the quadratic Taguchi loss function is as follows:

$$L = \begin{cases} k_c X^2, & if \ \ X < USL \\ A \quad\ , & if \ \ X > USL \end{cases} \quad (4\text{-}2)$$

where $USL = a_I b_I + \omega\sqrt{a_I b_I^2} \geq 0$ is the upper specification limit, $X > 0$ is the quality variable, k_c is the coefficient of loss function, and A is the cost of extra working for selling discount price P_U.

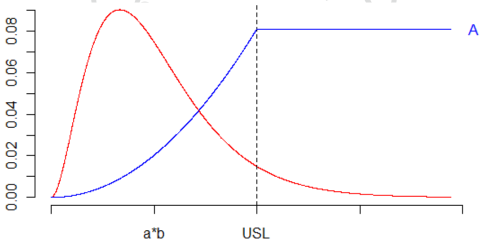

Figure 4-1. The Gamma Distribution and Taguchi Loss Function with Inspection

If the producer decides to inspect products, then the expected profit per unit item using the quadratic Taguchi loss function is

$$\int_0^{USL} (P_C - k_c x^2) f(x) dx + (P_U - A) \int_{USL}^{\infty} f(x) dx - IC \quad (4\text{-}3)$$

where $f(x)$ is the p.d.f of the gamma distribution and IC is the inspection cost.

Hence, in the in-control state, the expected profit per unit time is

$$
\begin{aligned}
EP_I &= \left[\int_0^{USL} (P_C - k_c x^2) f_I(x)dx + (P_U - A)\int_{USL}^{\infty} f_I(x)dx - IC \right] * R \\
&= \left[P_C \frac{\gamma(a_I, a_I + \omega\sqrt{a_I})}{\Gamma(a_I)} - \frac{k_c b_I^2}{\Gamma(a_I)} \gamma(a_I + 2, a_I + \omega\sqrt{a_I}) + (P_U - A)(1 - \frac{\gamma(a_I, a_I + \omega\sqrt{a_I})}{\Gamma(a_I)}) - IC \right] * R
\end{aligned}
\quad (4\text{-}4)
$$

where $\gamma(a,t) = \int_0^t x^{a-1} e^{-x} dx$ is the lower incomplete gamma function.

Similarly, in the out-of-control state, the expected profit per unit time is

$$
\begin{aligned}
EP_O &= \left[\int_0^{USL} (P_C - k_c x^2) f_O(x)dx + (P_U - A)\int_{USL}^{\infty} f_O(x)dx - IC \right] * R \\
&= \left[P_C \frac{\gamma(a_O, \frac{USL}{b_O})}{\Gamma(a_O)} - \frac{k_c b_O^2}{\Gamma(a_O)} \gamma(a_O + 2, \frac{USL}{b_O}) + (P_U - A)(1 - \frac{\gamma(a_O, \frac{USL}{b_O})}{\Gamma(a_O)}) - IC \right] * R
\end{aligned}
\quad (4\text{-}5)
$$

Hence, the expected cycle profit is

$$
EP = EP_I \frac{1}{\theta} - \frac{T}{\theta h ARL_0} + EP_O \left[h(ARL_1 - \frac{1}{2} + \frac{\theta h}{12}) + en + D \right] - \frac{(s_0 + s_1 n)ET}{h} - W \quad (4\text{-}6)
$$

Therefore, the expected profit per unit time is

$$
EAP = \frac{EP}{ET} \quad (4\text{-}7)
$$

4.3 Determining the Optimum Producer Inspection and Design Parameter of the Economic EWMA$_{\text{X-bar}}$ Control Chart

The procedure to determine n^*, h^*, ω^*, and control limits of the economic EWMA$_{\text{X-bar}}$ control chart with producer inspection is as follows:

Step1. Let n=2.

Step2. Determine the *UCL* coefficient (L_1) of the EWMA$_{\text{X-bar}}$ control chart. With a, b, and λ, let LCL=0 and ARL_0=740 to solve L_1 using the routine “uniroot” in the R program. Hence, *UCL* is determined.

Step3. Determine the *LCL* coefficient (L_2) of the $EWMA_{X\text{-}bar}$ control chart. With *UCL*, let ARL_0=370 to solve L_2 using the routine "uniroot" in the R program. Hence, the economic $EWMA_{X\text{-}bar}$ control chart is constructed.

Step4. We use the routine "DEoptim" of the R program to maximize *EAP*, subject to $0.5 \leqq h \leqq 8$ and $2 \leqq \omega$. We let $2 \leqq \omega$ to ensure that the yield is more than 0.95 for a_I =1.5 and b_I=2. Hence, h^* and ω^* are determined. If *EAP(n+1)* is greater than *EAP(n)*, then we choose *EAP(n+1)* to become *EAP**.

Step5. Let n=n+1, $3 \leqq n \leqq 25$. Proceed to Step2.

4.4 Example and Optimum Results Comparison for with and without Producer Inspection

The same as Section 3.4, for the gamma distribution parameters, we let a_I =1.5, b_I =2, δ_1=0.1, and δ_2=0.05. For cycle time and profit parameters, we let θ=0.01, e=0.05, D=20, T=250, s_0=5, s_1=0.1, and W=500. We also let k_c=10, A=600, IC=0.1, P_C=300, P_U=150, and R=200.

Similarly, we choose $EWMA_{X\text{-}bar}$ chart with three different λ, and let g=101. We compare tolerance, design parameters, *EAP**, and ARL_1 as follows:

Table 4-1. Optimum Results of Profit Model under Three Different λ

λ	1	0.5	0.05
L_1	3.438	3.303	2.604
L_2	2.571	2.668	2.387
n^*	25	25	25
h^*	0.5	0.5	0.5
ω^*	2.311	2.311	2.311
*USL**	8.66	8.66	8.66
Yield	0.965834	0.965834	0.965834
*EAP**	31379.35	31860.24	**32238.47**
ARL_1	135.66	63.02	**22.94**
*UCL**	4.684	3.934	3.204
*LCL**	1.741	2.245	2.813

According to Table 4-1, n*, h*, and ω* are the same in three types of optimum results, but we have the largest *EAP**, the smallest ARL_1, and the narrowest chart when we use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.05. The table shows that λ affects the optimum result significantly. Therefore, we suggest that the producer takes inspection with *USL**=8.66, use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.05 and take 25 samples every 0.5 unit time to obtain 32238.47 profits per unit time.

For comparison of without inspection, we merged Tables 3-1 and 4-1 as follows:

Table 4-2. Merging Tables 3-1 and 4-1

λ	1		0.5		0.05	
Inspection	Without	With	Without	With	Without	With
L_1	3.438	3.438	3.303	3.303	2.604	2.604
L_2	2.571	2.571	2.668	2.668	2.387	2.387
n*	25	25	25	25	25	25
h*	0.5	0.5	0.5	0.5	0.5	0.5
*EAP**	26621.1	31379.35	27246.47	31860.24	27738.33	32238.47
ARL_1	135.66	135.66	63.02	63.02	22.94	22.94
*UCL**	4.684	4.684	3.934	3.934	3.204	3.204
*LCL**	1.741	1.741	2.245	2.245	2.813	2.813

According to Table 4-2, with and without inspection, n*, h*, *UCL**, *LCL**, and ARL_1 are the same at each λ. However, the *EAP**, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 16.2% profit per unit time when we have an inspection.

(2) If λ=0.5, we increase 16.9% profit per unit time when we have an inspection.

(3) If λ=1, we increase 17.9% profit per unit time when we have an inspection.

Therefore, we suggest that the producer takes inspection with *USL**=8.66, use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.05 and take 25 samples every 0.5 unit time to obtain 32238.47 profit per unit time.

4.5 Sensitivity Analysis and Comparing the Results of EWMA$_{X\text{-}bar}$ Chart with λ=1

In sensitivity analysis, we use the same combinations of parameters in Table 3-3.

In Table 4-3, we let a_I =25 and b_I =0.2 maximize *EAP* and determine the optimum *n**, *h**, and ω* at each combination of parameters, subject to 2≦*n*≦25, 0.5≦*h*≦8, and 2≦ω. The optimum results are solved as follows:

Table 4-3. Optimum Result in Each Experiment

	Optimum results for profit model with λ=1							Optimum results for profit model with λ=0.05						
Exp.	ω*	*EAP**	ARL_1	*n**	*h**	*UCL**	*LCL**	ω*	*EAP**	ARL_1	*n**	*h**	*UCL**	*LCL**
1	4.487	39686.39	1.09	5	0.5	6.449	3.764	4.487	38646.06	2.06	6	0.5	5.185	4.852
2	3.660	-941.02	1.09	5	0.5	6.449	3.764	3.660	-1133.29	2.06	6	0.5	5.185	4.852
3	3.660	**1779.87**	4.51	6	0.5	6.314	3.864	3.660	**2191.23**	6.07	3	0.5	5.249	4.784
4	3.660	**3632.39**	4.51	6	0.5	6.314	3.864	3.660	**3739.40**	6.07	3	0.5	5.249	4.784
5	4.487	47087.44	1.45	14	0.5	5.840	4.236	4.487	46949.93	2.72	15	0.5	5.109	4.902
6	3.660	-11075.92	2.29	4	0.5	6.634	3.632	3.660	-11142.79	4.64	2	0.5	5.298	4.732
7	4.487	42181.72	1.19	8	0.5	6.128	4.006	4.487	41410.73	2.21	10	0.5	5.143	4.885
8	4.487	41144.76	2.29	4	0.5	6.634	3.632	4.487	41106.61	4.64	2	0.5	5.298	4.732
9	3.660	**3930.40**	10.99	18	0.5	5.737	4.322	3.660	**4807.32**	11.46	6	0.5	5.185	4.852
10	4.487	**46232.68**	**6.47**	25	0.5	5.621	4.421	4.487	**46289.11**	**5.67**	18	0.5	5.092	4.904
11	3.944	343452.49	2.38	2	0.5	7.390	3.142	3.944	341621.67	3.24	2	0.5	5.298	4.732
12	3.944	345900.25	2.38	2	0.5	7.390	3.142	3.944	345512.01	3.24	2	0.5	5.298	4.732
13	2.071	167806.31	1.09	5	0.5	6.449	3.764	2.071	167337.93	2.06	6	0.5	5.185	4.852
14	3.944	**339667.17**	4.51	6	0.5	6.314	3.864	3.944	**340063.00**	6.07	3	0.5	5.249	4.784
15	2.071	146191.39	1.45	14	0.5	5.840	4.236	2.071	145807.34	2.82	14	0.5	5.115	4.900
16	2.071	134792.50	1.19	8	0.5	6.128	4.006	2.071	134277.64	2.21	10	0.5	5.143	4.885
17	2.071	167045.58	2.29	4	0.5	6.634	3.632	2.071	166939.27	4.64	2	0.5	5.298	4.732
18	3.944	**358006.36**	**6.47**	25	0.5	5.621	4.421	3.944	**358195.16**	**5.50**	19	0.5	5.087	4.902
19	3.944	**366754.99**	10.99	18	0.5	5.737	4.322	3.944	**367715.87**	10.45	7	0.5	5.172	4.863
20	2.071	**169650.57**	**6.47**	25	0.5	5.621	4.421	2.071	**169792.39**	**5.39**	20	0.5	5.082	4.899

According to Table 4-3, at Experiment 3, 4, 9, 10, 14, 18, 19, and 20 the profit model with λ=0.05 has larger *EAP** than the profit model with λ=1. We find δ_1=3.5 and δ_2=-0.01 are very small shift at Experiment 9, 10, 18, 19, and 20. The δ_1=6.5, δ_2=-0.01 and *e*=0.5 are small shift, but *e* is large at Experiments 3, 4, and 14.

At Experiment 10, 18, and 20, the profit model with λ=0.05 has larger *EAP** and smaller ARL_1 than the profit model with λ=1. For these three experiments with larger *EAP**, we find δ_1=3.5 and δ_2=-0.01 are very small, also e=0.05 and s_0=0.5, are small.

The ω*, (P_C ,P_U), and (k_c, A, IC, R) at Experiment 2, 3, 4, 6, and 9 are same, respectively. The ω*, (P_C ,P_U), and (k_c, A, IC, R) at Experiment 1, 5, 7, 8, and 10, are same, respectively. The ω*, (P_C ,P_U), and (k_c, A, IC, R) at Experiment 11, 12, 14, 18, and 19, are same, respectively. The ω*, (P_C ,P_U), and (k_c, A, IC, R) at Experiment 13, 15, 16, 17, and 20, are same, respectively. Hence, the ω* depends only on the values of (P_C ,P_U) and (k_c, A, IC, R).

We use the optimum results for profit model with λ=0.05 in Table 4-3 to plot the response figures (from Figure 4-2. to 4-8.) and determine the parameters that affects optimum value significantly.

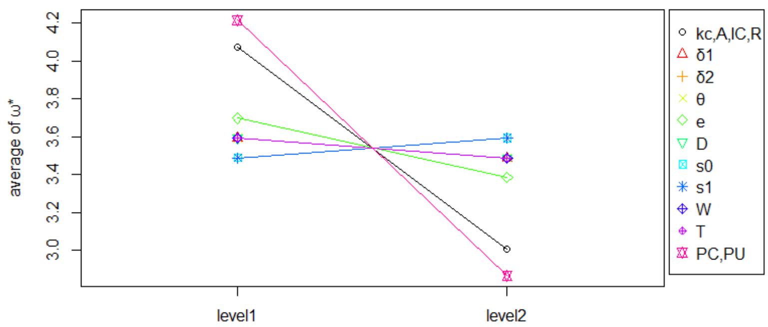

Figure 4-2. Response Figure of $\overline{\omega^*}$

According to Figure 4-2, (P_C ,P_U) and (k_c, A, IC, R) are the most significant. The larger (P_C ,P_U) or (k_c, A, IC, R), the larger ω*. This means that the larger the selling price or cost, the larger the *USL** is.

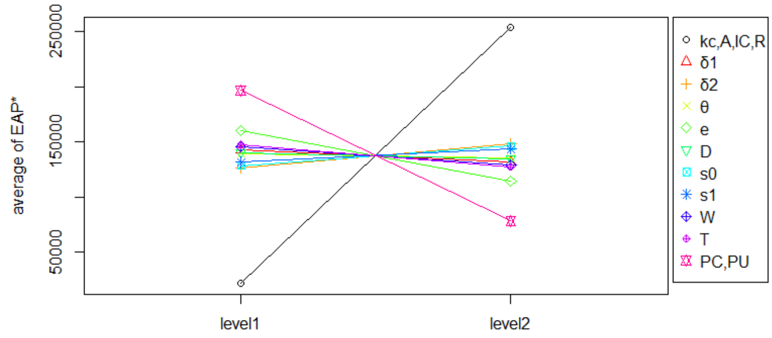

Figure 4-3. Response Figure of $\overline{EAP^*}$

According to Figure 4-3, (k_c, A, IC, R) and (P_C ,P_U) are the most significant. The smaller the (k_c, A, IC, R), the larger the EAP^*, and the larger the (P_C ,P_U), the larger the EAP^*. The smaller the cost, the larger the profit is, and the larger the selling price, the larger the profit is.

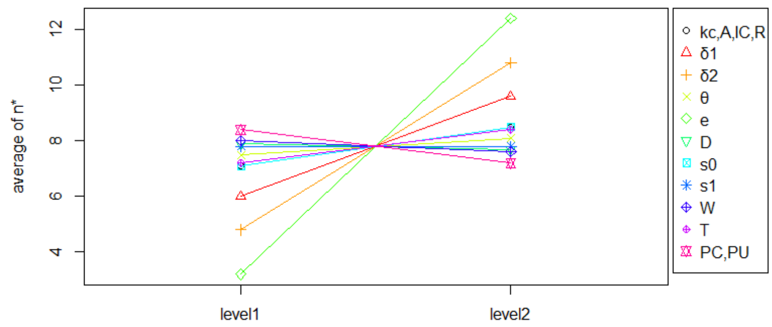

Figure 4-4. Response Figure of $\overline{n^*}$

According to Figure 4-4, δ_1, δ_2, and e are the most significant. The smaller the δ_1, δ_2 or e, the larger the n^* is. The smaller shift in products necessitates more samples for testing. The term e^*n causes the parameter e to affect the optimum value n^* significantly.

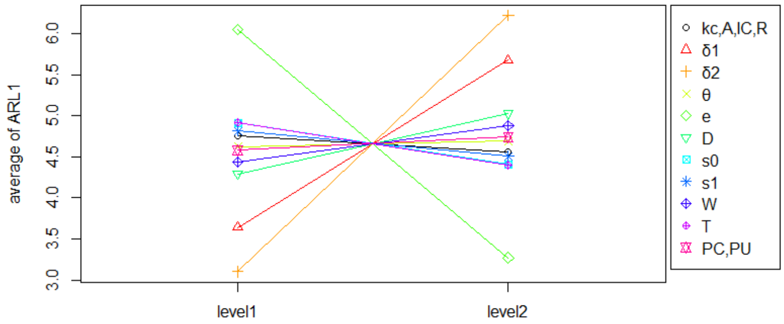

Figure 4-5. Response Figure of ARL_1

According to Figure 4-5, δ_1, δ_2, and e are the most significant. The larger the δ_1 or δ_2, the smaller the ARL_1 is, and also the smaller the e, the smaller the ARL_1 is. A larger shift results in larger power; hence, the smaller the ARL_1. The smaller the e, the larger the n^* is; hence, the smaller the ARL_1.

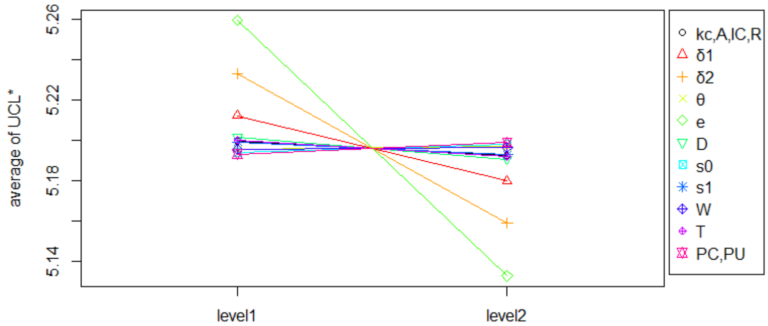

Figure 4-6. Response Figure of $\overline{UCL^*}$

According to Figure 4-6, δ_1, δ_2, and e are the most significant. The smaller the δ_1, δ_2 or e, the smaller the UCL^* is. The smaller shift in products necessitates a narrower chart to test; hence, the smaller the UCL^*. The smaller the e, the larger the n^* is; hence, the smaller the UCL^*.

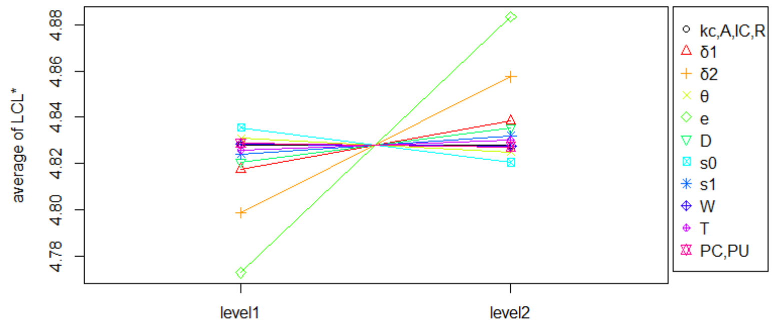

Figure 4-7. Response Figure of $\overline{LCL*}$

According to Figure 4-7, δ_2 and e are the most significant. The smaller the δ_2 or e, the larger the $LCL*$ is. The smaller shift in products necessitates a narrower chart to test; hence, the larger the $LCL*$. The smaller the e, the larger the $n*$ is; hence, the larger the $LCL*$.

We use the value of $EAP*$ of the EWMA$_{\text{X-bar}}$ chart with λ=0.05 minus the $EAP*$ of the EWMA$_{\text{X-bar}}$ chart with λ=1 to plot the response figure and to determine the significant parameters.

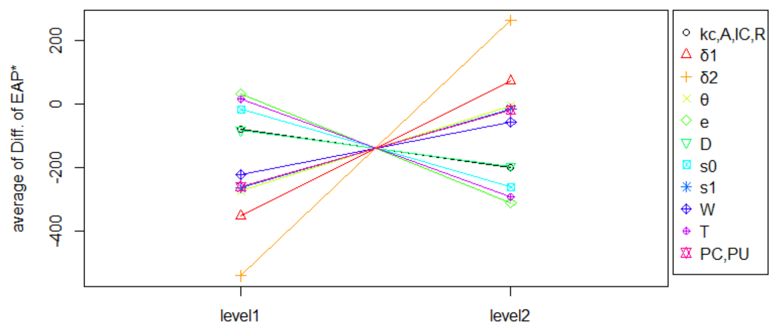

Figure 4-8. Response Figure of $\overline{\text{Difference of } EAP*}$

According to Figure 4-8, δ_2 is the most significant. The smaller the δ_2, the larger the difference of $EAP*$ is. The smaller the shift in b, the better performance of the EWMA$_{\text{X-bar}}$ chart with λ=0.05. This is because the smaller the shift, the smaller λ we need.

CHAPTER 5. DETERMINING THE BEST λ OF THE ECONOMIC EWMA$_{\text{X-bar}}$ CONTROL CHART UNDER DIFFERENT SHIFT SCALES IN THE MEAN AND VARIANCE

5.1 Data Description and Determining the Optimum Producer Inspection and the Design Parameters of the Economic EWMA$_{\text{X-bar}}$ Control Chart

In this section, we use routine "rgamma" to simulate one type of in-control gamma distribution and five types of out-of-control gamma distributions.

We first simulate 25 samples of size 4 data from the in-control gamma distribution with *a*=25 and *b*=0.2, with a mean and variance of 5 and 1, respectively.

Table 5-1. In-control Data with Gamma (*a*=25, *b*=0.2)

No.	simulation data with *n*=4				$\bar{X}$	No.	simulation data with *n*=4				$\bar{X}$
1	6.283	6.397	4.894	5.691	5.816	14	5.849	6.036	5.879	3.814	5.394
2	4.071	5.513	4.908	5.222	4.929	15	6.359	4.882	4.89	2.939	4.767
3	5.222	4.037	5.591	5.076	4.982	16	6.288	3.448	6.242	2.928	4.727
4	2.54	5.11	3.753	5.155	4.14	17	5.595	5.352	4.419	4.756	5.03
5	6.439	4.634	5.628	4.692	5.348	18	5.124	4.054	5.542	4.358	4.77
6	5.2	5.276	3.601	2.725	4.201	19	5.816	4.549	5.241	5.435	5.26
7	5.265	4.934	5.253	3.719	4.793	20	4.593	4.515	5.584	5.111	4.951
8	3.954	5.004	5.628	6.364	5.237	21	6.918	5.952	5.245	4.799	5.728
9	4.04	3.106	5.334	4.119	4.15	22	4.881	4.955	6.066	4.761	5.166
10	4.102	7.045	4.045	5.118	5.077	23	4.525	5.358	5.983	3.144	4.753
11	4.317	4.943	5.044	4.384	4.672	24	5.442	4.729	5.419	5.011	5.15
12	3	5.595	5.794	4.064	4.613	25	7.023	5.363	5.197	3.921	5.376
13	4.992	5.155	7.028	5.743	5.729	$\bar{\bar{X}}=4.99$					

As a producer, we do not know the parameters of the gamma distribution; thus, we used the MLE method, which maximizes cumulative products of p.d.f for given data to estimate $\hat{a}$ and $\hat{b}$ of the simulation data.

With 100 data in Table 5-1, we estimate the parameters of in-control data, and obtain $\hat{a}_I = 24.349$, $\hat{b}_I = 0.205$, $\widehat{\text{Mean}} = \hat{a}_I * \hat{b}_I = 4.99$, and $\widehat{\text{Var}} = \hat{a}_I * \hat{b}_I^2 = 1.023$.

For the following, we let n=4, θ=0.01, e=0.05, D=20, T=250, s_0=5, s_1=0.1, W=500, k_c=10, A=600, IC=0.1, P_C=300, P_U=150, and R=200. If the producer decides not to inspect, we maximize *EAP* (Equation 3-12) to determine the optimum h^*, subject to $0.5 \leqq h \leqq 8$. If the producer decides to inspect, we maximize *EAP* (Equation 4-7) to determine the optimum h^* and ω^*, subject to $0.5 \leqq h \leqq 8$ and $2 \leqq \omega$.

I. To compare the profit model with different λ, we adopt moderate shifts in the mean and variance of out-of-control gamma data.

Let a=26 and b=0.25, that is, the out-of-control mean and variance are 6.5 and 1.625, respectively, which means δ_1=1 and δ_2=0.05. We simulate 15 samples of size 4 data from the out-of-control gamma distribution with a=26 and b=0.25, as follows:

Table 5-2. Out-of-control Data with Gamma (a=26, b=0.25)

No.	simulation data with n=4				$\bar{X}$
1	5.745	7.612	4.765	6.292	6.103
2	6.441	4.498	5.032	9.175	6.287
3	7.145	6.166	5.624	7.499	6.608
4	7.21	5.55	7.1	5.784	6.411
5	5.971	6.097	7.355	5.414	6.209
6	6.292	7.096	9.027	5.149	6.891
7	4.741	6.698	7.71	6.21	6.34
8	6.76	5.287	11.144	5.372	7.141
9	5.447	6.346	6.172	6.578	6.136
10	7.465	7.072	7.945	9.309	7.948
11	8.298	4.874	5.87	6.009	6.263
12	10.221	8.085	5.179	7.298	7.696
13	5.417	6.453	6.746	6.935	6.388
14	7.509	6.938	5.835	5.22	6.375
15	9.154	7.477	8.304	5.888	7.706
					$\bar{\bar{X}} = 6.7$

With 60 data in Table 5-2, we estimate the parameters of out-of-control data, and obtain $\hat{a}_O = 25.268$, $\hat{b}_O = 0.265$, $\hat{\delta}_1 = 0.919$, $\hat{\delta}_2 = 0.006$, $\widehat{\text{Mean}} = 6.7$, and $\widehat{\text{Var}} = 1.777$. Hence, we have a 1.685 mean shift scale and a 1.317 s.d. shift scale.

According to Table 2-17, to minimize ARL_1, λ=0.6 is the best at n=4. Thus, we choose λ=1, 0.6, 0.05 and g=101. We compare the optimum results of profit model with λ=1, 0.6, and 0.05, as follows:

Table 5-3. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.6	0.05	1	0.6	0.05
L_1	3.272	3.211	2.743	3.272	3.211	2.743
L_2	2.732	2.772	2.303	2.732	2.772	2.303
h^*	0.5	0.5	0.5	0.5	0.5	0.5
ω^*	-	-	-	3.627	3.627	3.627
USL^*	-	-	-	8.66	8.66	8.66
Yield	-	-	-	0.998884	0.998884	0.998884
EAP^*	920.55	**952.33**	755.91	1327.2	**1357.31**	1171.23
P_W	299.833	299.833	299.833	-	-	-
ARL_1	1.94	**1.71**	3.11	1.94	**1.71**	3.11
UCL	6.646	6.055	5.214	6.646	6.055	5.214
LCL	3.61	4.074	4.805	3.61	4.074	4.805
first true alarm on which sample	No.6 (5 outliers)	No.3 (13 outliers)	No. 4 (12 outliers)	No.6 (5 outliers)	No.3 (13 outliers)	No. 4 (12 outliers)

According to Table 5-3, with and without inspection, h^*, EWMA$_{\text{X-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 54.9% profit per unit time when we have an inspection.

(2) If λ=0.6, we increase 42.5% profit per unit time when we have an inspection.

(3) If λ=1, we increase 44.2% profit per unit time when we have an inspection.

If we use the economic EWMA$_{\text{X-bar}}$ chart with λ=0.6, we have the largest EAP^* and smallest ARL_1 for the moderate shifts in the mean and variance. Therefore, we suggest that the producer takes inspection with USL^*=8.66, use the economic EWMA$_{\text{X-bar}}$ chart with λ=0.6 and take four samples every 0.5 unit time.

To find the detection ability for the three types of $EWMA_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

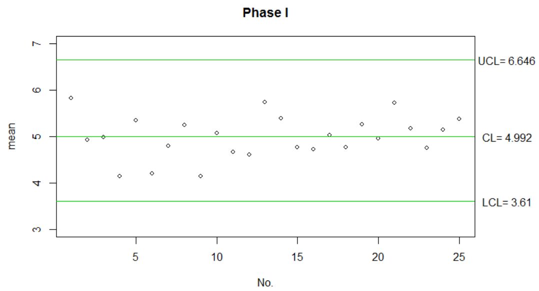

Figure 5-1. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=1, Figure 5-1 shows that no points are out of limits for in-control samples.

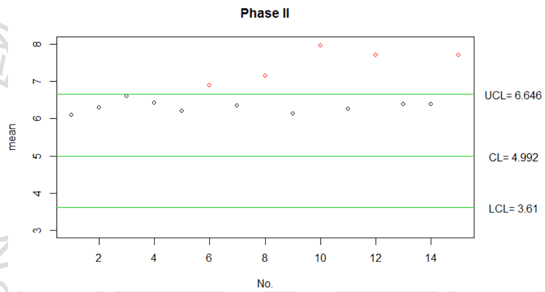

Figure 5-2. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=1, Figure 5-2 shows that No. 6, 8, 10, 12, and 15 are out of limits; the first true alarm is on No. 6.

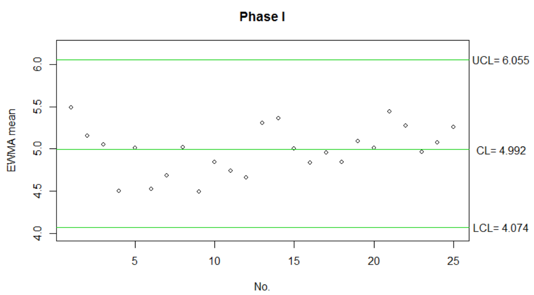

Figure 5-3. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.6) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=0.6, Figure 5-3 shows that no points are out of limits for in-control samples.

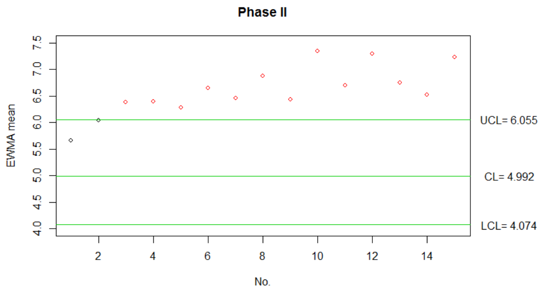

Figure 5-4. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.6) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=0.6, Figure 5-4 shows that No. 3 to No. 15 are out of limits; the first true alarm is on No. 3.

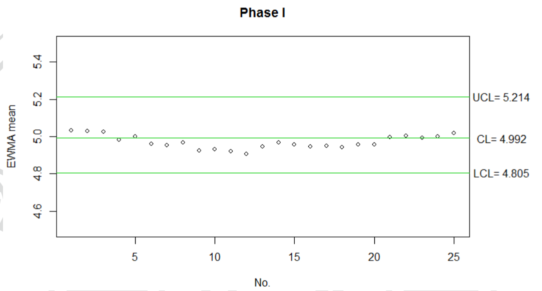

Figure 5-5. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.05) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=0.05, Figure 5-5 shows that no points are out of limits for in-control samples.

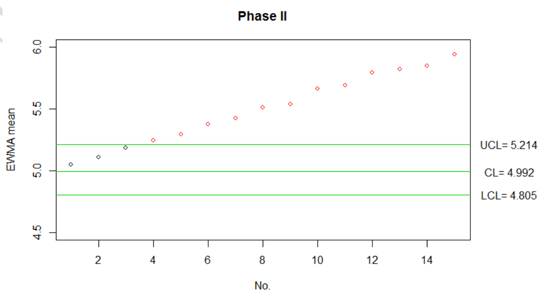

Figure 5-6. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.05) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=0.05, Figure 5-6 shows that No. 4 to No. 15 are out of limits; the first true alarm is on No. 4.

II. To compare the profit model with different λ, we adopt small shifts in the mean and variance of out-of-control gamma data.

Let a=28 and b=0.21, that is, the out-of-control mean and variance are 5.88 and 1.2348, respectively, which means δ_1=3 and δ_2=0.01. We simulate 15 samples of size 4 data from the out-of-control gamma distribution with a=28 and b=0.21, as follows:

Table 5-4. Out-of-control Data with Gamma (a=28, b=0.21)

No.	simulation data with n=4				$\overline{X}$
1	6.156	5.224	7.595	6.145	6.28
2	5.026	4.593	8.429	6.001	6.012
3	5.205	5.889	4.731	3.281	4.776
4	4.828	5.989	4.256	4.911	4.996
5	7.144	7.129	5.382	6.513	6.542
6	5.664	5.578	6.608	6.595	6.111
7	7.939	4.867	5.966	6.424	6.299
8	5.524	5.661	7.156	6.023	6.091
9	5.271	5.992	5.396	7.1	5.94
10	7.212	5.952	7.16	5.058	6.345
11	7.683	5.009	6.831	5.237	6.19
12	6.226	4.889	4.416	5.096	5.157
13	5.478	4.99	5.898	6.196	5.641
14	6.961	6.639	4.215	6.964	6.195
15	5.797	7.595	7.614	7.204	7.053
					$\overline{\overline{X}} = 5.975$

With 60 data in Table 5-4, we estimate the parameters of out-of-control data, and obtain $\hat{a}_O = 30.883$, $\hat{b}_O = 0.193$, $\hat{\delta}_1 = 6.534$, $\hat{\delta}_2 = -0.012$, $\widehat{\text{Mean}} = 5.975$, and $\widehat{\text{Var}} = 1.156$. Hence, we have a 0.974 mean shift scale and a 1.063 s.d. shift scale.

We choose λ=1, 0.5, 0.05 and g=101, where the EWMA$_{\text{X-bar}}$ chart with λ=1 is the same as the X-bar probability chart. We compare the optimum results of profit model with λ=1, 0.5, and 0.05, as follows:

Table 5-5. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.5	0.05	1	0.5	0.05
L_1	3.272	3.178	2.743	3.272	3.178	2.743
L_2	2.732	2.784	2.303	2.732	2.784	2.303
h^*	0.5	0.5	0.5	0.5	0.5	0.5
ω^*	-	-	-	3.627	3.627	3.627
USL^*	-	-	-	8.66	8.66	8.66
Yield	-	-	-	0.998884	0.998884	0.998884
EAP^*	3763.32	**4134.25**	4039.73	3832.9	**4200.45**	4106.79
P_W	299.833	299.833	299.833	-	-	-
ARL_1	9.16	**4.01**	5.3	9.16	**4.01**	5.3
UCL	6.646	5.92	5.214	6.646	5.92	5.214
LCL	3.61	4.179	4.805	3.61	4.179	4.805
first true alarm on which sample	No.15 (1 outlier)	No.6 (8 outliers)	No.7 (9 outliers)	No.15 (1 outlier)	No.6 (8 outliers)	No.7 (9 outliers)

According to Table 5-5, with and without inspection, h^*, EWMA$_{\text{X-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 1.66% profit per unit time when we have an inspection.

(2) If λ=0.5, we increase 1.6% profit per unit time when we have an inspection.

(3) If λ=1, we increase 1.85% profit per unit time when we have an inspection.

If we use the economic EWMA$_{\text{X-bar}}$ chart with λ=0.5, we have the largest EAP^* and smallest ARL_1 for the small shifts in the mean and variance. Therefore, we suggest that the producer takes inspection with USL^*=8.66, use the economic EWMA$_{\text{X-bar}}$ chart with λ=0.5 and take four samples every 0.5 unit time.

To find the detection ability for the three types of EWMA$_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

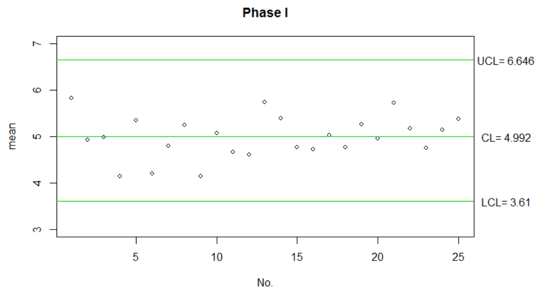

Figure 5-7. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=1) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=1, Figure 5-7 shows that no points are out of limits for in-control samples.

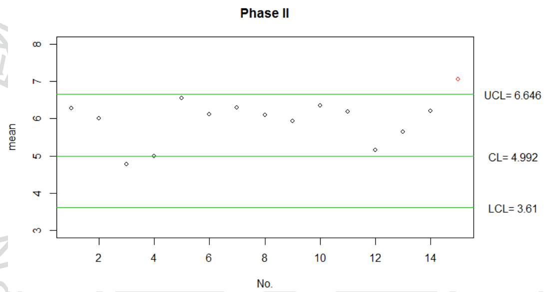

Figure 5-8. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=1, Figure 5-8 shows that No. 15 is out of limits; the first true alarm is on No. 15.

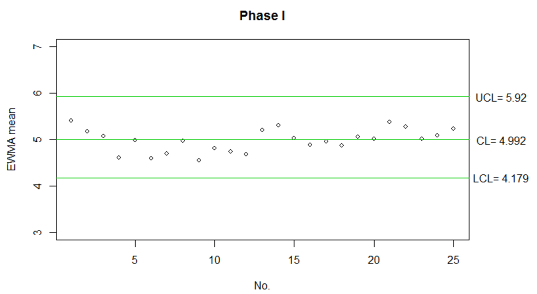

Figure 5-9. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.5) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=0.5, Figure 5-9 shows that no points are out of limits for in-control samples.

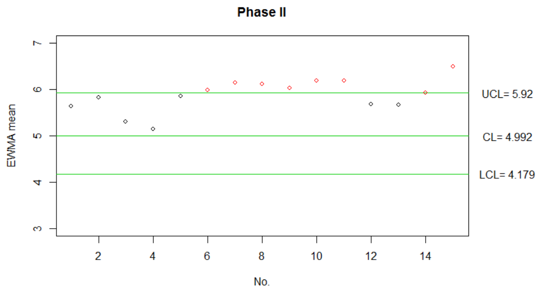

Figure 5-10. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.5) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=0.5, Figure 5-10 shows that No. 6 to No. 11, 14, and 15 are out of limits; the first true alarm is on No. 6.

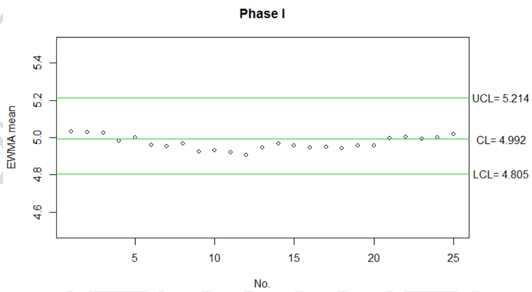

Figure 5-11. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.05) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=0.05, Figure 5-11 shows that no points are out of limits for in-control samples.

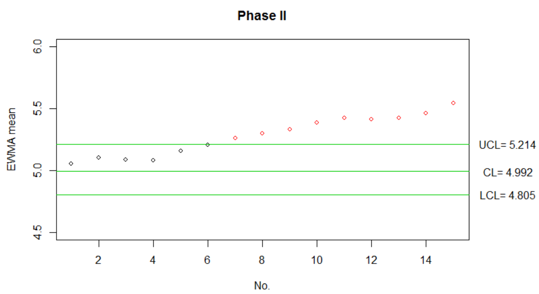

Figure 5-12. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.05) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=0.05, Figure 5-12 shows that No. 7 to No. 15 are out of limits; the first true alarm is on No. 7.

III. To compare the profit model with different λ, we adopt only moderate shifts in the mean of out-of-control gamma data.

Let a=42.45 and b=0.154, that is, the out-of-control mean and variance are 6.5 and 1, respectively, which meansδ_1=17.25 and δ_2=-0.046. We simulate 15 samples of size 4 data from the out-of-control gamma distribution with a=42.45 and b=0.154, as follows:

Table 5-6. Out-of-control Data with Gamma (a=42.45, b=0.154)

No.	simulation data with n=4				$\bar{X}$
1	5.211	6.206	8.77	6.833	6.755
2	6.232	6.273	7.556	6.781	6.71
3	7.759	6.392	4.876	6.144	6.293
4	8.373	7.25	6.74	5.986	7.087
5	5.014	7.998	5.377	5.547	5.984
6	7.65	5.385	5.822	6.622	6.37
7	7.911	6.213	10.067	6.721	7.728
8	7.416	5.252	6.553	6.646	6.467
9	6.259	7.502	5.828	7.118	6.677
10	4.747	5.193	7.432	6.134	5.876
11	5.911	7.173	6.807	6.404	6.574
12	7.335	6.388	6.383	6.643	6.687
13	6.699	8.889	5.93	5.889	6.852
14	6.41	4.786	5.894	7.576	6.167
15	6.871	6.969	7.435	5.815	6.773
					$\bar{\bar{X}} = 6.6$

With 60 data in Table 5-6, we estimate the parameters of out-of-control data, and obtain $\hat{a}_O = 41.331$, $\hat{b}_O = 0.16$, $\hat{\delta}_1 = 16.983$, $\hat{\delta}_2 = -0.045$, $\widehat{\text{Mean}} = 6.6$, and $\widehat{\text{Var}} = 1.05$. Hence, we have a 1.603 mean shift scale and a 1.017 s.d. shift scale.

According to Table 2-17, to minimize ARL_1, λ=0.6 is the best at n=4. Thus, we choose λ=1, 0.6, 0.05 and g=101, where the $EWMA_{X\text{-bar}}$ chart with λ=1 is the same as the X-bar probability chart. We compare the optimum results of profit model with λ=1, 0.6, and 0.05, as follows:

Table 5-7. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.6	0.05	1	0.6	0.05
L_1	3.272	3.211	2.743	3.272	3.211	2.743
L_2	2.732	2.772	2.303	2.732	2.772	2.303
h^*	0.5	0.5	0.5	0.5	0.5	0.5
ω^*	-	-	-	3.627	3.627	3.627
*USL**	-	-	-	8.66	8.66	8.66
Yield	-	-	-	0.998884	0.998884	0.998884
*EAP**	1569.48	**1622.3**	1436.81	1691.04	**1743.11**	1560.23
P_W	299.833	299.833	299.833	-	-	-
ARL_1	2.2	**1.78**	3.24	2.2	**1.78**	3.24
UCL	6.646	6.055	5.214	6.646	6.055	5.214
LCL	3.61	4.074	4.805	3.61	4.074	4.805
first true alarm on which sample	No.1 (8 outliers)	No.2 (14 outliers)	No.3 (13 outliers)	No.1 (8 outliers)	No.2 (14 outliers)	No.3 (13 outliers)

According to Table 5-7, with and without inspection, h^*, $EWMA_{X\text{-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 8.6% profit per unit time when we have an inspection.

(2) If λ=0.6, we increase 7.4% profit per unit time when we have an inspection.

(3) If λ=1, we increase 7.7% profit per unit time when we have an inspection.

If we use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.6, we have the largest *EAP** and smallest ARL_1 for the only moderate shifts in the mean. Therefore, we suggest that the producer takes inspection with *USL**=8.66, use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.6 and take four samples every 0.5 unit time.

To find the detection ability for the three types of EWMA$_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

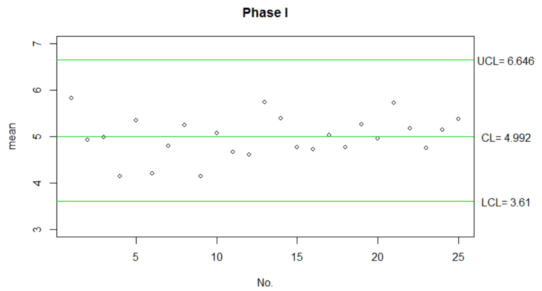

Figure 5-13. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=1) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=1, Figure 5-13 shows that no points are out of limits for in-control samples.

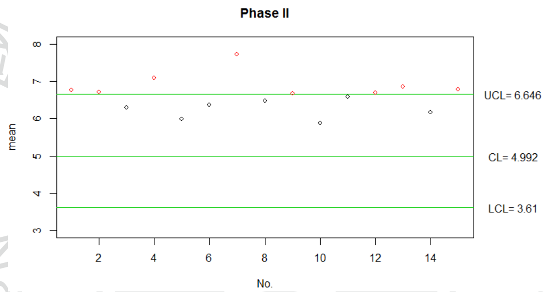

Figure 5-14. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=1, Figure 5-14 shows that No. 1, 2, 4, 7, 9, 12, 13, and 15 are out of limits; the first true alarm is on No. 1.

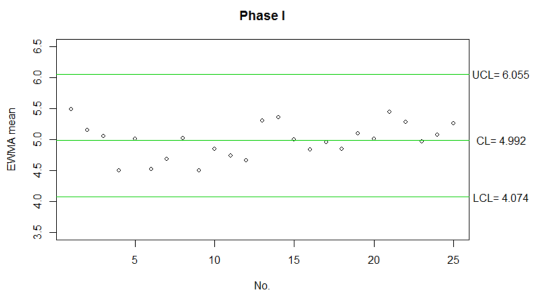

Figure 5-15. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.6) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=0.6, Figure 5-15 shows that no points are out of limits for in-control samples.

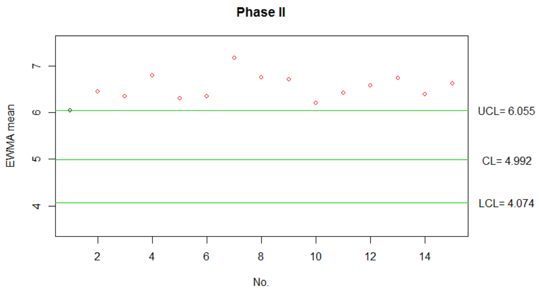

Figure 5-16. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.6) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=0.6, Figure 5-16 shows that No. 2 to No. 15 are out of limits; the first true alarm is on No. 2.

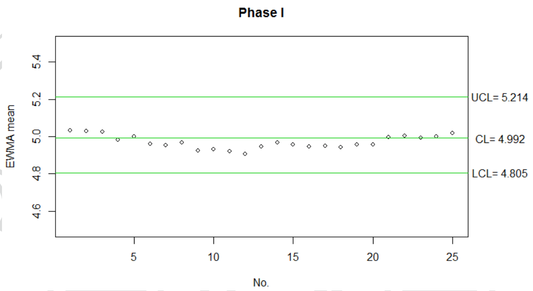

Figure 5-17. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.05) with In-control Data

For EWMA$_{X\text{-bar}}$ chart with λ=0.05, Figure 5-17 shows that no points are out of limits for in-control samples.

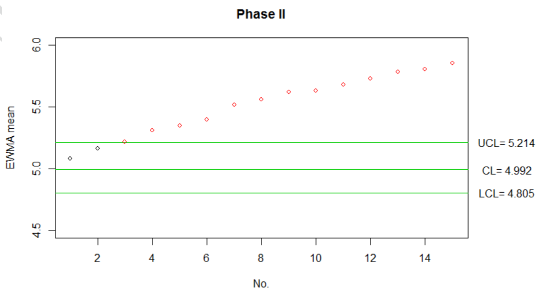

Figure 5-18. The Economic EWMA$_{X\text{-bar}}$ Chart (λ=0.05) with Out-of-control Data

Plots the out-of-control statistics on the EWMA$_{X\text{-bar}}$ Chart with λ=0.05, Figure 5-18 shows that No. 3 to No. 15 are out of limits; the first true alarm is on No. 3.

IV. To compare the profit model with different λ, we adopt only moderate shifts in the variance of out-of-control gamma data.

Let a=15.385 and b=0.325, that is, the out-of-control mean and variance are 5 and 1.625, respectively, which means δ_1=-9.615 and δ_2=0.125. We simulate 15 samples of size 4 data from the out-of-control gamma distribution with a=15.385 and b=0.325, as follows:

Table 5-8. Out-of-control Data with Gamma (a=15.385, b=0.325)

No.	simulation data with n=4				$\overline{X}$
1	5.783	4.26	7.003	4.471	5.379
2	5.485	4.479	7.158	3.814	5.234
3	4.143	7.228	7.417	4.916	5.926
4	6.487	5.547	2.772	4.822	4.907
5	5.967	4.88	5.348	3.423	4.904
6	4.063	3.395	5.573	2.929	3.99
7	5.252	4.555	6.222	4.037	5.016
8	3.769	4.488	2.513	4.25	3.755
9	4.227	7.095	4.826	5.706	5.463
10	4.821	4.497	6.032	4.128	4.87
11	5.136	4.276	7.218	5.864	5.624
12	4.27	4.185	7.084	5.688	5.307
13	6.958	3.505	6.282	4.767	5.378
14	4.593	3.912	4.769	7.98	5.313
15	5.699	7.59	4.129	5.879	5.824
					$\overline{\overline{X}} = 5.126$

With 60 data in Table 5-8, we estimate the parameters of out-of-control data, and obtain $\hat{a}_O = 15.608$, $\hat{b}_O = 0.328$, $\hat{\delta}_1 = -8.741$, $\hat{\delta}_2 = 0.123$, $\widehat{\text{Mean}} = 5.126$, and $\widehat{\text{Var}} = 1.684$. Hence, we have a 0.126 mean shift scale and a 1.281 s.d. shift scale.

According to Table 2-17, to minimize ARL_1, λ=0.3 is the best at n=4. Thus, we choose λ=1, 0.3, 0.05 and g=101, where the $EWMA_{X\text{-bar}}$ chart with λ=1 is the same as the X-bar probability chart. We compare the optimum results of profit model with λ=1, 0.3, and 0.05,, as follows:

Table 5-9. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.3	0.05	1	0.3	0.05
L_1	3.272	3.071	2.743	3.272	3.071	2.743
L_2	2.732	2.784	2.303	2.732	2.784	2.303
h^*	0.5	0.5	0.5	0.5	0.5	0.5
ω^*	-	-	-	3.627	3.627	3.627
USL^*	-	-	-	8.66	8.66	8.66
Yield	-	-	-	0.998884	0.998884	0.998884
EAP^*	6800.75	**6828.18**	6725.84	6895.95	**6921.99**	6824.86
P_W	299.833	299.833	299.833	-	-	-
ARL_1	51.4	**48.56**	59.45	51.4	**48.56**	59.45
UCL	6.646	5.644	5.214	6.646	5.644	5.214
LCL	3.61	4.4	4.805	3.61	4.4	4.805
first true alarm on which sample	No true alarm	No true alarm	No true alarm	No true alarm	No true alarm	No true alarm

According to Table 5-9, with and without inspection, h^*, $EWMA_{X\text{-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 1.5% profit per unit time when we have an inspection.
(2) If λ=0.3, we increase 1.4% profit per unit time when we have an inspection.
(3) If λ=1, we increase 1.4% profit per unit time when we have an inspection.

If we use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.3, we have the largest EAP^* and smallest ARL_1 for the only moderate shifts in the variance. Therefore, we suggest that the producer takes inspection with USL^*=8.66, use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.3 and take four samples every 0.5 unit time.

To find the detection ability for the three types of $EWMA_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

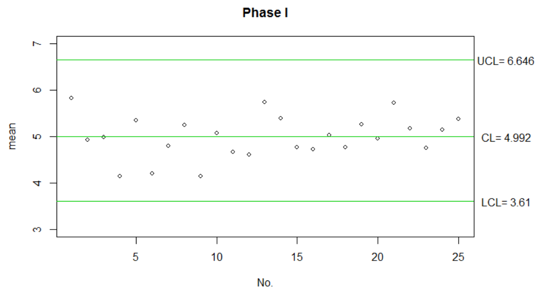

Figure 5-19. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=1, Figure 5-19 shows that no points are out of limits for in-control samples.

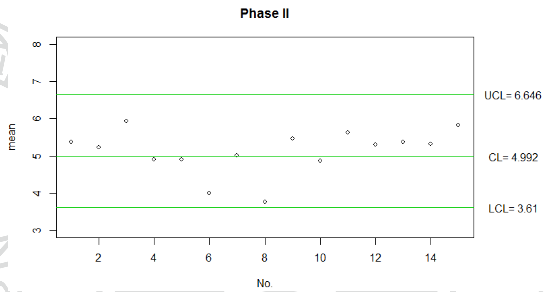

Figure 5-20. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=1, Figure 5-20 shows that no points are out of limits; it has no true alarm.

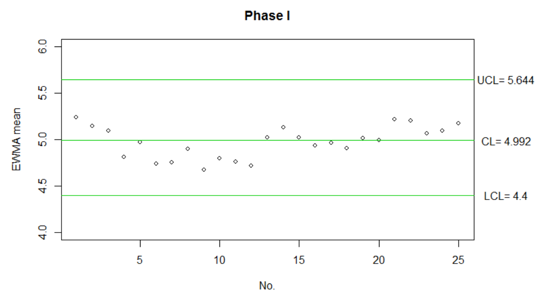

Figure 5-21. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.3) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=0.3, Figure 5-21 shows that no points are out of limits for in-control samples.

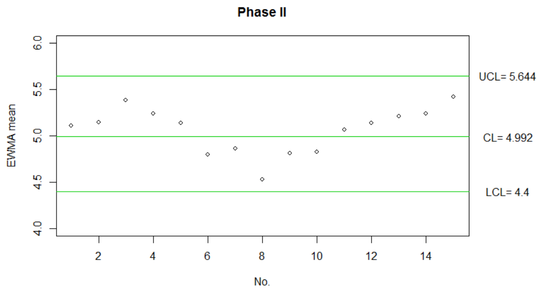

Figure 5-22. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.3) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.3, Figure 5-22 shows that no points are out of limits; it has no true alarm.

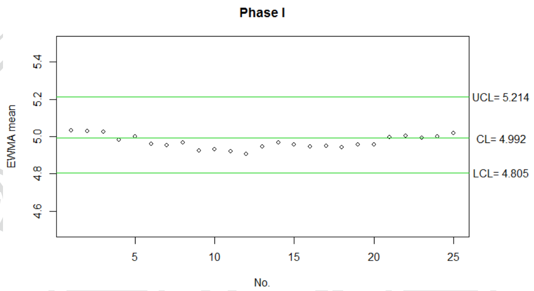

Figure 5-23. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.05) with In-control Data

For $EWMA_{X\text{-}bar}$ chart with λ=0.05, Figure 5-23 shows that no points are out of limits for in-control samples.

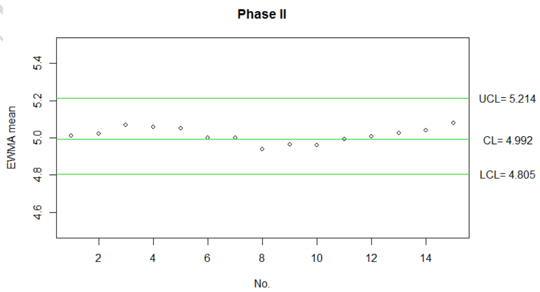

Figure 5-24. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.05) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.05, Figure 5-24 shows that no points are out of limits; it has no true alarm.

V. To compare the profit model with different λ, we adopt only small shifts in the variance of out-of-control gamma data.

Let a=15.385 and b=0.325, that is, the out-of-control mean and variance are 5 and 1.625, respectively, which means δ_1=-9.615 and δ_2=0.125. We simulate 15 samples of size 4 data from the out-of-control gamma distribution with a=15.385 and b=0.325, as follows:

Table 5-10. Out-of-control Data with Gamma (a=15.385, b=0.325)

No.	simulation data with n=4				$\overline{X}$
1	3.461	5.507	6.388	3.679	4.759
2	3.152	3.912	3.297	4.57	3.733
3	4.175	3.619	3.699	5.961	4.363
4	5.62	5.381	6.398	3.62	5.255
5	4.68	3.814	5.188	5.141	4.706
6	3.949	5.531	6.154	5.891	5.381
7	5.054	4.684	6.065	5.079	5.22
8	6.155	3.538	3.748	4.434	4.469
9	7.259	5.163	5.59	5.38	5.848
10	6.283	6.908	5.931	6.023	6.287
11	4.941	4.361	6.621	3.688	4.903
12	4.891	5.098	5.064	4.271	4.831
13	4.771	5.773	7.21	4.154	5.477
14	4.146	6.678	4.142	4.289	4.814
15	5.892	5.977	5.689	4.045	5.401
					$\overline{\overline{X}} = 5.03$

With 60 data in Table 5-10, we estimate the parameters of out-of-control data, and obtain $\hat{a}_O = 22.087$, $\hat{b}_O = 0.228$, $\hat{\delta}_1 = -2.261$, $\hat{\delta}_2 = 0.023$, $\widehat{\text{Mean}} = 5.03$, and $\widehat{\text{Var}} = 1.145$. Hence, we have a 0.039 mean shift scale and a 1.058 s.d. shift scale.

We choose λ=1, 0.2, 0.05 and g=101, where the $EWMA_{X\text{-bar}}$ chart with λ=1 is the same as the X-bar probability chart. We compare the optimum results of profit model with λ=1, 0.2, and 0.05, as follows:

Table 5-11. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.2	0.05	1	0.2	0.05
L_1	3.272	2.976	2.743	3.272	2.976	2.743
L_2	2.732	2.747	2.303	2.732	2.747	2.303
h^*	0.5	0.5	0.5	0.5	0.5	0.5
ω^*	-	-	-	3.627	3.627	3.627
USL^*	-	-	-	8.66	8.66	8.66
Yield	-	-	-	0.998884	0.998884	0.998884
EAP^*	7488.64	**7491.52**	7456.01	7528.7	**7531.53**	7496.61
P_W	299.833	299.833	299.833	-	-	-
ARL_1	223.6	**220.6**	260.48	223.6	**220.6**	260.48
UCL	6.646	5.493	5.214	6.646	5.493	5.214
LCL	3.61	4.528	4.805	3.61	4.528	4.805
first true alarm on which sample	No true alarm	No true alarm	No true alarm	No true alarm	No true alarm	No true alarm

According to Table 5-11, with and without inspection, h^*, $EWMA_{X\text{-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.05, we increase 0.5% profit per unit time when we have an inspection.
(2) If λ=0.2, we increase 0.5% profit per unit time when we have an inspection.
(3) If λ=1, we increase 0.5% profit per unit time when we have an inspection.

If we use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.2, we have the largest EAP^* and smallest ARL_1 for the only small shifts in the variance. Therefore, we suggest that the producer takes inspection with USL^*=8.66, use the economic $EWMA_{X\text{-bar}}$ chart with λ=0.2 and take four samples every 0.5 unit time.

To find the detection ability for the three types of $EWMA_{X\text{-}bar}$ chart, we plot the in-control and out-of-control statistics on them.

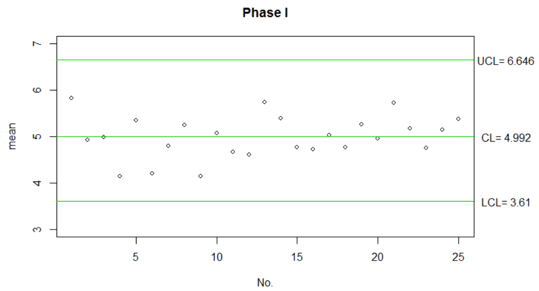

Figure 5-25. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=1) with In-control Data

For $EWMA_{X\text{-}bar}$ chart with λ=1, Figure 5-25 shows that no points are out of limits for in-control samples.

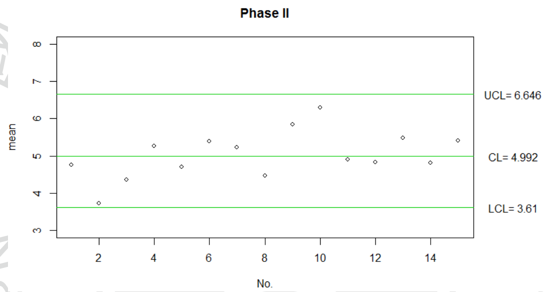

Figure 5-26. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=1, Figure 5-26 shows that no points are out of limits; it has no true alarm.

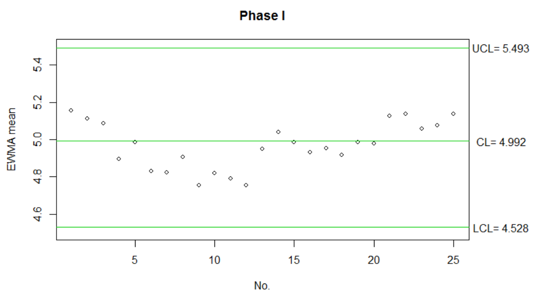

Figure 5-27. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.2) with In-control Data

For $EWMA_{X\text{-}bar}$ chart with λ=0.2, Figure 5-27 shows that no points are out of limits for in-control samples.

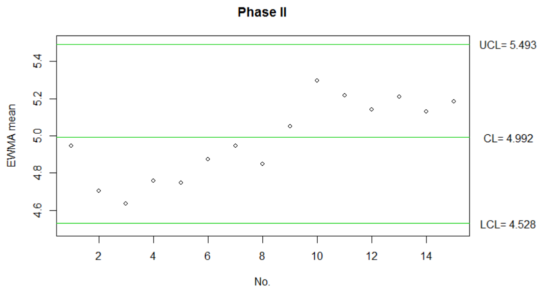

Figure 5-28. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.2) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.2, Figure 5-28 shows that no points are out of limits; it has no true alarm.

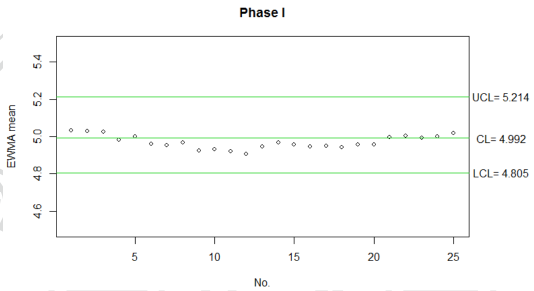

Figure 5-29. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.05) with In-control Data

For $EWMA_{X\text{-}bar}$ chart with λ=0.05, Figure 5-29 shows that no points are out of limits for in-control samples.

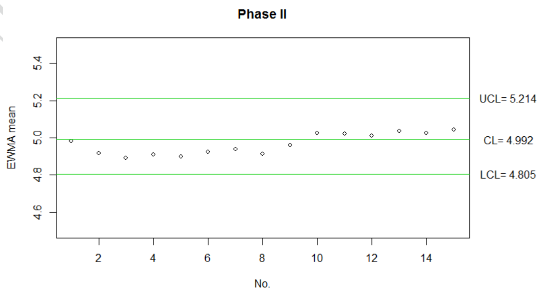

Figure 5-30. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.05) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.05, Figure 5-30 shows that no points are out of limits; it has no true alarm.

VI. Comparing the profit model with different λ using an example of service time data.

We consider a quality variable with an exponential distribution, and take the service time data from Yang et al. (2012). The service time is an important quality characteristic for a bank branch in Taiwan. To measure the efficiency in the service system of a bank branch, the sampling service times (in minutes) are measured from 10 counters every 2days for 30days; that is, 15 samples of size n=10 are taken from an in-control service system. These data have been analyzed and have a right-skewed distribution, as shown in Table 5-12.

Table 5-12. In-control Service Time Data.

No.	In-control service data with n=10										$\bar{X}$
1	0.88	0.78	5.06	5.45	2.93	6.11	11.59	1.2	0.89	3.21	3.81
2	3.82	13.4	5.16	3.2	32.27	3.68	3.14	1.58	2.72	7.71	7.67
3	1.4	3.89	10.88	30.85	0.54	8.4	5.1	2.63	9.17	3.94	7.68
4	16.8	8.77	8.36	3.55	7.76	1.81	1.11	5.91	8.26	7.19	6.95
5	0.24	9.57	0.66	1.15	2.34	0.57	8.94	5.54	11.69	6.58	4.73
6	4.21	8.73	11.44	2.89	19.49	1.2	8.01	6.19	7.48	0.07	6.97
7	15.08	7.43	4.31	6.14	10.37	2.33	1.97	1.08	4.27	14.08	6.71
8	13.89	0.3	3.21	11.32	9.9	4.39	10.5	1.7	10.74	1.46	6.74
9	0.03	12.76	2.41	7.41	1.67	3.7	4.31	2.45	3.57	3.33	4.16
10	12.89	17.96	2.78	3.21	1.12	12.61	4.23	6.18	2.33	6.92	7.02
11	7.71	1.05	1.11	0.22	3.53	0.81	0.41	3.73	0.08	2.55	2.12
12	5.81	6.29	3.46	2.66	4.02	10.95	1.59	5.58	0.55	4.1	4.50
13	2.89	1.61	1.3	2.58	18.65	10.77	18.23	3.13	3.38	6.34	6.89
14	1.36	1.92	0.12	11.08	8.85	3.99	4.32	1.71	1.77	1.94	3.71
15	21.52	0.63	8.54	3.37	6.94	3.44	3.37	6.37	1.28	12.83	6.83
											$\bar{\bar{X}} = 5.766$

Since the 150 in-control data follows exponential distribution, we estimate the parameters of the exponential distribution, and obtain $\hat{b}_I = 5.766$, $\widehat{\text{Mean}} = 5.766$, and $\widehat{\text{Var}} = 33.244$. We use the routine "ks.test" to test in-control data with Kolmogorov-Smirnov test method and have a p-value = 0.6714; therefore, we do not reject the data drawn from the exponential distribution with b_I=5.766.

The new data set of service times from a new automatic service system of the bank branch, 10 new samples of size 10, were collected and listed in Table 5-13.

The out-of-control service time data are as follows:

Table 5-13. Out-of -control Service Time Data.

No.	Out-of-control service data with *n*=10										$\bar{X}$
1	3.54	0.01	1.33	7.27	5.52	0.09	1.84	1.04	2.91	0.63	2.42
2	0.86	1.61	1.15	0.96	0.54	3.05	4.11	0.63	2.37	0.05	1.53
3	1.45	0.19	4.18	0.18	0.02	0.7	0.8	0.97	3.6	2.94	1.50
4	1.37	0.14	1.54	1.58	0.45	6.01	4.59	1.74	3.92	4.82	2.62
5	3	2.46	0.06	1.8	3.25	2.13	2.22	1.37	2.13	0.25	1.87
6	1.59	3.88	0.39	0.54	1.58	1.7	0.68	1.25	6.83	0.31	1.88
7	5.01	1.85	3.1	1	0.09	1.16	2.69	2.79	1.84	2.62	2.22
8	4.96	0.55	1.43	4.12	4.06	1.42	1.43	0.86	0.67	0.13	1.96
9	1.08	0.65	0.91	0.88	2.02	2.88	1.76	2.87	1.97	0.62	1.56
10	4.56	0.44	5.61	2.79	1.73	2.46	0.53	1.73	7.02	2.13	2.90
											$\bar{\bar{X}} = 2.045$

With 100 data in Table 5-13, we estimate the parameters of out-of-control data, and obtain $\hat{b}_O = 2.045$, $\hat{\delta}_2 = -3.72$, $\widehat{\text{Mean}} = 2.045$, and $\widehat{\text{Var.}} = 4.184$. Hence, we have a -0.645 small mean shift scale and a 0.355 small s.d. shift scale. During calculation, the out-of-control mean and variance are smaller than the in-control mean and variance.

We use the routine "ks.test" to test out-of-control data and we have a *p*-value=0.4182; therefore, we do not reject the data drawn from the exponential distribution with b_O=2.045.

We let a_I =1, n=10, θ=0.01, e=0.05, D=20, T=250, s_0=5, s_1=0.1, W=500, k_c=10, A=600, IC=0.1, P_C=300, P_U=150, and R=200. If the producer decides not to inspect, we maximize *EAP* (Equation 3-12) to determine the optimum h^*, subject to $0.5 \leqq h \leqq 8$. If the producer decides to inspect, we maximize *EAP* (Equation 4-7) to determine the optimum h^* and ω^*, subject to $0.5 \leqq h \leqq 8$ and $2 \leqq \omega$.

We choose λ=1, 0.4, and 0.1 and g=101, where the EWMA$_{X\text{-bar}}$ chart with λ=1 is the same as the X-bar probability chart. We compare the optimum results of profit model with λ=1, 0.4, and 0.1, as follows:

Table 5-14. The Optimum Results Comparison of Profit Model with Different λ

Inspection	Without			With		
λ	1	0.4	0.1	1	0.4	0.1
L_1	3.85	3.517	2.902	3.85	3.517	2.902
L_2	2.187	2.451	2.506	2.187	2.451	2.506
h^*	8	8	8	8	8	8
ω^*	-	-	-	2	2	2
USL^*	-	-	-	17.297	17.297	17.297
Yield	-	-	-	0.950213	0.950213	0.950213
EAP^*	-42444.27	-43125.13	**-36886.39**	-5577.36	-5973.12	**-2346.79**
P_W	292.532	292.532	292.532	-	-	-
ARL_1	2.69	**2.55**	3.9	2.69	**2.55**	3.9
UCL	12.786	8.972	6.98	12.786	8.972	6.98
LCL	1.778	3.531	4.718	1.778	3.531	4.718
first true alarm on which sample	No.2 (3 outliers)	No.2 (9 outliers)	No.3 (8 outliers)	No.2 (3 outliers)	No.2 (9 outliers)	No.3 (8 outliers)

According to Table 5-14, with and without inspection, h^*, EWMA$_{X\text{-bar}}$ chart, and ARL_1 are the same at each λ. However, with inspection, we increased the profit per unit time as follows:

(1) If λ=0.1, we increase 93.6% profit per unit time when we have an inspection.
(2) If λ=0.4, we increase 86.1% profit per unit time when we have an inspection.
(3) If λ=1, we increase 86.9% profit per unit time when we have an inspection.

If we use the economic EWMA$_{X\text{-bar}}$ chart with λ=0.1, we have the largest EAP^*, but largest ARL_1. If we use the economic EWMA$_{X\text{-bar}}$ chart with λ=0.4, we have the smallest EAP^*, but smallest ARL_1. To maximize EAP^* for small shifts in the mean and variance we suggest that the producer takes inspection with USL^*=17.297, use the economic EWMA$_{X\text{-bar}}$ chart with λ=0.1 and take 10 samples every 8 unit time.

To find the detection ability for the three types of $EWMA_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

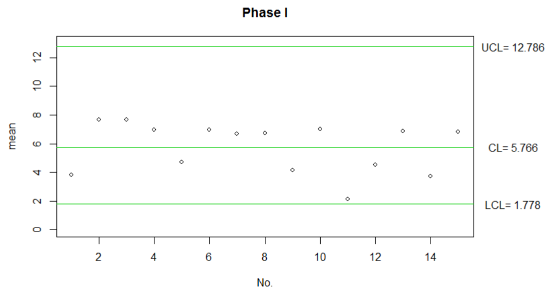

Figure 5-31. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=1, Figure 5-31 shows that no points are out of limits for in-control samples.

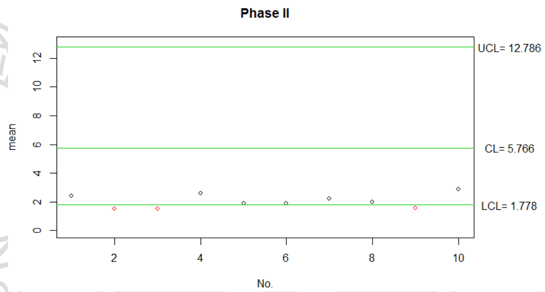

Figure 5-32. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=1) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=1, Figure 5-32 shows that No. 2, 3, and 9 are out of limits; the first true alarm is on No. 2.

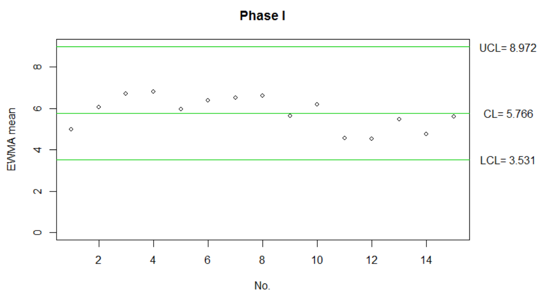

Figure 5-33. The Economic $EWMA_{X\text{-bar}}$ Chart (λ=0.4) with In-control Data

For $EWMA_{X\text{-bar}}$ chart with λ=0.4, Figure 5-33 shows that no points are out of limits for in-control samples.

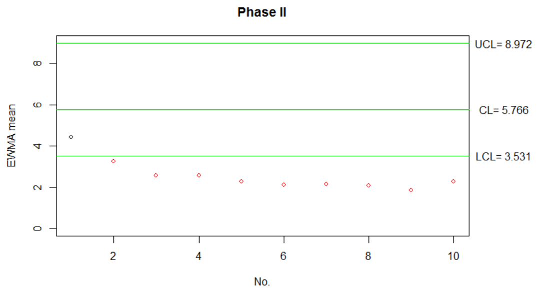

Figure 5-34. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.4) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.4, Figure 5-34 shows that No. 2 to No. 10 are out of limits; the first true alarm is on No. 2.

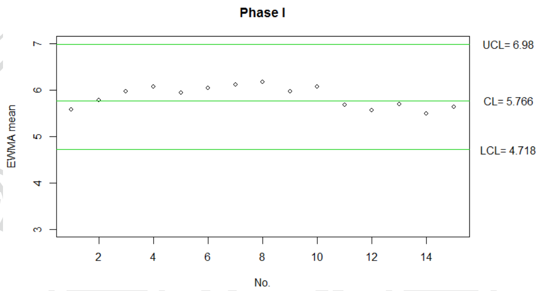

Figure 5-35. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.1) with In-control Data

For $EWMA_{X\text{-}bar}$ chart with λ=0.1, Figure 5-35 shows that no points are out of limits for in-control samples.

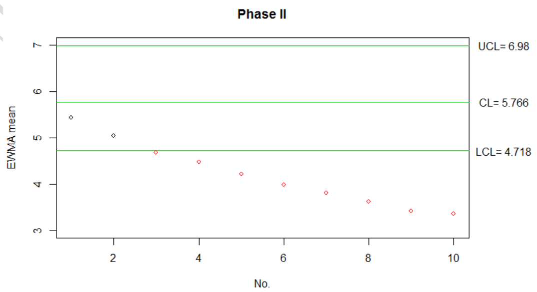

Figure 5-36. The Economic $EWMA_{X\text{-}bar}$ Chart (λ=0.1) with Out-of-control Data

Plots the out-of-control statistics on the $EWMA_{X\text{-}bar}$ Chart with λ=0.1, Figure 5-36 shows that No. 3 to No. 10 are out of limits; the first true alarm is on No. 3.

5.2 Performance Comparison of Six Numerical Examples

According to Section 5.1, we have five numerical examples of gamma distribution and one exponential numerical example. In each numerical example, we choose three λ to determine the optimum results and compare them with each other. In Table 5-15, we summarize all six numerical examples as follows:

Table 5-15. Comparison of Six Numerical Examples

$\hat{a}_I$	$\hat{b}_I$	$\hat{\delta}_1$	$\hat{\delta}_2$	mean shift scale	s.d. shift scale	λ	Summary
24.349	0.205	0.919	0.006	1.685	1.317	0.05 (λ is arbitrarily choose) 0.6 (The best λ in table 2-17) 1 (Reduce to X-bar probability chart)	For max. *EAP** and min. ARL_1, λ=0.6 is the best, but λ=0.05 is the worst.
24.349	0.205	6.534	-0.012	0.974	1.063	0.05 (λ is arbitrarily choose) 0.5 (λ is arbitrarily choose) 1 (Reduce to X-bar probability chart)	For max. *EAP** and min. ARL_1, λ=0.5 is the best, but λ=1 is the worst.
24.349	0.205	16.983	-0.045	1.603	1.017	0.05 (λ is arbitrarily choose) 0.6 (The best λ in table 2-17) 1 (Reduce to X-bar probability chart)	For max. *EAP** and min. ARL_1, λ=0.6 is the best, but λ=0.05 is the worst.
24.349	0.205	-8.741	0.123	0.126	1.281	0.05 (λ is arbitrarily choose) 0.3 (The best λ in table 2-17) 1 (Reduce to X-bar probability chart)	For max. *EAP** and min. ARL_1, λ=0.3 is the best, but λ=0.05 is the worst.
24.349	0.205	-2.261	0.023	0.039	1.058	0.05 (λ is arbitrarily choose) 0.2 (λ is arbitrarily choose) 1 (Reduce to X-bar probability chart)	For max. *EAP** and min. ARL_1, λ=0.2 is the best, but λ=0.05 is the worst.
1	5.766	0	-3.72	-0.645	0.355	0.1 (λ is arbitrarily choose) 0.4 (λ is arbitrarily choose) 1 (Reduce to X-bar probability chart)	For max. *EAP**, λ=0.1 is the best, but λ=0.4 is the worst. For min. ARL_1, λ=0.4 is the best, but λ=0.1 is the worst.

According to Table 5-15 and Table 2-17, we use the best λ to obtain the largest EAP^* and smallest ARL_1 in three of the six numerical examples. However, in the example of service time data, the choice λ, could not simultaneously obtain the largest EAP^* and smallest ARL_1.

The best λ becomes smaller when the mean shift scale becomes smaller. The result is reasonable.

CHAPTER 6. DETERMINING THE OPTIMUM PRODUCER INSPECTION AND THE ECONOMIC VSI EWMA$_{\text{X-bar}}$ CONTROL CHART

6.1 VSI EWMA$_{\text{X-bar}}$ Control Chart and *ATS* Calculation

In this chapter, we consider the variable sampling interval (VSI) EWMA$_{\text{X-bar}}$ control chart, which partitions the region between the two control limits into two sub-regions as follows:

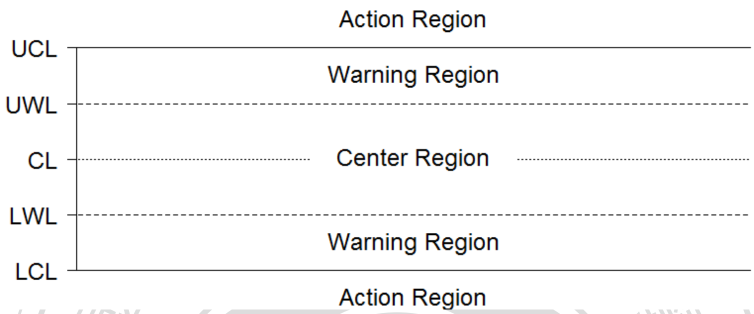

Figure 6-1. VSI Control Chart

The VSI EWMA$_{\text{X-bar}}$ control chart based on the sampling distribution is

$$UCL = a_I b_I + L_1 \sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

$$UWL = a_I b_I + W_1 \sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

$$CL = a_I b_I$$

$$LWL = a_I b_I - W_2 \sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

$$LCL = a_I b_I - L_2 \sqrt{\frac{\lambda}{2-\lambda}\frac{a_I b_I^2}{n}}$$

where $0 \leqq W_1 \leqq L_1$ and $0 \leqq W_2 \leqq L_2$.

When the last sample point falls within the warning region, take the next sample after h_1 unit time; and when the last sample point falls within the center region, take the next sample after h_2 unit time, where $h_1 \leqq h_2$.

To measure the performance of the economic VSI $\text{EWMA}_{\text{X-bar}}$ control chart, we calculate ATS_1 as follows:

Step1. Divide the interval between the upper and lower control limits into g=2m+1, the number of states, subintervals of width 2δ, where $\delta = \dfrac{UCL - LCL}{2g}$.

Step2. Define state j=($S_j - \delta$, $S_j + \delta$) , j=-m,…-1,0,1,…,m, and S_j as the midpoint for the j-th interval.

Step3. The statistic $Z_{t,j}$ is in transient state j at time t, if

$$S_j - \delta < Z_{t,j} \le S_j + \delta \;\; for \; -m \le j \le m$$

Step4. The transition probability matrix for the transient state calculated by out-of-control gamma distribution is

$$R_O = [p_{t-1,t}(jk)]$$

where
$$\begin{aligned}
p_{t-1,t}(jk) &= P(S_k - \delta < Z_{t,k} \le S_k + \delta \mid S_j - \delta < Z_{t-1,j} \le S_j + \delta) \\
&= P(S_k - \delta < Z_{t,k} \le S_k + \delta \mid Z_{t-1,j} = S_j) \\
&= P(S_k - \delta < \lambda \overline{X}_{t,k} + (1-\lambda) Z_{t-1,k} \le S_k + \delta \mid Z_{t-1,j} = S_j) \\
&= P\left(\frac{(S_k - \delta) - (1-\lambda) S_j}{\lambda} < \overline{X}_t \le \frac{(S_k + \delta) - (1-\lambda) S_j}{\lambda}\right) \\
&= \frac{r\left(na_O, \frac{n}{b_O} \frac{(S_k + \delta) - (1-\lambda) S_j}{\lambda}\right)}{\Gamma(na_O)} - \frac{r\left(na_O, \frac{n}{b_O} \frac{(S_k - \delta) - (1-\lambda) S_j}{\lambda}\right)}{\Gamma(na_O)}
\end{aligned}$$

Step5. Assume that the process begins from state 0; thus, $p_{zs} = (\overbrace{0,0,\ldots,0}^{m},1,\overbrace{0,\ldots,0,0}^{m})$.

Step6. Calculate Zero-state ATS_1

$$ATS_1 = p_{zs}^T (I - R_O)^{-1} \vec{h} \tag{6-1}$$

where R_O is the transition probability matrix calculated by out-of-control gamma distribution, I is the g*g dimension identity matrix, and h_j, the element of $\vec{h}$, is defined as follows:

$$h_j = \begin{cases} h_1 & \text{if } S_j \in [UWL, UCL] \cup [LCL, LWL] \\ h_2 & \text{if } S_j \in (LWL, UWL) \end{cases}$$

6.2 Derivation of the Profit Model without Producer Inspection

For expected cycle time, because the sampling interval is not fixed, again we use ATS_1, which is Equation 6-1, instead of $h/(1-\beta)-\tau$ in Equation 3-3.

Hence, the expected cycle time is

$$ET = \frac{1}{\theta} + ATS_1 + en + D \tag{6-2}$$

where ATS_1 is calculated using Equation 6-1.

For expected cycle profit, because the sampling interval is not fixed, we use ARL_0 instead of $1/\theta h$ and use (ARL_0+ARL_1) instead of ET/h in Equation 3-11.

Hence, the expected cycle profit is

$$EP = EP_I \frac{1}{\theta} - T + EP_O * ATS_1 - (s_0 + s_1 n)(ARL_0 + ARL_1) - W \tag{6-3}$$

where ARL_0, ARL_1, EP_I, EP_O, and ATS_1 are calculated using Equation 2-3, 2-4, 3-9, 3-10, and 6-1, respectively.

Therefore, the expected profit per unit time is

$$EAP = \frac{EP}{ET} \tag{6-4}$$

where ET is calculated using Equation 6-2 and EP is calculated using Equation 6-3.

6.3 Derivation of the Profit Model with Producer Inspection

Similarly, for expected cycle time, we have the same reason in Section 6.2 and the same formula of expected cycle time, with or without inspection.

Hence, the same as Equation 6-2, the expected cycle time is

$$ET = \frac{1}{\theta} + ATS_1 + en + D \tag{6-5}$$

For expected cycle profit, we have the same reason in Section 6.2 and a similar formula of expected cycle profit with or without inspection, except for the calculation of EP_I and EP_O.

Hence, the expected cycle profit is

$$EP = EP_I \frac{1}{\theta} - T + EP_O * ATS_1 - (s_0 + s_1 n)(ARL_0 + ARL_1) - W \qquad (6\text{-}6)$$

where ARL_0, ARL_1, EP_I, EP_O, and ATS_1 are calculated using Equation 2-3, 2-4, 4-4, 4-5, and 6-1, respectively.

Therefore, the expected profit per unit time is

$$EAP = \frac{EP}{ET} \qquad (6\text{-}7)$$

where ET is calculated using Equation 6-5 and EP is calculated using Equation 6-6.

6.4 Determining Optimum Parameters of the Economic VSI EWMA$_{\text{X-bar}}$ Control Chart with and without producer tolerance

The procedures to determine n, warning limits, control limits, and ATS_1 of the economic VSI EWMA$_{\text{X-bar}}$ control chart without producer inspection is as follows:

Step1. Let n=2.

Step2. Determine the UCL coefficient (L_1) of the EWMA$_{\text{X-bar}}$ control chart. With λ, let LCL=0 and $ATS_0 = p_{zs}^T (I - R_I)^{-1} \vec{h}_0 = 740$ to solve L_1 using the routine "uniroot" in the R program, where $\vec{h}_0$ is the $g*1$ dimension vector with all components are h_0, which is the FSI sampling time. Hence, UCL is determined.

Step3. Determine the LCL coefficient (L_2) of the EWMA$_{\text{X-bar}}$ control chart. With UCL, let $ATS_0 = p_{zs}^T (I - R_I)^{-1} \vec{h}_0 = 370$ to solve L_2 using the routine "uniroot" in the R program.

Step4. With UCL and LCL, from $ATS_0 = p_{zs}^T (I - R_I)^{-1} \vec{h} = 370$, W_2 is a function of W_1. To determine optimum W_1, we use the routine "DEoptim" of the R program to maximize EAP in Equation 6-4, subject to $0 \leqq W_1 \leqq L_1$ with specified parameters.

Hence, the coefficient of the upper warning control limits (*UWL*) of the $EWMA_{X\text{-}bar}$ control chart is determined.

If $EAP(n+1)$ is greater than $EAP(n)$, then we choose $EAP(n+1)$ to become EAP^*.

Step5. With W_1, let $ATS_0 = p_{zs}^T (I - R_I)^{-1} \vec{h} = 370$ to solve W_2 using the routine "uniroot" in the R program. Hence, lower warning control limits (*LWL*) of the $EWMA_{X\text{-}bar}$ control chart is determined.

The economic VSI $EWMA_{X\text{-}bar}$ control chart is then constructed.

Step6. Let n=n+1, 3≦n≦25. Proceed Step2.

The procedures to determine n, ω^*, warning limits, control limits, and ATS_1 of the economic VSI $EWMA_{X\text{-}bar}$ control chart with producer inspection is instead Step 4 of the previous procedure as follows:

Step4. With *UCL* and *LCL*, from $ATS_0 = p_{zs}^T (I - R_I)^{-1} \vec{h} = 370$, W_2 is a function of W_1. To determine optimum ω and W_1, we use the routine "DEoptim" of the R program to maximize *EAP* in Equation 6-7, subject to $2 \leqq \omega$ and $0 \leqq W_1 \leqq L_1$ with specified parameters.

If we let $h_1=h_2=h_0$, $W_1=0$, and $W_2=0$, then the VSI $EWMA_{X\text{-}bar}$ control chart becomes the FSI $EWMA_{X\text{-}bar}$ control chart.

6.5 Two Numerical Examples and the Results Comparison with the FSI $EWMA_{X\text{-}bar}$ Control Chart

For the first numerical example, we use the procedures in Section 6.4 with the same specified parameters in Section 3.4 and choose $\lambda=0.05$, $h_0=1$, $h_1=0.5$, and $h_2=2$ to compare the optimum results of the VSI $EWMA_{X\text{-}bar}$ control chart with the FSI $EWMA_{X\text{-}bar}$ control chart, as follows:

Table 6-1. Comparison of the optimum results of VSI and FSI $EWMA_{X\text{-}bar}$ control charts

Inspection	Without		With	
Chart	FSI	VSI	FSI	VSI
L_1	2.604	2.604	2.604	2.604
L_2	2.387	2.387	2.387	2.387
W_1	-	0.002	-	0.002
W_2	-	0.929	-	0.929
$n*$	25	25	25	25
$\omega*$	-	-	2.311	2.311
*USL**			8.66	8.66
Yield			0.965834	0.965834
*EAP**	27428.1	27677.6	31998.4	**32190**
ATS_1	22.929	13.119	22.929	**13.119**
*UCL**	3.2043	3.2043	3.2043	3.2043
*UWL**	-	3.0002	-	3.0002
*LWL**	-	2.9271	-	2.9271
*LCL**	2.8127	2.8127	2.8127	2.8127

According to Table 6-1, $n*$ and $\omega*$ are the same as in the model with inspection, but different FSI and VSI $EWMA_{X\text{-}bar}$ charts. We have the largest *EAP** and the smallest ATS_1 when we use the economic VSI $EWMA_{X\text{-}bar}$ chart with λ=0.05. With or without inspection, $n*$, $EWMA_{X\text{-}bar}$ chart, and ATS_1 are the same. However, for *EAP**, we increase the profit per unit time as follows:

(1) If we use the VSI $EWMA_{X\text{-}bar}$ control chart, we increase 16.3% profit per unit time when we have an inspection.

(2) If we use the FSI $EWMA_{X\text{-}bar}$ control chart, we increase 16.66% profit per unit time when we have an inspection.

(3) If we use the VSI $EWMA_{X\text{-}bar}$ chart, we increase 0.6% profit per unit time more than the FSI $EWMA_{X\text{-}bar}$ chart when the producer decides to inspect.

(4) If we use the VSI $EWMA_{X\text{-}bar}$ chart, we increase 0.91% profit per unit time more than the FSI $EWMA_{X\text{-}bar}$ chart when the producer decides not to inspect.

Therefore, we suggest that the producer takes inspection with *USL**=8.66, use the economic VSI $EWMA_{X\text{-bar}}$ chart with λ=0.05 and take 25 samples for having better performance. If the last sample point falls within the warning region, then take the next sample after 0.5 unit time; if the last sample point falls within the center region, then take the next sample after 2 unit time. We then obtain 32190 profits per unit time.

For the second numerical example, we consider a special case of exponential distribution using the same service time data as that in the end of Section 5.1. We also use the same specified parameters and choose the same λ=0.1 and 0.4.

We compare the optimum results of the VSI $EWMA_{X\text{-bar}}$ control chart with the FSI $EWMA_{X\text{-bar}}$ control chart at λ=0.4, as follows:

Table 6-2. Comparison of the optimum results of the VSI and FSI $EWMA_{X\text{-bar}}$ charts at λ=0.4

Inspection	Without		With	
Chart	FSI	VSI	FSI	VSI
L_1	3.5169	3.5169	3.5169	3.5169
L_2	2.4510	2.4510	2.4510	2.4510
W_1	-	0.5403	-	0.5403
W_2	-	0.2943	-	0.2943
ω^*	-	-	2	2
*USL**			17.297	17.297
Yield			0.950213	0.950213
*EAP**	-52723.27	-52546.4	-11560.23	**-11457.41**
ATS_1	2.55	2.778	2.55	2.778
UCL	8.972	8.972	8.972	8.972
UWL	-	6.258	-	6.258
LWL	-	5.497	-	5.497
LCL	3.531	3.531	3.531	3.531
first true alarm on which sample	No.2 (9 outliers)	No.2 (9 outliers)	No.2 (9 outliers)	No.2 (9 outliers)
first true alarm on which time	2 unit time	2.5 unit time	2 unit time	2.5 unit time

According to Table 6-2, ω* is the same as in the model with inspection, but different FSI and VSI $EWMA_{X\text{-bar}}$ charts. We have the largest *EAP**, but largest ATS_1 when we use the economic VSI $EWMA_{X\text{-bar}}$ chart with λ=0.4. With or without inspection, ATS_1 is the same. However, for *EAP**, we increase the profit per unit time as follows:

(1) If we use the VSI $EWMA_{X\text{-bar}}$ control chart with λ=0.4, we increase 78.1% profit per unit time when we have an inspection.

(2) If we use the FSI $EWMA_{X\text{-bar}}$ control chart with λ=0.4, we increase 86.1% profit per unit time when we have an inspection.

(3) If we use the VSI $EWMA_{X\text{-bar}}$ chart, we increase 0.89% profit per unit time more than the FSI $EWMA_{X\text{-bar}}$ chart when the producer decides to inspect.

(4) If we use the VSI $EWMA_{X\text{-bar}}$ chart, we increase 0.34% profit per unit time more than the FSI $EWMA_{X\text{-bar}}$ chart when the producer decides not to inspect.

Therefore, for maximum *EAP*, we suggest that the producer takes inspection with *USL**=17.297, use the economic VSI $EWMA_{X\text{-bar}}$ chart with λ=0.4 and take 10 samples for every 0.5 or 2 unit time.

To find the detection ability for the three types of FSI $EWMA_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

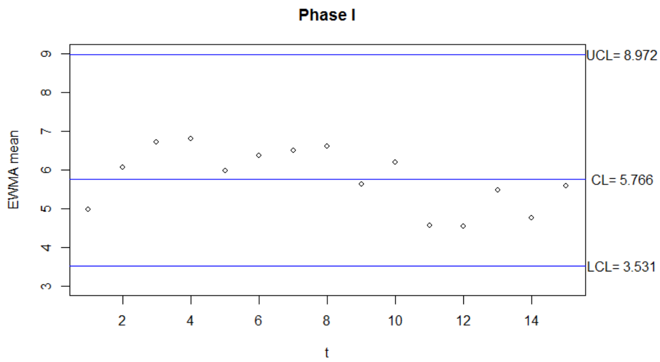

Figure 6-2. The Economic FSI $EWMA_{X\text{-bar}}$ Chart (λ=0.4) with In-control Data

For FSI $EWMA_{X\text{-bar}}$ chart with λ=0.4, Figure 6-2 shows that no points are out of limits for in-control samples.

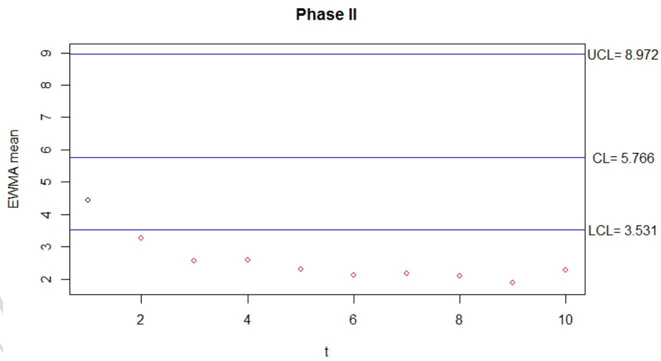

Figure 6-3. The Economic FSI $EWMA_{X\text{-bar}}$ Chart (λ=0.4) with Out-of-control Data

Plots the out-of-control statistics on the FSI $EWMA_{X\text{-bar}}$ Chart with λ=0.4, Figure 6-3 shows that No. 2 to No. 10 are out of limits; the first true alarm is on No. 2.

To find the detection ability for the three types of VSI EWMA$_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

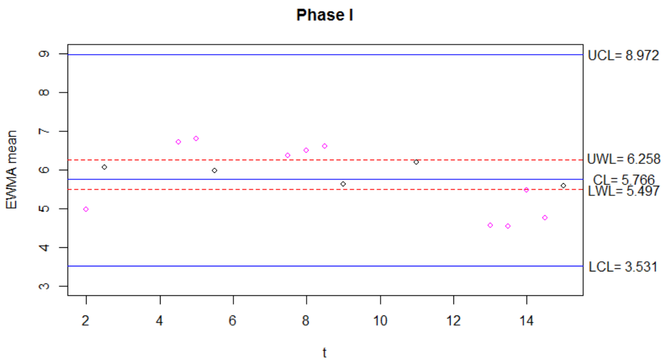

Figure 6-4. The Economic VSI EWMA$_{X\text{-bar}}$ Chart (λ=0.4) with In-control Data

According to the falling region of each in-control plotted point, we determine the sampling time as follows:

Table 6-3. Region and Sampling Time of Each In-control Statistic (λ=0.4)

No.	X-bar	EWMA	Region	h_i
1	3.81	4.98	W.R.	2
2	7.67	6.06	C.R.	0.5
3	7.68	6.71	W.R.	2
4	6.95	6.80	W.R.	0.5
5	4.73	5.97	C.R.	0.5
6	6.97	6.37	W.R.	2
7	6.71	6.51	W.R.	0.5
8	6.74	6.60	W.R.	0.5
9	4.16	5.63	C.R.	0.5
10	7.02	6.18	C.R.	2
11	2.12	4.56	W.R.	2
12	4.50	4.54	W.R.	0.5
13	6.89	5.48	W.R.	0.5
14	3.71	4.77	W.R.	0.5
15	6.83	5.59	C.R.	0.5

For VSI EWMA$_{X\text{-bar}}$ chart with λ=0.4, Figure 6-4 and Table 6-3 show that no points are out of limits for in-control samples.

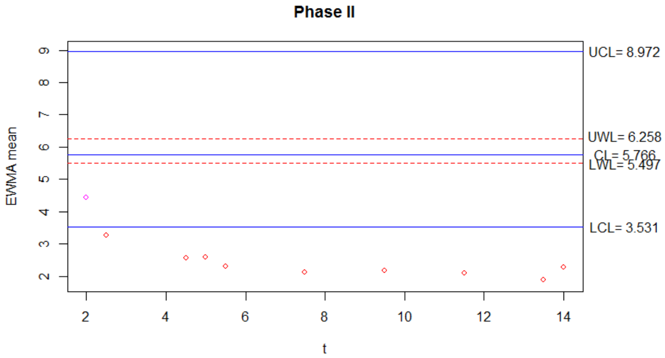

Figure 6-5. The Economic VSI $EWMA_{X\text{-bar}}$ Chart (λ=0.4) with Out-of-control Data

According to the falling region of each out-of-control plotted point, we determine the sampling time as follows:

Table 6-4. Region and Sampling Time of Each Out-of-control Statistic (λ=0.4)

No.	X-bar	EWMA	Region	h_i
1	2.42	4.43	W.R.	2
2	1.53	3.27	A.R.*	0.5
3	1.50	2.56	A.R.*	0.5
4	2.62	2.58	A.R.*	2
5	1.87	2.30	A.R.*	2
6	1.88	2.13	A.R.*	0.5
7	2.22	2.16	A.R.*	0.5
8	1.96	2.08	A.R.*	0.5
9	1.56	1.88	A.R.*	2
10	2.90	2.29	A.R.*	2

If the plotted point falls inside the action region, then we randomly choose the sampling time.

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=0.4, Figure 6-5 and Table 6-4 show that No. 2 to No. 10 out of limits; the first true alarm is on No. 2.

We compare the optimum results of the VSI $EWMA_{X\text{-bar}}$ control chart with the FSI $EWMA_{X\text{-bar}}$ control chart at λ=0.1, as follows:

Table 6-5. Comparison of the optimum results of the VSI and FSI $EWMA_{X\text{-bar}}$ charts at λ=0.1

Inspection	Without		With	
Chart	FSI	VSI	FSI	VSI
L_1	2.9015	2.9015	2.9015	2.9015
L_2	2.5052	2.5052	2.5052	2.5052
W_1	-	0.2088	-	0.2088
W_2	-	0.6048	-	0.6048
ω^*	-	-	2	2
*USL**			17.297	17.297
Yield			0.950213	0.950213
*EAP**	-51692.3	-51823.37	**-10960.91**	-11037.11
ATS_1	3.9	3.73	3.9	3.73
UCL	6.98	6.98	6.98	6.98
UWL	-	5.853	-	5.853
LWL	-	5.513	-	5.513
LCL	4.718	4.718	4.718	4.718
first true alarm on which sample	No.3 (8 outliers)	No.3 (8 outliers)	No.3 (8 outliers)	No.3 (8 outliers)
first true alarm on which time	3 unit time	3 unit time	3 unit time	3 unit time

According to Table 6-5, ω^* is the same in the model with inspection, but different FSI and VSI $EWMA_{X\text{-bar}}$ charts. We have the largest *EAP**, but largest ATS_1 when we use the economic FSI $EWMA_{X\text{-bar}}$ chart with λ=0.1. With or without inspection, ATS_1 is the same. However, for *EAP**, we increase the profit per unit time as follows:

(1) If we use the VSI $EWMA_{X\text{-bar}}$ control chart with λ=0.1, we increase 78.8% profit per unit time when we have an inspection.

(2) If we use the FSI $EWMA_{X\text{-bar}}$ control chart with λ=0.1, we increase 93.6% profit per unit time when we have an inspection.

(3) If we use the FSI EWMA$_{X\text{-bar}}$ chart, we increase 0.7% profit per unit time more than the VSI EWMA$_{X\text{-bar}}$ chart when the producer decides to inspect.

(4) If we use the FSI EWMA$_{X\text{-bar}}$ chart, we increase 0.25% profit per unit time more than the VSI EWMA$_{X\text{-bar}}$ chart when the producer decides not to inspect.

Therefore, for maximum *EAP*, we suggest that the producer takes inspection with *USL**=17.297, use the economic FSI EWMA$_{X\text{-bar}}$ chart with λ=0.1 and take 10 samples every 0.5 or 2 unit time.

To find the detection ability for the three types of FSI EWMA$_{X\text{-bar}}$ chart, we plot the in-control and out-of-control statistics on them.

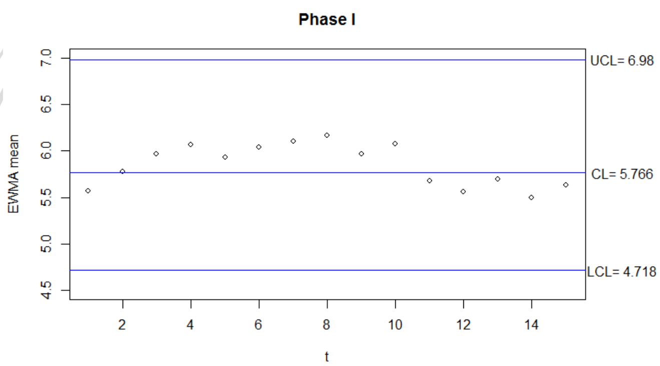

Figure 6-6. The Economic FSI EWMA$_{X\text{-bar}}$ Chart (λ=0.1) with In-control Data

For FSI EWMA$_{X\text{-bar}}$ chart with λ=0.1, Figure 6-6 shows that no points are out of limits for in-control samples.

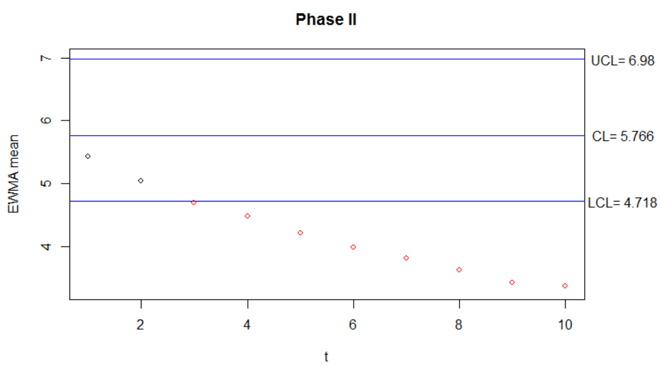

Figure 6-7. The Economic FSI EWMA$_{X\text{-bar}}$ Chart (λ=0.1) with Out-of-control Data

Plots the out-of-control statistics on the FSI EWMA$_{X\text{-bar}}$ Chart with λ=0.1, Figure 6-7 shows that No. 3 to No. 10 are out of limits; the first true alarm is on No. 3.

To find the detection ability for the three types of VSI $EWMA_{X\text{-}bar}$ chart, we plot the in-control and out-of-control statistics on them.

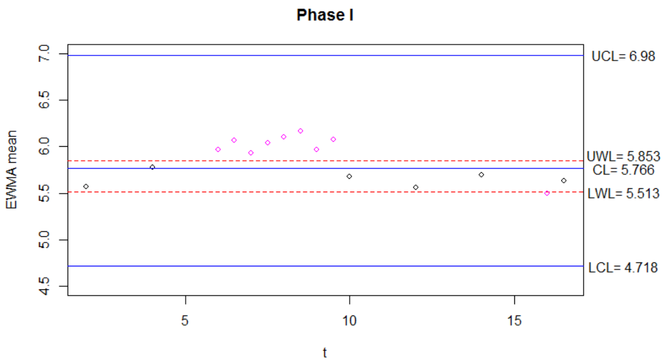

Figure 6-8. The Economic VSI $EWMA_{X\text{-}bar}$ Chart (λ=0.1) with In-control Data

According to the falling region of each in-control plotted point, we determine the sampling time as follows:

Table 6-6. Region and Sampling Time of Each In-control Statistic (λ=0.1)

No.	X-bar	EWMA	Region	h_i
1	3.81	5.57	C.R.	2
2	7.67	5.78	C.R.	2
3	7.68	5.97	W.R.	2
4	6.95	6.07	W.R.	0.5
5	4.73	5.93	W.R.	0.5
6	6.97	6.04	W.R.	0.5
7	6.71	6.10	W.R.	0.5
8	6.74	6.17	W.R.	0.5
9	4.16	5.97	W.R.	0.5
10	7.02	6.07	W.R.	0.5
11	2.12	5.68	C.R.	0.5
12	4.50	5.56	C.R.	2
13	6.89	5.69	C.R.	2
14	3.71	5.49	W.R.	2
15	6.83	5.63	C.R.	0.5

For VSI $EWMA_{X\text{-}bar}$ chart with λ=0.1, Figure 6-8 and Table 6-6 show that no points are out of limits for in-control samples.

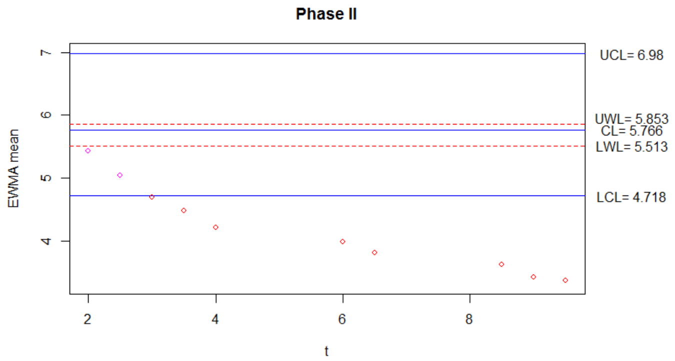

Figure 6-9. The Economic VSI $EWMA_{X\text{-bar}}$ Chart (λ=0.1) with Out-of-control Data

According to the falling region of each out-of-control plotted point, we determine the sampling time as follows:

Table 6-7. Region and Sampling Time of Each Out-of-control Statistic (λ=0.1)

No.	X-bar	EWMA	Region	h_i
1	2.42	5.43	W.R.	2
2	1.53	5.04	W.R.	0.5
3	1.50	4.69	A.R.*	0.5
4	2.62	4.48	A.R.*	2
5	1.87	4.22	A.R.*	2
6	1.88	3.98	A.R.*	2
7	2.22	3.81	A.R.*	2
8	1.96	3.62	A.R.*	0.5
9	1.56	3.42	A.R.*	0.5
10	2.90	3.37	A.R.*	0.5

If the plotted point falls inside the action region, then we randomly choose the sampling time.

Plots the out-of-control statistics on the $EWMA_{X\text{-bar}}$ Chart with λ=0.1, Figure 6-9 and Table 6-7 show that No. 3 to No. 10 out of limits; the first true alarm is on No. 3.

In the second numerical example, the *EAP** of FSI $EWMA_{X\text{-bar}}$ chart with λ=0.1 in Table 6-5 is larger than that of VSI $EWMA_{X\text{-bar}}$ chart with λ=0.4 in Table 6-2. The FSI $EWMA_{X\text{-bar}}$ chart is slightly better than the VSI $EWMA_{X\text{-bar}}$ chart when we fix h_0=1, h_1=0.5, and h_2=2.

6.6 Sensitivity Analysis and the Optimum Results Comparison between the FSI $EWMA_{X\text{-}bar}$ Chart and VSI $EWMA_{X\text{-}bar}$ Chart

For sensitivity analysis in this section, we consider only the producer decides to inspect. We use the same combinations of parameters in Table 3-3.

In Table 6-8, for improving computing efficiency of optimum solutions, we choose n equal to the optimum n of the profit model in Table 4-3. We let a_I=25, b_I =0.2, h_0=1, h_1=0.5, h_2=2, and λ=0.05 to maximize *EAP* and to determine the optimum ω^* and W_1^* at each experiment, subject to $2 \leqq \omega$ and $0 \leqq W_1 \leqq L_1$. The optimum results are solved using the procedure in Section 6.4, as follows:

Table 6-8. Optimum Results in Each Experiment

				Economic VSI $EWMA_{X\text{-}bar}$ chart					Economic FSI $EWMA_{X\text{-}bar}$ chart				
Exp.	Fix n	L_1	L_2	ω^*	*EAP**	ATS_1	*UWL**	*LWL**	ω^*	*EAP**	ARL_1	*UWL**	*LWL**
1	6	2.83	2.26	4.747	37947.78	1.03	5.002	4.939	4.747	36108.05	2.06	5.185	4.852
2	6	2.83	2.26	3.660	-1229.16	1.03	5.002	4.939	3.660	-1591.71	2.06	5.185	4.852
3	3	2.69	2.33	3.367	2069.35	3.06	5.001	4.915	3.367	530.25	6.06	5.249	4.784
4	3	2.69	2.33	3.660	3697.28	3.06	5.002	4.915	3.660	3286.46	6.06	5.249	4.784
5	15	2.63	2.38	4.487	46890.20	1.36	5.000	4.962	4.487	46624.74	2.72	5.109	4.902
6	2	2.63	2.37	3.367	-10900.88	2.34	5.003	4.897	3.367	-11715.25	4.64	5.298	4.732
7	10	2.83	2.27	4.747	41062.06	1.11	5.000	4.953	4.747	39779.87	2.21	5.143	4.885
8	2	2.63	2.37	4.487	41034.47	2.34	5.002	4.897	4.487	40491.73	4.64	5.298	4.732
9	6	2.83	2.26	3.660	4633.30	5.85	5.002	4.939	3.660	3837.41	11.46	5.185	4.852
10	18	2.44	2.55	4.487	46165.21	4.45	5.001	4.968	4.487	46098.63	5.67	5.092	4.904
11	2	2.63	2.37	3.944	340559.84	1.62	5.003	4.897	3.944	334357.06	3.24	5.298	4.732
12	2	2.63	2.37	4.487	344608.20	1.62	5.002	4.897	4.487	343148.26	3.24	5.298	4.732
13	6	2.83	2.26	2.000	169797.16	1.03	5.001	4.939	2.000	169162.21	2.06	5.185	4.852
14	3	2.69	2.33	4.487	339756.57	3.06	5.000	4.915	4.487	338207.39	6.06	5.249	4.784
15	14	2.68	2.34	2.000	150700.08	1.41	5.001	4.960	2.000	150067.93	2.82	5.115	4.900
16	10	2.83	2.27	2.000	143564.82	1.11	5.001	4.953	2.000	142870.64	2.21	5.143	4.885
17	2	2.63	2.37	2.071	166758.15	2.34	5.003	4.897	2.071	165342.69	4.64	5.298	4.732
18	19	2.36	2.66	4.487	357615.26	4.43	5.001	4.969	4.487	357398.08	5.50	5.087	4.902
19	7	2.85	2.26	3.944	367648.58	5.31	5.002	4.945	3.944	366796.14	10.45	5.172	4.863
20	20	2.28	2.83	2.071	169471.13	4.24	5.020	4.990	2.071	169274.14	5.39	5.082	4.899

According to Table 6-8, in all of the experiments, the VSI $EWMA_{X\text{-}bar}$ chart has a larger *EAP** and smaller ATS_1 than the FSI $EWMA_{X\text{-}bar}$ chart.

We can use the optimum results of the economic VSI $EWMA_{X\text{-bar}}$ chart with λ=0.05 in Table 6-8 to plot response figures (from Figure 6-10. to 6-15.) and determine the significant parameters that affects optimum value.

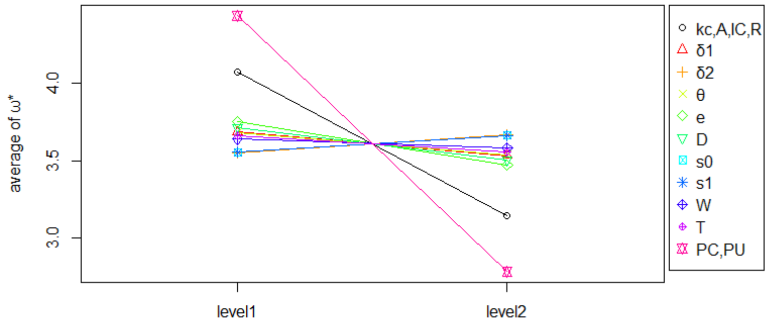

Figure 6-10. Response Figure of $\overline{\omega^*}$

According to Figure 6-10, (P_C ,P_U) and (k_c, A, IC, R) are the most significant. The larger (P_C ,P_U) or (k_c, A, IC, R), the larger ω^*. This means that the larger the selling price or cost, the larger the USL^* is.

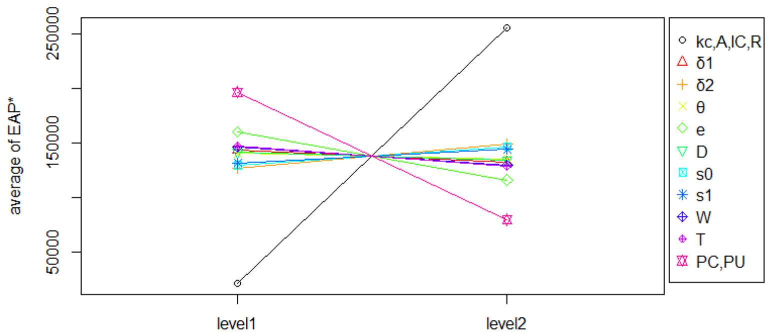

Figure 6-11. Response Figure of $\overline{EAP^*}$

According to Figure 6-11, (k_c, A, IC, R) and (P_C ,P_U) are the most significant. The smaller the (k_c, A, IC, R), the larger the EAP^*, and the larger the (P_C ,P_U), the larger the EAP^*. The smaller the cost, the larger the profit is, and the larger the selling price, the larger the profit is.

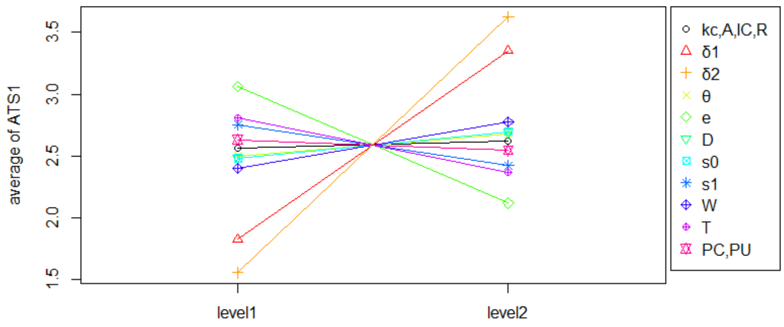

Figure 6-12. Response Figure of ATS_1

According to Figure 6-12, δ_1, δ_2, and e are the most significant. The larger δ_1 or δ_2, the smaller ATS_1 is, and also the smaller e, the smaller ATS_1 is. A larger shift results in larger power; hence, the smaller ATS_1. The smaller the e, the larger the n is; hence, the smaller the ATS_1.

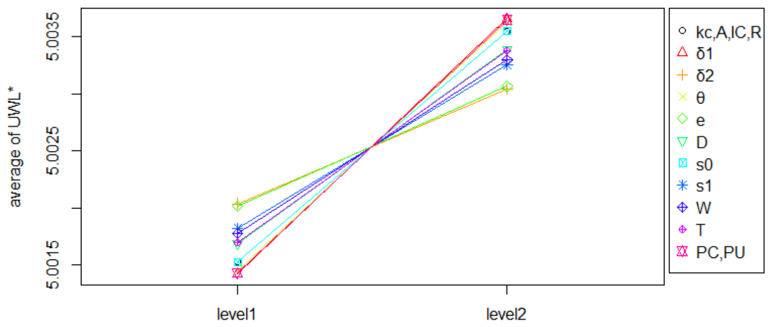

Figure 6-13. Response Figure of $\overline{UWL^*}$

According to Figure 6-13, although all of the parameters seem significant, the value of the y-axis is too small. Therefore, not all of the parameters are significant.

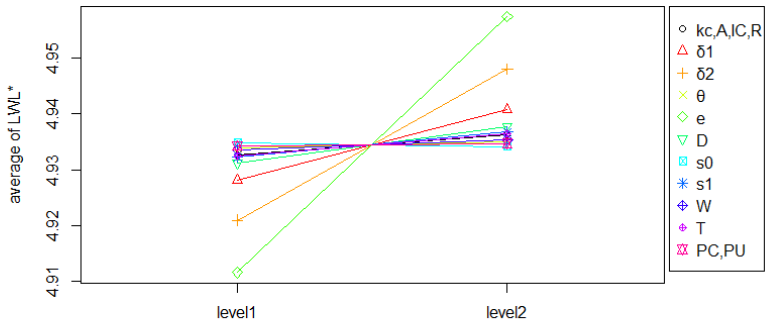

Figure 6-14. Response Figure of $\overline{LWL*}$

According to Figure 6-14, δ_2 and *e* are the most significant. The smaller the δ_2 or *e*, the larger the *LWL** is. The smaller shift in products necessitates a narrower chart to test; hence, the larger the *LWL**. The smaller the *e*, the larger the *n* is; hence, the larger the *LWL**.

We use the value of the *EAP** of the VSI EWMA$_{\text{X-bar}}$ chart minus the *EAP** of the FSI EWMA$_{\text{X-bar}}$ chart to plot the response figure and to determine the significant parameters.

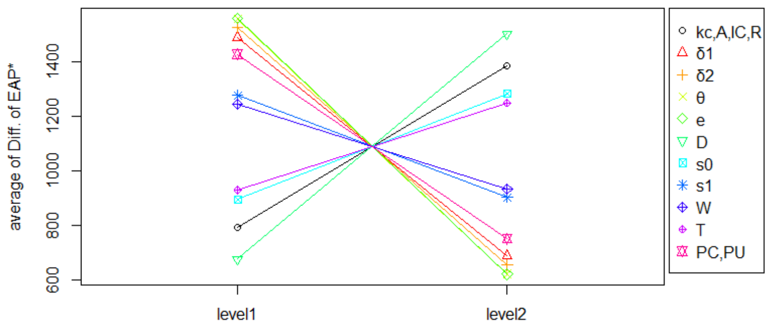

Figure 6-15. Response Figure of $\overline{\text{Difference of } EAP*}$

According to Figure 6-15, all of the parameters are significant. However, *e* and θ are the most significant. The larger the *e* or θ, the larger the difference of *EAP** is. If process has a big *e* or θ, than using the economic VSI EWMA$_{\text{X-bar}}$ chart is considerably better than using the FSI EWMA$_{\text{X-bar}}$ chart.

CHAPTER 7. SUMMARY

In this study, we simultaneously determine the upper specification limit and the design parameters of $EWMA_{X\text{-bar}}$ control chart with maximal profit.

For the smaller the better quality variable of gamma distribution, because its parameters a and b both exist in the mean and variance; hence, the $EWMA_{X\text{-bar}}$ chart detects both mean and variance. We use the $EWMA_{X\text{-bar}}$ chart to simultaneously detection process mean and variance and calculate *ARL* using the Markov chain approach to measure the performance of the $EWMA_{X\text{-bar}}$ chart. We set ARL_0=370 to compare ARL_1 with different shift scales, λ, and n. We then have the best λ, which minimizes ARL_1 at each shift scale and n. We find out that n significantly affects the value of the best λ when both the mean shift scale and the s.d. shift scale have a large value. In addition, the larger the mean shift scale or the s.d. shift scale, the larger the value of the best λ.

If the producer decides not to inspect products and the distribution parameters are known, we follow the procedure in Section 3.3 to determine the optimum parameters of the $EWMA_{X\text{-bar}}$ chart. If the producer decides to inspect products and the distribution parameters are known, then we follow the procedure in Section 4.3 to determine the upper specification limit and the optimum parameters of the $EWMA_{X\text{-bar}}$ chart. To obtain additional profit, to inspect is better than not to inspect in our example in Section 4.4. In the sensitivity analysis of Chapters 3 and 4, (P_C ,P_U) and (k_c, A, IC, R) significantly affect ω^* and EAP^*. However, δ_1, δ_2, and e significantly affect n^*, ARL_1, UCL^*, and LCL^*. We also find out that the ω^* only dependent on the values of (P_C ,P_U) and (k_c, A, IC, R). The smaller the shift in b, the better the performance of the $EWMA_{X\text{-bar}}$ chart with λ=0.05.

Most examples show that we can simultaneously obtain the largest EAP^* and the smallest ARL_1. However, in service time data, any one of three different λ could not obtain the largest EAP^* and smallest ARL_1. According to the comparison in Section 5.2, the mean shift scale affects the choice of the best λ significantly.

We also consider the VSI $EWMA_{X\text{-bar}}$ chart, and calculate *ATS* to measure the performance of the VSI $EWMA_{X\text{-bar}}$ chart. We follow the procedure in Section 6.4 to determine the upper specification limit and the optimum parameters of the $EWMA_{X\text{-bar}}$ chart. The first numerical example in Section 6.5 shows that by inspecting and using the VSI $EWMA_{X\text{-}bar}$ chart, we can obtain the greatest profits per unit time. However, in the example of service time data, the FSI $EWMA_{X\text{-}bar}$ chart is slightly better than the VSI $EWMA_{X\text{-}bar}$ chart. In the sensitivity analysis of Chapter 6, the VSI $EWMA_{X\text{-}bar}$ chart has a larger *EAP** and a smaller ATS_1 than the FSI $EWMA_{X\text{-}bar}$ chart in all of the experiments. Also, (P_C ,P_U) and (k_c, A, IC, R) significantly affect ω* and *EAP**. However, δ_1, δ_2, and e significantly affect ATS_1 and *LWL**. If the process has a large e or θ, than using the economic VSI $EWMA_{X\text{-}bar}$ chart is considerably better than using the FSI $EWMA_{X\text{-}bar}$ chart.

In the future, the study can be extend to determine the optimum h_1 and h_2 to get significant difference of EAP* between VSI chart and FSI chart. Also, the study can be extend to determine the optimum gamma distribution.

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